An Analytical Model and Investigation of Induction Motor Drive Fed from Three-Level Space-Vector Modulated VSI

JIRI KLIMA
Department of Electrical Engineering and Automation
Technical Faculty of CZU in Prague
CZECH REPUBLIC

Abstract: This paper presents an analytical model enabling us to investigate the currents and electromagnetic torque of an induction motor drive that is fed from three-level space-vector PWM voltage source inverter. The mathematical model is based on the mixed p-z approach (mixed the Laplace and Modified Z-transform).
The three-level space-vector PWM VSI using in each sampling period the method of the three nearest vectors (NTV), enabling us to improve harmonic spectrum.

Key-Words: - Induction motor, three-level inverter, space-vector modulation

1. Introduction

The three-level inverter as shown in Fig.1,significantly enhances the handling capacity of power electronics using currently available power devices like GTO´s.The three-level topology doubles the forward blocking voltage capability and reduces harmonics, comparing with a conventional two-level topology [1],[2],[3],[4],[5].
Consequently, three-level inverter topology is promising, especially for high voltage-high power traction applications.
Among various PWM techniques controlling three-level inverter, the Space Vector PWM (SVPWM) is often preferred. The reason is its simplicity both in hardware and software.
The paper presents a mathematical model that enables one to obtain a closed form analytical solution of the steady-state and electromagnetic transient processes in induction motor drive fed from three-level SVPWM inverter. The mathematical model uses the Laplace and Modified z transform (mixed description in the p and z domains.)

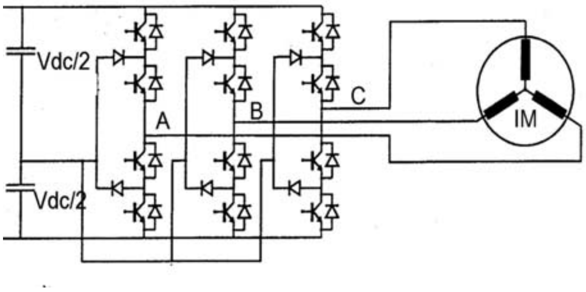

Fig-1 Three-level voltage source inverter feeding an induction motor

2. Voltage Space Vectors

Motor phase voltages V_{in}, i=a,b,c can be expressed as space vector:

$$\mathbf{V}_S(t)=2/3(V_{an}+a.V_{bn}+a^2.V_{cn}) \;,\quad a=e^{j2\pi/3} \tag{1}$$

On substituting the possible values of V_{in} into (1), the space vector of output voltage can be drawn as in the Fig.2 Space voltage vectors with their switching states form

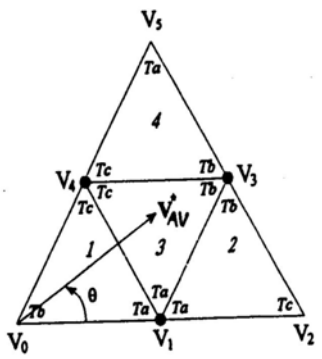

Fig.2 First sixth of the hexagonal contour of the voltage space-vector trajectory in complex plane

well known hexagon contour. Its first sixth of output period (sector) is shown in Fig.2 .In the subsequent sector the direction of the voltage vectors are rotated through $\pi/3$.It means, that SVPWM is a periodical with $\pi/3$.
In view of the discontinuous behavior of the stator voltage space vectors with six-fold symmetry, we shall express time as

$$t = (n+\varepsilon)T = (n+\varepsilon)\frac{T_1}{6} \quad (2)$$

where T is period of the modulation, which is called sector.

n is number of sector, n=0,1,2,….. $0 \le \varepsilon < 1$ (3)

With a three-level inverter, three different states can be switched on to each of the three motor terminals so that a total of 27 switching states occur. But several of the output voltage space vectors resulting from different combinations of switching states are identical and so only 19 output voltage space vectors can be distinguished, as shown in Fig.1.

The stator voltage space vector under SVM can be expressed, in the n-the sectors as:

$$\mathbf{V}_S(n,\varepsilon) = \frac{2.V_{DC}}{3}\exp(n.\frac{j\pi}{3}) \sum_{k=1}^{M}\left[\ \exp\left\{\frac{j\pi\alpha(k)}{3}\right\} f(\varepsilon,k)\beta(k)\right] \quad (4)$$

within sector n

$f(\varepsilon,k)$ takes values 1 or 0,and it is the switching function

$\alpha(k)$ defines the sequence of the phase shifts of the space vectors within sector,

V_{DC} is DC bus voltage.

