Effect of Odd Number of Rotor Slits on the Performance of a High-Speed, High-Power, Solid-Rotor Induction Motor

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Abstract: - The effect of the number of the rotor slits on the performance of a high-speed, high-power, solidrotor induction motor is numerically studied using two-dimensional finite element analysis. The theoretical background on the selection of the rotor slit number is presented. The effect of the odd number of the rotor slits on the motor torque ripple as well as on the unbalanced magnetic pull between the stator and the rotor is discussed in detail.

Key-Words: - Solid rotor, induction motor, numerical analysis, unbalanced magnetic pull

1 Introduction

Industrial high-speed applications are becoming more and more popular. For achieving a high speed, high frequency electrical machines are nowadays a preferable choice. Using a high-speed machine power electronic drive replaces a traditional standard motor and a large step-up gear. The load machinery is in such a case directly attached on the motor shaft. The losses due to the mechanical gearbox are thus avoided and the full speed control of the drive is achieved. The other advantages of using high-speed electrical machines are the reduction in size per unit output power and an improvement in the drive train efficiency. The highspeed electrical machines find their applications e.g. in high-pressure pumps, compressors as well as in small energy conversion units.

The most common types of small high-speed machines used in industrial applications are the induction motor with a thick shaft and a cage winding or a permanent magnet synchronous motor. A comprehensive analysis on these machine types in a medium-speed range is presented in [1]. In both motor types the rotor structure must be strong enough to tolerate the excessive centrifugal forces due to the high circumferential velocity. This requirement excludes the use of the traditional induction motor structure, i.e. laminated rotor core with a squirrel cage. Furthermore, the cooling of a small but powerful permanent magnet machine is very challenging. Thus, in the mechanical and thermal point of view, the most attractive solution for a large power high-speed machine is a polyphase solid-rotor induction motor. Solid-rotor induction motors are built with a rotor core made of a solid single piece of ferromagnetic material. The simplest solid-rotor is, in fact, a smooth steel cylinder. This kind of a rotor is easy to manufacture and it has the best mechanical and fluid dynamical properties. Its chemical resistance is also quite good and may be increased by different coating if necessary. The electromagnetic properties of such a rotor are, however, quite poor, as, e.g. the slip of the motor is large and the power factor low.

The performance of a solid-rotor induction motor can be boosted by using copper end-rings. According to [2], a two-pole smooth solid-rotor equipped with copper end-rings produces twice as much torque at a certain slip as the same rotor without end-rings. The second performance improvement in a solid-rotor can be achieved by slitting the cross section of the rotor in such a way that a better magnetic flux penetration into the rotor will be enabled. Slitting the rotor decreases the low frequency impedance of the rotor thus producing more torque, and increases the high-frequency surface impedance of the rotor thus decreasing the rotor eddy-current loss. The drawback of the axial slitting is that the ruggedness of the solid rotor is partly lost and at very high speeds the friction between the rotating rotor and air increases remarkably [3]. However, the slitting intensifies the cooling of the rotor, because the cooling surface of the rotor increases.

The effect of the number of the rotor slits on the torque of the solid-rotor induction motor was numerically studied in [2]. The number of the rotor slits was varied between 28 and 52 and the number

of the stator slots was 48 in all the calculations. It was shown that the output torque of the motor as a function of the number of the rotor slits had a form of a downwards-opening parabola the maximum value of which was achieved when the number of the rotor slits was between 34 and 42, sliding to a higher number when the slip was increased. However, the study was restricted to only even numbers of the rotor slits. This was mainly due to the fact that the use of odd numbers of rotor slits are banned in most of the recommendations for slit numbers given in textbooks on the construction and operation of electrical machines. The effect of an odd number of rotor slots on the motor performance characteristics in the case of a conventional four pole 15 kW induction motor was studied in [4]. It was shown that the magnitude of the unbalanced magnetic pull caused by the odd numbers of the rotor slots could be extremely large. It was also shown that an odd number of rotor slots is a good choice when the synchronous torques and rotor losses are considered.

In this paper the effect of the odd number of rotor slits on the torque ripple, rotor losses and unbalanced magnetic pull of a high-speed, high power solid-rotor induction motor are numerically studied. The results obtained from the odd numbers of the rotor slits are compared with the corresponding values obtained by using even numbers of rotor slits. In order to take the effect of the loading into account the calculations were performed at three different slip values. Time-stepping calculations in all cases are necessary when using odd numbers of rotor slits since the time-harmonic solution fails in these cases.

