# **Research on the Robust Estimation for Image Geometric Correction**

WEILI JIAO, XIUPENG JIA, GUOJIN HE, WEI WANG, BO CHENG China Remote Sensing Satellite Ground Station Chinese Academy of Sciences P. O. Box 2434, Beijing 100086 CHINA http://www.rsgs.ac.cn

*Abstract*: - The robust estimator algorithms are proposed to be used for automatically detecting and eliminating the GCP gross errors in the rectification process. New weights of the GCP data are computed based on equivalent weight principle and residue errors. The experiments indicate that robust estimation can be used in the gross error detection and elimination. The accuracy using this method is better than the common ones.

Key Words: - Remote sensing, Ground control points (GCPs), Gross errors, Robust estimation, Equivalent weight, Geometric correction

## **1** Introduction

Remote sensing is a kind of science and technique to collecting the data of objects from remote distance by sensor. We can get the information of the objects, the area, or the phenomena by analyzing the data. The position information of the object is a kind of important information. For the earth observation, it is to get the objects coordinates of each pixel.

Remote sensing geometric rectification is to establish the model between the surface coordinate and the images, in order to make each pixel in the image corresponding to each object in the surface. It can be classified as system geometric rectification and precision geometric rectification. System geometric rectification is to correct the system errors by the model established by the parameters of the aerocraft's sensor and ephemeris. It is a gross correction due to the errors in the parameters. The ground control points are used to make more precise rectification. The polynomial rectification method can be used to improve the position accuracy, if there are some GCPs. This model can not correct the displacement of the relief. Both GCPs and DEM are used for the rectification of the displacement caused by the viewing angle and the rough surface. This is called orthorectification.

No mater which kind of method, the polynomial method or the orthorectification method, is used to make a precise geometric correction, the GCPs are necessary for calculating the model parameters for modeling the relationship between the image and the ground coordinate systems, in order to get the more precise geometric position for the image. With the development of intelligent image processing, ground control points and tie points (TPs) can be automatically selected by the imagery matching (image to image matching, or image to map matching, or through GCP imagery chip database). So far, any kind of matching algorithms can not avoid the mismatching, which induces the gross errors of GCP or TP. Although the GCPs can be selected by people, there are gross errors. The gross errors of GCP make the rectification model unstable and unbelievable, and reduce the accuracy of the rectification image. How to eliminate the influence of gross error is a research issue.

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In this paper, based on the analyzing GCP gross errors and the normal detection methods, the robust estimator algorithms are proposed to be used for automatically detecting and eliminating the GCP gross errors in the rectification process.

## 2 The GCP Gross Errors Understanding

In order to improve the correction accuracy, the GCPs are used for calculating the model parameters. Normally there are several methods for GCPs selection. (1) Keyboard Input: find the GCP's position in the image, and input its geometric coordinates by keyboard. The geometric coordinates can be measured by GPS in the field work, or directly measured from the map. (2)Image Matching: the image to be corrected matched with the geometric corrected image, or with the raster or vector map, or with the GCP imagery chips stored in the chip database, to get the TPs imagery coordinates and their geometric coordinate.

No mater which kind of selection method, the gross errors of GCPs can not be avoided. The purpose of

the research is to automatically find the gross errors, to correctly position the errors, and to eliminate the errors from the normal observation data.

At present, the normal method to be used for measuring the GCPs accuracy is the root mean square (RMS) calculated by Least Squares Estimation (LS) model in the software of remote sensing image processing. The LS algorithm can withstand the influence of a great deal of small errors, and the estimation value is unbiased and the standard variation is least. But for the data of outliers it is very sensitive [1]. The reasons are: (1) The process of estimation can not separate from the gross error elimination. The residues are derived from the least squares. The estimation is not stable with the statistics including gross errors in the data. (2) Some gross errors can not be detected out by the LS method for some complicated data, especially in the situation of high automatic system and large volume of data. Gross errors often show themselves as outliers, but not all outliers are gross errors. Some outliers are genuine and may be the most important observations of the sample. The estimation will be worse if these data are eliminated. During the process of image correction, each GCP can be taken as the information from which the parameter estimation of correction model can be obtained. To classify the information based on the quality of GCP data, they can be classified as valid information, information able to be used and wrong information. Related to distribution of GCPs with errors, valid data means data with normal They fit the pattern set by the distribution. majority of the data. Information able to be used means the data with symmetric distribution, but unknown precision distribution model. Wrong information means data with large deviation, or anti-symmetric distribution. No mater with any estimation method. the wrong information introduces deviation value only. The principle to estimate parameter of correction model by use of GCP is to fully use valid information, restrictively use of information able to be used, exclude wrong information.