$\beta(k)$ is the sequence of the amplitudes of the space vectors within sector

From (4) it can be clearly seen that a modulation strategy given for one sector (from k=1 to k=M) is a periodical with angles: n.π /3.

We can distinguish four types of voltage vectors:

a) **zero voltage vectors** $\alpha(k) = \beta(k) = 0$
   half voltage vectors $\beta(k) = 0.5, \alpha(k) = 0.or.1$

b) **intermediate voltage vectors** (5)
   $\beta(k) = \sqrt{3}/2, \alpha(k) = 0.5$

c) **full voltage vectors** $\beta(k) = 1, \alpha(k) = 0.or.1$

As can be seen from Fig.1 (first sector) a triangle formed by the vectors $\mathbf{V}_0$,$\mathbf{V}_2$ and $\mathbf{V}_5$ can be divided into four smaller regions 1,2,3,4 .SVPWM strategy with Nearest Three Vectors(NTV) selection is using the closed three vectors to command $\mathbf{V}_{AV}$ as shown for region 3(vectors $\mathbf{V}_1$,$\mathbf{V}_3$,$\mathbf{V}_4$).

The duration of each voltage vector can be calculated as for two-level inverter with SVPWM [6].

A command vector VAV is given by modulation factor

$g=\dfrac{V_{AV}}{2\sqrt{3}V_{DC}}$ and polar angle to $\mathbf{V}_1$(real axis)

$$\rho = \frac{\pi(2m-1)}{6N_1}$$

m=1,2,…N_1

N_1 is a number of sampling intervals ΔT within a sector T

$N_1=T/\Delta T$ (6)

and per unit switching times (duty ratios) are defined as follows:

$\Delta\varepsilon_A=\Delta T_A/T$, $\Delta\varepsilon_B=\Delta T_B/T$, $\Delta\varepsilon_C=\Delta T_C/T$ (7)

Since the dwell time cannot be smaller than zero, the limits of modulation factor g for this vector selection can be calculated as follow (Fig.1)

Region 1: $\Delta\varepsilon_B=0$, $g=1/(2\sin(60^o+\rho))$

Region 2: $\Delta\varepsilon_C=0$, $g=1/(2\sin(60^o-\rho))$

Region 3: $\Delta\varepsilon_A=0$, $g=1/(2\sin\rho)$ (8)

Region 4: $\Delta\varepsilon_C=0$, $g=1/(\sin(60^o+\rho))$

Now, from (8) and Fig.1 we can derive conditions for boundary of regions to which reference vector (given by g andρ) belongs:

Region 1: $0\le\rho\le60^o$,
$0\le g\le 1/(2\sin(60^o+\rho))$

Region 2: $0\le\rho\le30^o$,
$1/(2\sin(60^o-\rho)) \le g\le 1/(\sin(60^o+\rho))$

Region 3: $0^o\le\rho\le30^o$,
$1/(\sin(60^o+\rho)) \le g\le 1/(2\sin(60^o-\rho))$ (9)

$30^o\le\rho\le60^o$, $1/(2\sin(60^o+\rho)) \le g\le 1/(2\sin(\rho)$ (9)

Region 4: $30^o\le\rho\le60^o$,
$1/(2\sin(\rho) \le g\le 1/(2\sin(60^o+\rho))$

In the subsequent sector Eqv. (9) are still valid; however the direction of the individual vectors has been rotated through 60^o.

3. Analytical Results

To find the Laplace transform of (4) we can use relation between the Laplace and modified Z transform [6].
Using (3) , and its derivation

$dt=T_1.d\varepsilon$

we can write for the Laplace transform of the periodic voltage vector:

$$\mathbf{V}(p) = \sum_{n=0}^{\infty}\int_0^1 \mathbf{V}(n,\varepsilon).e^{-p(n+\varepsilon)T}.T_1.d\varepsilon = T_1.\int_0^1 \mathbf{V}(z,\varepsilon).e^{-p.T.\varepsilon}.d\varepsilon \text{ ,(10)}$$

Where $z=e^{pT}$ and

$$\mathbf{V}(z,\varepsilon) = \sum_{n=0}^{\infty} \mathbf{V}(n,\varepsilon).z^{-n} \qquad (11)$$

$\mathbf{V}(z,\varepsilon)$ is the modified Z transform of $\mathbf{V}(n,\varepsilon)$.