2 Selection of the Rotor Slit Number

In the following the selection of the rotor slit number in the case of two-pole motor with 48 stator slots is discussed. The analysis is restricted to the rotor slit numbers between 28 and 39. The upper limit is due to the saturation of the rotor teeth.

The stator and the rotor magneto-motive forces create together the air-gap magnetic flux density distribution which varies in time and space. The magnitudes of the harmonic fields affecting in the air-gap region depend highly on the ratio of the slot numbers of the stator and the rotor. In order to avoid the disturbance caused by asynchronous and synchronous torques and vibration for the operation of an induction motor, the rotor slit number Q_r has to be selected with special care. The following conditions are presented in the practical work by Richter [5]. It must be noticed that all the avoidable rotor slit numbers are basically valid when there is no rotor skewing.

To reduce the asynchronous super harmonic torque, the rotor slit number has to be as small as possible. It is generally recommended that

$$Q_{\rm r} < 1.25 Q_{\rm s}$$
, (1)

where Q_s is the number of the stator slots. Therefore, in the case of 48 stator slots the number of rotor slits should be smaller than 60.

Let us consider only machines with three-phase windings. A three-phase winding creates only harmonics of odd numbers. In order to limit the synchronous torques when the motor is at stall, the slit number of the rotor has to meet the condition

$$Q_{\rm r} \neq 6 pg, \qquad (2)$$

where p is the number of pole pairs and g may be any positive integer. It can be seen in (2) that in the studied range of rotor slits, the rotor slit numbers 30 and 36 are not recommended. However, in the speed controlled electrical drives, the supply frequency of the motor is increased gradually. Thus the situation where the rotor is at stall and the stator frequency has its nominal value can not exist.

To avoid synchronous torques created by slot super harmonics, the selection of the rotor slit number has to meet the following conditions

$$Q_{\rm r} \neq Q_{\rm s}; Q_{\rm r} \neq \frac{1}{2}Q_{\rm s}; Q_{\rm r} \neq 2Q_{\rm s}.$$
 (3)

None of the studied rotor slit numbers equals the conditions in (3).

To avoid synchronous torques during running, the following inequality has to be in effect

$$Q_{\rm r} \neq 6pg \pm 2p , \qquad (4)$$

where g is again any positive integer. The + sign is valid for positive rotation speeds and – sign is valid for negative rotation speeds. It can be seen in (4) that the rotor slit numbers 32 and 38 could create synchronous torques at positive rotation speeds and the rotor slit numbers 28 and 34 could create synchronous torques at negative rotation speeds.

To avoid dangerous slot super harmonics, the following inequalities have to be in effect

$$Q_{\rm r} \neq Q_{\rm s} \pm 2p; Q_{\rm r} \neq 2Q_{\rm s} \pm 2p;$$

$$Q_{\rm r} \neq Q_{\rm s} \pm p; Q_{\rm r} \neq \frac{Q_{\rm s}}{2} \pm p.$$
(5)

It is seen that none of the studied rotor slit numbers fulfills the conditions of (5).

In order to avoid mechanical vibrations, the following inequalities have to be in effect

$$Q_{\rm r} \neq 6pg \pm 1; Q_{\rm r} \neq 6pg \pm 2p \pm 1;$$

$$Q_{\rm r} \neq 6pg \pm 2p \mp 1.$$
(6)

As it is seen in (6) all odd rotor slit numbers covered in this study cause mechanical vibrations. This can be explained as follows. When the number of rotor slits is an even number, the large local forces between the stator teeth and the rotor teeth cancel each-other on the perimeter on the air-gap, and the net force acting on the rotor is zero. If the number of rotor slits is an odd number, the motor geometry is not symmetric enough to eliminate the forces and a net force, which is called an unbalanced magnetic pull, usually occurs. The estimation of the magnitude of the unbalanced magnetic pull due to the odd number of rotor slits is, however, very difficult, and therefore odd numbers of rotor slits are usually avoided by designers.