## **3** Robust Estimation and Its Application on the Image Geometric Correction

With gross errors in the observation data, the least square estimation value is apart from the true value. The robust estimation is developed by mathematicians to eliminate the gross errors in the contaminated observation data. In the situation of gross errors un-avoidable, robust estimation is to choose an optimal estimation method to limit the influence by the gross errors in the unknown variables, and to derive the optimal estimation. Robust estimation started in 19th century. But until 1950s, with the development of computer, it was studied comprehensively. The concept of robustness was proposed by G.E.P.Box. Tukev. Huber, etc. have researched in further detail. Since 1980s, Huber, Hamper and Rousseeuw etc. published influencing papers [2] [3] [4], which established the base of robust theory. So far, robust estimation is mature for one dimensional parameter in theory and practice.

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The robust theory also has been paid much attention in the field of surveying and mapping in China. Based on the summary of robust estimation theory, Zhou etc. introduced the robust least squares estimator in detail, which is used to solve the related problems in the geodetic survey [1]. In recent years, the parameter estimation problem when outliers and ill-conditioning exist simultaneously is researched. A class of new estimators, shrunken type robust estimators, is proposed by grafting the biased estimation techniques philosophy into the robust estimator, and their properties are discussed A set of self-contained theory system on [5]. robustified least squares estimator is researched based on equivalent variance-covariance [6]. With the fast development, the robust estimation has wider applications. The estimator has been improved in the application process.

#### **3.1 Robust Least Squares Estimator**

Robust least squares estimator belongs to the M-estimators category. It is the same with the classical least squares estimator except in the weight function. The weights are prior in the least squares estimator, while the weights are a function of residues in the robust least squares estimator. Robust least squares estimator is integrated with the least squares estimator by the equivalent weights [1].

The linearized error equation is V = AX - L. M-estimators can be described by minimization of

 $\rho$ -function ( $\sum_{i=1}^{n} \rho(v_i) = \min$ ), or by its derivative, a  $\varphi$ -function ( $\sum \varphi(v_i)a_i = 0$ , where  $\varphi(v_i) = \rho'(v_i)$ ,

 $a_i$  is the *i*th line in the coefficient matrix,  $v_i$  is residue of the *i*th observation).

Let 
$$\frac{\varphi(v_i)}{v_i} = w_i$$
 (1)

Then 
$$\sum w_i v_i a_i = 0$$
 (2)

Where  $W_i$ is the factor of weight. Let  $\overline{P} = \{\overline{P}_i\} = \{P_i w_i\}$ ,  $\overline{P}$  is the equivalent weight. Then equation (2) can be written as:  $A^T \overline{P} V = 0$ . Its normal equation is: (3)

 $\mathbf{A}^{\mathrm{T}}\overline{\mathbf{P}}\mathbf{A}\hat{\mathbf{X}} = \mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{L}$ 

The estimation parameter can be described as:

$$\mathbf{X} = (\mathbf{A}^{\mathrm{T}} \mathbf{P} \mathbf{A})^{-1} \mathbf{A}^{\mathrm{T}} \mathbf{P} \mathbf{L}$$
(4)

Apparently, the mainly difference between least squares estimator and robust least squares estimator is the equivalence weight. When  $w_i = 1$ , equation (3) and (4) become least squares estimator. The factor of weight  $W_i$  is the kernel variable, which is related to the residues of the observations. It is derived from many times iteration according to the iterative result of adjustment to ensure the equivalence weight suitable to the actual value, and in this way the Т

influence of outliers is controlled.