So, if we know $\mathbf{V}(n,\varepsilon)$, we can find from (10) the Laplace transform of the discontinuous space vectors
Using (4) and (10) we get for the Laplace transform of the stator voltage vector:

$$\mathbf{V}(p) = \frac{2.V_{dc}}{3}.\frac{1}{p}.\frac{e^{pT}}{e^{pT}-e^{j\pi/3}}.\sum_{k=1}^{M}\beta(k).e^{j\pi\alpha(k)/3}(e^{-pT\varepsilon_{kA}}-e^{-pT\varepsilon_{kB}})\text{.......(12)}$$

where $\varepsilon_{kA}.T$ and $\varepsilon_{kB}.T$ are repectively,the beginning and the end of application of k-th vector.
From equations of an induction motor we can derive for the Laplace transform of the stator and rotor currents, respectively:

$$\begin{aligned}\mathbf{I}_S(p) &= \mathbf{V}(p)\frac{A_S(p)}{B_S(p)} = \mathbf{V}(p)\frac{k_R+p-j\omega}{\sigma L_S(p-p_1)(p-p_2)}\\ \mathbf{I}_R(p) &= \mathbf{V}(p)\frac{A_R(p)}{B_R(p)} = -\mathbf{V}(p)\frac{L_m(p-j\omega)}{\sigma L_S L_R(p-p_1)(p-p_2)}\end{aligned} \qquad (13)$$

where $p_{1,2}$ ate the roots of the characteristic equation B(p)=0

$$p_{1,2} = \frac{-(k_S+k_R-j\sigma\omega)}{2\sigma} \pm \sqrt{\left(\frac{k_S+k_R-j\sigma\omega}{2\sigma}\right)^2+\frac{j\omega k_S}{\sigma}} \qquad (14)$$

where : $\sigma = 1-\frac{L^2{}_m}{L_S L_R}$, $k_S = \frac{R_S}{L_S}$, $k_R = \frac{R_R}{L_R}$

By substituting (12) into (13) we arrive at Laplace transform of the space vectors of the stator and rotor currents.

The inverse Laplace transform of (13) can not be solved directly, using the residue theorem, as in (13) are infinite numbers of poles given by

$$\exp(pT)-\exp(j\pi/3)=0 \qquad (15)$$

But we can transform (13) into modified Z-transform. If doing that, we arrive at

$$I_y(z,\varepsilon) = \frac{2V_{DC}}{3}\frac{z}{(z-\exp(j\pi/3))} Z_m\left\{\frac{A_y(p)}{pB_y(p)}\sum_{k=1}^{M}\left[\beta(k)\exp\frac{j\pi\alpha(k)}{3}\left(\exp(-pT\varepsilon_{kA})-(\exp(-pT\varepsilon_{kB})\right)\right]\right\} \qquad (16)$$

where:

y=S (stator) or R (rotor),

$Z_m\{X(p)\} = X(z,\varepsilon)$ denoting the modified Z transform operator
Solving (16) we must use the translation theorem in Z-space which holds:

$$Z_m\{\exp(-pa).F(p)\} = z^{-x}.F(z,\varepsilon-a+x) \qquad (17)$$

where parameter x is given by

$$x = \begin{cases} 1 & \text{for } 0<\varepsilon\leq a \\ 0 & \text{for } a<\varepsilon\leq 1 \end{cases} \qquad (18)$$