3 Method of Analysis

A two-dimensional, non-linear, time-stepping finite element analysis was utilized in the calculation of the performance characteristics of the solid rotor induction motor. The calculation is based on the numerical solution of the magnetic vector potential A in the two-dimensionally modeled core region of the machine. The electromagnetic field of the motor in the Cartesian plane can be described in terms of magnetic vector potential as

$$\nabla v \nabla A + \sigma \left(\frac{\partial A}{\partial t} \right) = J , \qquad (7)$$

where ν is the magnetic reluctivity, σ is the electric conductivity, *t* is time and *J* is the current density. In order to model the power supply as well as the end-winding effects of the stator coils (7) is coupled with the circuit equation

$$u = Ri + L_{\rm ew} \,\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{\mathrm{d}\psi}{\mathrm{d}t}\,,\tag{8}$$

where u and i are the voltage and the current of the winding, R is the resistance of the winding, ψ is the flux linkage associated with the two-dimensionally modelled magnetic field and L_{ew} is the end-winding inductance, representing the part of flux linkage, which is not included in ψ . The end-effects of the solid rotor are taken into account by decreasing the electrical conductivity of the rotor core material by a factor k [6]

$$k = 1 - \frac{2\tau}{\pi L} \tanh\left(\frac{\pi L}{2\tau}\right), \qquad (9)$$

where τ is the pole pitch and *L* is the length of the rotor. It must be noticed that the inductance of the rotor end is not taken into account in the 2-D modelling. Furthermore, the stator iron was modelled as non-conducting, non-linear electrical steel, and the non-linearity is modelled by a single valued magnetization curve. Thus the stator iron losses are neglected when solving the magnetic field. In order to speed up the dampening of the transient effects a magneto-dynamic solution was used as a start-up for the actual transient calculation.

The force calculation is based on the principle of virtual work [7]. In this method, the force is calculated as a partial derivative of the magnetic coenergy W_{co} with respect to the virtual displacement *s*

$$\boldsymbol{F} = \frac{\partial W_{\rm co}}{\partial \boldsymbol{s}} = \frac{\partial}{\partial \boldsymbol{s}} \int_{V}^{H} \boldsymbol{B} \mathrm{d} \boldsymbol{H} \mathrm{d} V , \qquad (10)$$

where *B* is the magnetic flux density, *H* is the magnetic field intensity and *V* is the volume. It must be noticed that in the two-dimensional case, the volume integral in (10) becomes a surface integration over the air-gap finite elements. In the case of the electromagnetic torque *T*, the virtual displacement in (10) is replaced by the virtual angular displacement θ as

$$T = \frac{\partial W_{co}}{\partial \theta} = \frac{\partial}{\partial \theta} \int_{V} \int_{0}^{H} \boldsymbol{B} d\boldsymbol{H} dV. \qquad (11)$$

4 The Motor

The motor studied was a three-phase, 9 MW, 200 Hz, two-pole, solid-rotor, high-speed, high-power induction motor. The solid rotor was axially slitted

and six different odd rotor slit numbers, i.e. 29, 31, 33, 35, 37 and 39 were analyzed. In order to compare the effect of the odd number of the rotor slits on the performance of the motor, the calculations were also performed with six different even rotor slit numbers, i.e. 28, 30, 32, 34, 36 and 38. In all calculations the shape of the rotor slit was selected to be rectangular. The width of a slit was 2.5 mm and the depth of a slit was 60 mm. The width of the slit was selected to be 2.5 mm because of practical manufacturing reasons. The rotor core material was structural steel (S355J0 / EN 10025) and the outer diameter of the rotor was 330 mm. The stator stack length was 850 mm and the number of the stator slots was 48, respectively. A two layer, 5/6-short pitch winding was used in the stator to reduce the effect of the 5th and 7th winding harmonics on the torque waveform and on the rotor surface losses. In order to get comparable results the same stator with an identical mesh was used in all the calculations. The cross-sectional geometry and the flux plot of the studied motor with 33 rotor slits is presented in Fig. 2. All FEM calculations were performed using Flux-2D[™] software package from CEDRAT Ltd.



Fig. 2. The cross-sectional geometry and the flux plot of the studied solid-rotor induction motor with 33 rotor slits at a slip of 1 per cent. Flux 2DTM software was used.

5 Results

Let us first consider the effect of the number of rotor slits on the torque ripple of the motor. The comparison is done at the nominal operation point of the motor, i.e. the slip was 1.2 per cent. The calculation results, where the electromagnetic torque of the motor with different rotor slit numbers is plotted as a function of the rotor rotation angle, are presented in Figs. 3-5. It is clearly seen in Figs. 3-5 that the torque ripple of the motor is considerably smaller when the number of the rotor slits is an odd number. The presence of the sixth torque harmonic is also clearly seen especially in Fig. 3. As it was earlier mentioned the rotor slit numbers 32 and 38 could create synchronous torques at positive rotation speeds. The effect of the synchronous torque on the electromagnetic torque waveform in the case of the 32 rotor slits is presented in Fig. 5.