Equivalence weight controls the gross errors by the following way. For the normal observations it keeps the same weight:  $w_i = 1$ . For the un-normal but could be used data, the weight is degraded:  $w_i < 1$ . For the gross errors, the weight falls into 0:  $w_i = 0$ . Equivalence weight makes the adjustment smooth between outliers and normal data. It does not reject or accept a data directly, so it is superior to the least square estimator. The properties of the parameters of the robustified least square estimator depends on the function of  $\rho(\bullet)$  and  $\phi(\bullet)$ . How to select the equivalent weight is depends on the requirement of the robust estimator. Table 1 show some estimation methods in common use. The detailed description of these methods can be found in [1]

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Estimation method	$ ho(\ ullet\ )$ Function	Equivalent Weight $w( \cdot )$	Parameter Description
Median method	$\rho(u) =  u $	$w(u) = sign(u) / u = \frac{1}{ u }$	$u_i = v_i / MAD$ where $MAD = med_i  v_i $
Huber method	$\rho(u) = \begin{cases} u^2 / 2, &  u  \le k \\ k  u  - \frac{1}{2} k^2, &  u  > k \end{cases}$	$w(u) = \begin{cases} 1, &  u  \le k \\ k \cdot sign(u) / u = k /  u ,  u  > k \end{cases}$	$u_{i} = v_{i} / s$ $S = \begin{cases} \hat{\sigma}, &  u  \le k \\ M A D, &  u  > k \end{cases}$
Tukey method	$\rho(u) = \begin{cases} \frac{1}{6} [1 - (1 - u^2)^3],  u  \le 1\\ \frac{1}{6}, &  u  > 1 \end{cases}$	$w(u) = \begin{cases} (1-u^2)^2,  u  \le 1\\ 0,   u  > 1 \end{cases}$	$u_i = v_i / (c \cdot MAD) , \text{ where } C \text{ is}$ regression factor , $MAD = med_i  v_i $
Hampel method	$\rho(u) = \begin{cases} u^2 / 2,  u  \le a \\ a u  - \frac{1}{2}a^2, a <  u  \le b \\ ab - \frac{1}{2}a^2 + (c-b)\frac{a}{2}[1 - (\frac{c- u }{c-b})^2], b <  u  \le c \\ ab - \frac{1}{2}a^2 + (c-b)\frac{a}{2},  u  > c \end{cases}$	$w(u) = \begin{cases} 1,  u  \le a \\ a /  u , a <  u  \le b \\ \frac{a \cdot (c -  u )}{(c - b) \cdot  u }, b <  u  \le c \\ 0,  u  > c \end{cases}$	$u_i = v_i / MAD$ where $MAD = med_i  v_i $
IGG method	valid data: $ v  \le 1.5\sigma_0$ data able to be used: $1.5\sigma_0 <  v  \le 2.5\sigma_0$ wrong data: $ v  > 2.5\sigma_0$	$w(u) = \begin{cases} 1,  u  \le 1.5 \\ k /  u , 1.5 <  u  \le 2.5 \\ 0,  u  > 2.5 \end{cases}$	The normal probability distribution of the data is as: 87% in $\pm 1.5\sigma_0$ , 99% in $\pm 2.5\sigma_0$

able 1: Some Robust Estimation Methods in Common U	Use
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## 3.2 Application of Robust Least Square Estimator on the Elimination of GCP Gross **Errors in the Image Correction**

GCPs are used to calculate the parameters of the rectification model in the process of image

Due to the errors of the reference correction. image, the selection process, and so on, there are gross errors or outliers in the GCPs. The robust least square estimator can be used to eliminate these errors automatically. Fig.1 shows the flowchart of this method.





The polynomial model is used to demonstrate the process in this paper. Polynomial model can be

described as: 
$$\begin{cases} x = \sum_{i=0}^{N} \sum_{j=0}^{N-1} a_{ij} X^{i} Y^{j} \\ y = \sum_{i=0}^{N} \sum_{j=0}^{N-1} b_{ij} X^{i} Y^{j} \end{cases}$$
(5)

Where N is the order of the equation. N is usually chosen to be the 1st or the 2nd order in image processing, and more than the 3rd order is seldom chosen. The model of the 1st order can correct the image distortion of shift, rotation and scale. The model of the 2nd order can correct the distortion of partial torsion and relief. It does not improve the correction distortion with the increment of the polynomial order. Contrarily, it introduces the distortion between the pixels. So the model of more than the 3rd order is seldom chosen.

The process steps are: 1) To normalize the image coordinates and the geometric coordinates; 2) To establishing the coefficient matrix of the error equation with the GCPs; 3) To solve the coefficient by the least square method; 4) To Calculate the RMS of GCPs; 5) To solve the coefficient with the new equivalent weight; 6) To recalculate the RMS of GCPs and to compare with the result in step 4. If the variation is less than the threshold, end the iteration. Otherwise, goes to step 4 to continue the loop, until it is less than the threshold.