Using translation theorem and inverse z-transform we arrive at:

$$
i_y(n,\varepsilon)=\sum_{k=1}^{M}\left\{\begin{array}{l}\beta(k)\dfrac{2V_{DC}}{3}\exp(j\pi\alpha(k)/3)\left[\begin{array}{c}\dfrac{A(0)[\ \exp(-j\pi m_k/3)-\exp(-j\pi n_k/3)\ ]}{B(0)[\ \exp(j\pi/3)-1\ ]}\\ +\displaystyle\sum_{S=1}^{2}\dfrac{A(p_S)\exp(p_S T\varepsilon)}{p_S B'(p_S)[\ \exp(j\pi/3)-\exp(p_S T)\ ]}\\ [\ \exp((-j\pi m_k/3)+p_S T(m_k-\varepsilon_{kA}))\\ -\exp((-j\pi n_k/3)+p_S T(n_k-\varepsilon_{kB}))]\end{array}\right].\\ \exp(j\pi(n+1)/3)\end{array}\right\}
$$

$+$

$$(19)$$

$$
\sum_{k=1}^{M}\left\{\beta(k)\frac{2V_{DC}}{3}.\exp(j\pi\alpha(k)/3).\sum_{s=1}^{2}\begin{array}{l}\dfrac{A(p_s)}{p_s B'(p_s)}.\\ \dfrac{\exp(p_S T)}{[\ \exp(j\pi/3)-(\exp(p_S T)\ ]}.\\ [\ \exp(-p_S T\varepsilon_{kB})-\exp(-p_S T\varepsilon_{kA})\ ]\\ \exp[\ p_S T(n+\varepsilon)\ ]\end{array}\right\}
$$

$=\mathbf{i}_{yS}(n,\varepsilon)+\mathbf{i}_{yT}(n,\varepsilon)$

$\mathbf{i}_{yS}(n,\varepsilon)$ is the steady-state solution, $\mathbf{i}_{yT}(n,\varepsilon)$ is transient solution for stator y=S. or rotor y=R, respectively. Parameter m_k and n_k in (19) are given as follows:

$$
m_k=\begin{cases}1 & \text{for } \varepsilon<\varepsilon_{kA}\\ 0 & \text{for } \varepsilon\geq\varepsilon_{kA}\end{cases}\qquad n_k=\begin{cases}1 & \text{for } \varepsilon<\varepsilon_{kB}\\ 0 & \text{for } \varepsilon\geq\varepsilon_{kB}\end{cases}\qquad (20)
$$

The electromagnetic torque equation reads:

$$T_i=3/2p_pL_m\,\mathrm{Re}\{j\mathbf{I}^*_S.\mathbf{I}_R\}\qquad (21)$$

Where $\mathbf{I}^*_S$ is the vector complex conjugate to $\mathbf{I}_S$.

4. Time-Domain Analyses

The space vectors of stator voltage, stator current, phase stator current and electromagnetic torque

The space vectors of stator voltage, stator current, phase stator current and electromagnetic torque, each for steady-state components are presented in Fig.2

Parameters of SVPWM: f_1=50 Hz,N_1=4,g=0.8

Parameters of induction motor: R_S=0.02,R_R=0.017,X_S=X_R=3.0,X_m=2.9,slip s=0,025 (all in per units)

As modulating factor is g=0.8 we are processing through regions 2,3 and 4 inside of each sector.

Polar angle

$$\rho=\pi(2m-1)/24,\qquad m=1,2,\ldots\ldots 4\qquad (22)$$

Sample angles are :7,5° ,22,5°,37,5°,52,5°,

$\alpha(k)$ has values: 0, 0.5, 0 **in region 1**
$\beta(k)$ 0.5, √3/2, 1

$\alpha(k)$ 0, 0.5, 1 **in region 2**
$\beta(k)$ 0.5, √3/2, 0.5
$\alpha(k)$ 0, 0.5, 1 **in region 3**
$\beta(k)$ 0.5, √3/2, 0.5
$\alpha(k)$ 1, 0.5, 1 **in region 4**
$\beta(k)$ 1,√3/2, 0.5