Fig. 3. Electromagnetic torque of the studied solidrotor induction motor when the number of the rotor slits was an odd number.



Fig. 4. Electromagnetic torque of the studied solid rotor induction motor with the rotor slit numbers 28, 30, 34 and 36.



Fig. 5. Electromagnetic torque of the studied solid rotor induction motor with the rotor slit numbers 32 and 38.

It must also be mentioned that as seen in Figs. 3-5 the electromagnetic torque of the motor increases as the number of the rotor slits is increased.

The effect of the rotor slit number on the resistive rotor losses at the nominal operation point is presented in Fig. 6. The maximum value of the rotor loss is achieved when the number of rotor slits was 36. The rotor losses are smallest when the number of the rotor slits was 28. It must, however be noticed that the output power of the motor is also larger when the slit number is bigger.





The unbalanced magnetic pull in the case of odd number of the rotor slits at two different operation points, i.e. with the slip values of 0.5 and 1.2 per cent, are illustrated in Figs. 7 and 8. In this paper the average force is defined as the average of the absolute value of the unbalanced magnetic pull.



Fig. 7. Unbalanced magnetic pull due to the odd number of the rotor slits. The slip of the motor was 0.5 per cent.



Fig. 8. Unbalanced magnetic pull due to the odd number of the rotor slits. The slip of the motor was 1.2 per cent.

As it is seen in Figs. 7 and 8, the unbalanced magnetic pull between the stator and the rotor is affected by the loading of the motor. The maximum value of the unbalanced magnetic pull, i.e. 1107 N, is obtained at the nominal operation point when the number of the rotor slits was 31. However, with the rotor slit numbers 29 and 35, the maximum value of unbalanced magnetic pull was only 400 N. It must be mentioned that the weight of the analyzed rotor was 5500 N.

The behaviour of the radial force vector acting on the rotor with 31 rotor slits at the nominal operation point is shown in Fig. 9.



Fig. 9. The trace of the radial force vector acting on the rotor when the number of the rotor slits was 31. The trace contains 94 cycles. The slip of the motor was 1.2 per cent.

As it is seen in Fig. 9, the force vector rotates at a high frequency. This is mainly due to the presence of high order force harmonics. As an example the frequency of the force vector presented in Fig. 9 is approximately 18.5 kHz.

6 Conclusions

The effect of the number of the rotor slits on the performance of a high-speed, high-power, solid-rotor induction motor was numerically studied. It was shown, that the torque ripple of the motor is considerably smaller when the number of the rotor slits was selected to be an odd number. The unbalanced magnetic pull due to the odd number of the rotor slits was also studied. It was shown that the amplitude of the unbalanced magnetic pull depends on the loading of the motor. The maximum value of the unbalanced magnetic pull was achieved at the nominal operation point when the number of the rotor slits was 31.

References:

- Pyrhönen, J., Huppunen, J., Nerg, J., Niemelä, M., Kurronen, P., Medium-speed induction machine permanent magnet machine comparison, *Proc. 15th Int. Conf. Electrical Machines* (*ICEM*), Brügge, Belgium, 2002.
- [2] Huppunen, J., High-speed solid-rotor induction machine – Electromagnetic calculation and design, Acta Universitatis Lappeenrantaensis, no. 197, Diss, LUT, Lappeenranta, Finland, 2004.
- [3] Larjola, J., Sallinen, P., Lindgren, O., Esa, H., Pyrhönen, J., Lattu, J., Falck, R., Basic Research in High-Speed Technology, Research report EN B-71, LUT, 1991 (in Finnish).
- [4] Arkkio, A., Unbalanced Magnetic Pull in a Four-Pole Induction Motor with an Odd Number of Rotor Slots, *Proc. 2nd Chinese Int. Conf. Electrical Machines (CICEM)*, Hanzhou, China, 1995, pp. 343-348.
- [5] Richter, R., Electrische Machinen, Band IV. 2. Auflage. Birkhäuser Verlag, p. 440, Basel, 1954.
- [6] Russel, R. L., Norsworthy, K. H., Eddy current and wall losses in screened induction motors, *Proc. IEE*, Vol. 105A, No. 20, 1958, pp. 163-173.
- [7] Coulomb, J. L., A Methodology for the Determination of Global Quantities from a Finite Element Analysis and Its Applications to the Evaluation of Magnetic Forces, Torques and Stiffnesses, *IEEE Transactions on Magnetics MAG-19*, No. 6, pp. 2514 – 2519, 1983.