### **4** Experiments and Analysis

The experiment data is a scene of SPOT5 image in the south of Beijing. The main distortions are shift, rotation and scale in the image. So the 1<sup>st</sup> order of polynomial model is chosen for the correction. We selected 11 GCPs and 20 check points well distributed in the image. In order to verify the robust abilities, the gross errors are added in the GCPs. The error of -1pixel is added to the 2<sup>nd</sup> GCP in x direction, and +1pixel error is added to the 8<sup>th</sup> GCP in the y direction. To compare effects of the robust least squares estimator and the least squares estimator, the results of RMSs are showed in table 2, the equivalent weights are shown in table 3.

	LS estimation		Robust Least Squares estimation					
GCP No.	No gross errors	With gross errors	Median	Tukey	Huber	Hampel	IGG	
RMS of 11 GCPs								
1	0.16	0.27	0.17	0.17	0.19	0.18	0.17	
2	0.90	1.42	1.54	1.52	1.63	1.79	1.78	
3	0.68	0.70	0.75	0.87	0.65	0.55	0.62	
4	0.43	0.54	0.37	0.26	0.42	0.53	0.41	
5	0.60	0.50	0.52	0.66	0.39	0.31	0.33	
6	0.15	0.35	0.18	0.14	0.23	0.26	0.25	
7	0.48	0.43	0.54	0.62	0.46	0.47	0.50	
8	0.16	0.67	0.78	0.86	0.68	0.71	0.64	
9	0.10	0.26	0.11	0.05	0.21	0.13	0.22	
10	0.14	0.41	0.26	0.20	0.36	0.26	0.39	
11	0.09	0.14	0.00	0.06	0.08	0.05	0.11	
$\hat{\sigma}$	0.52	0.72	0.52	0.08	0.55	0.37	0.48	
RMS of 20 Check Points								
$\hat{\sigma}$	0.49	0.53	0.51	0.53	0.49	0.48	0.49	

Table 2: RMS from the Least Squares Estimator and the Robust Least Squares Estimator

GCP	Median		Tukey		Huber		Hampel		IGG	
No.	Px	Ру	Px	Ру	Px	Ру	Px	Ру	Px	Ру
1	0.0202	1	0.4781	0.9161	1	1	1	1	1	1
2	0.0026	0.0352	0	0	0.2538	0.5354	0	0	0	0.3747
3	0.6334	0.0404	0.9606	0	1	0.7652	1	0.6178	0.483 4	1
4	0.0116	0.1262	0	0.9287	1	1	1	0.8555	1	1
5	1	0.0611	0.9443	0	1	1	1	1	1	1
6	0.0202	0.3309	0.5265	0.9833	1	1	1	1	1	1
7	0.0121	0.0698	0	0	1	1	0.921 6	0.9897	0	1
8	0.0816	0.0404	0.9883	0	1	0.7446	1	0.3592	1	1
9	0.4292	0.1917	0.999	0.9113	1	1	1	1	1	1
10	0.2467	0.0949	0.9733	0.3466	1	1	1	1	1	1
11	0.6172	0.9671	0.9988	0.9647	1	1	1	1	1	1

Table 3: The Final Equivalent Weight Used In the Methods of the Robust Least Squares Estimator

From the experiments following knowledge can be drawn:

(1) It can be concluded from table 2 that the RMS derived from the robust least squares estimator is better than that from the classical least squares adjustment. The robust least squares estimator can decrease or eliminate the gross errors of GCP in the process, so that the accuracy is less influenced by the gross errors.

(2) It is shown from table 2 and table 3 that the robust least squares estimator reduces the RMS of none gross errors. The RMS of the gross errors are not reduced, on the contrary, they are increased. This is because the gross errors are detected through the residues by the robust estimation, and their equivalent weights are decreased. Then the GCPs with gross errors are not used or part used in the process.

(3) The digits in bold font in table 2 and 3 represent the contaminated data. Their RMSs are larger than others and the respect equivalent weights are lower down. The parameter estimation gained by the robust methods is less influenced by the gross errors.

(4) The robust estimation can automatically process the gross errors of GCPs. The main difference between robust estimation and classical error detection is that the equivalent weight is used in the robust estimation. Equivalence weight makes the adjustment smooth between gross errors and none gross errors data. It does not reject or accept a data directly, so it is superior to the classical least square estimator.

#### Acknowledgments

The research has been supported by the grant from the Knowledge Innovative Program of China RSGS (062103) and National Natural Science Foundation of China (60272032).

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