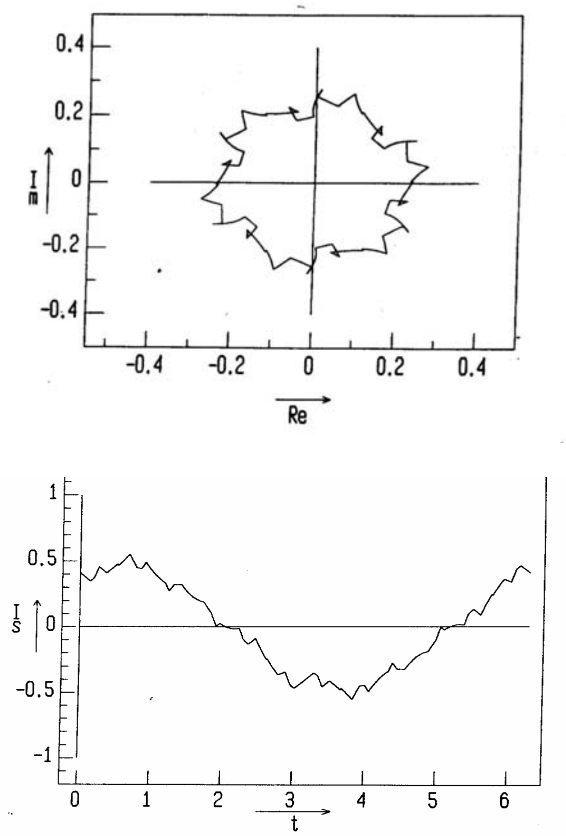

Fig.3 Space vector and phase stator steady-state current

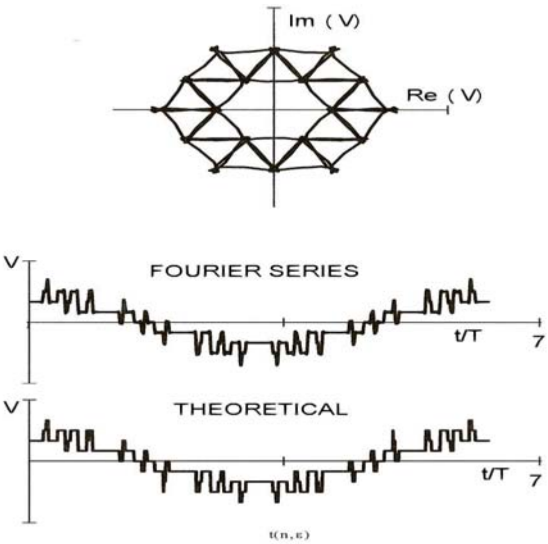

Fig.4 Space vector of the stator voltage vector and its phase waveform (theoretical and from Fourier expansion)

We can see from Fig.3, the current vector and phase current for simulated motor can be seen. The high current quality at a relative low-switching frequency can easily be seen. This dependency was calculated analytically from Eqv. (19) and (20). .As is shown steady-state current this dependency is given by the first part of (19).
Fig.4 shows the trajectory of space vector of the stator voltage in stator co-ordinate system, and also we can see both the theoretical and from the Fourier series expansion phase waveform. Fig.5 shows the stator voltage space vectors for three different values of the modulation index (g=0.3, 0.5, and 0.7)
Again, these dependencies were calculated analytically from Eva. (4).The strategy for SVPWM, (TNV) was described

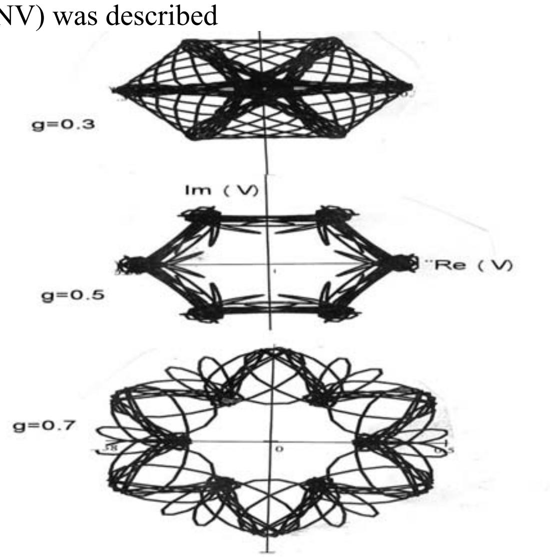

Fig.5 Voltage space vectors trajectories form different values of modulation index

5. Conclusion

The three-level inverters give good opportunities for switching frequency to be decreased.Namely,with these inverters good quality of AC drives can be achieved with high-power and high-voltage semiconductor devices.
Relations have been derived for the analytical study of three-level inverter fed induction motor drive with Space Vector PWM.The equations derived in the time domain are of a simple form. The change in the switching instants of the inverter is reflected in the solution by a change in the values m_k and n_k, respectively

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