# Optimization in Short-Term Operation of Hydro Power Systems

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**ABSTRACT:** The paper describes the use of an advanced optimization tool for short term scheduling of hydro power systems. The tool takes into account mixed-integer formulation and hence, start-up costs for the start-up of units can be accounted for. The goal has been to quantify the improvements in cost reduction by using such a tool, and to see how this will influence upon the daily production planning. The analyses are made from a post-spot-view, meaning that the spot market has been cleared and the selling and buying volumes for the next day are given. The problem formulation will then be to cover the load obligations for the next day, given a cost minimizing objective, all relevant constraints taken into account.

The analyses show a potential for significant cost reduction. This cost reduction is mainly due to improved total efficiency, and a more optimal dispatch and a reduction in start and stop costs.

*Keywords:* Short-term hydro scheduling, start/stop costs, optimal use of resources, successive linear programming

# **1 INTRODUCTION**

The New Norwegian Energy Act came into force on January 1, 1991. This is the legal basis to open all networks and create competition in generation and retail supply aiming at a more flexible and efficient electricity supply industry. The Norwegian Energy Act requires a separation between competitive and monopoly activities [1].

This marked liberalization has increased the focus on profit and revenues. Each producer is making their own plans based on commercial terms, and are making their bids into the market depending on their expected prices in the short and long term. The power producer has in principle no obligation to serve any particular consumer. The objective is to generate and sell electricity with maximum profits. To obtain this, the total efficiency of the production system will be of major significance. But also, a direct connection between the forecast of the marked prices and the marginal cost of production will be important to make sure the resource is used to maximize the income. In such a scenario, the modeling of start-up cost of units will be important to obtain a good balance between cost due to reduced efficiency, and cost due to start and stop of a unit.

The generation planning process for a hydro power producer is a very complex task. The resource is limited over the year, and typically, the inflow comes in times when the consumption is lowest. In the long term the inflow can be described as a stochastic variable, while in the short term detailed scheduling issues including start and stop of units must be considered. In addition to this, the price varies not only over the year, but also over the day and between day and night. Due to the complexity of the planning process ranging from short term to long term considerations, it is common to divide the overall task into three different stages; long term generation planning with a time horizon up to five years, medium term generation planning with a time horizon 3-18 months depending on the time of the year, and the short term generation planning which will be to schedule the generation for the next 24 hours to 2 weeks ahead. All these stages are connected such that the short term scheduling gets its boundary conditions from the medium term planning, which again gets its boundary conditions from the long term planning.

The variables that connect all these stages are the water-values. The water-values, which might be thought of as the fuel price for the different reservoirs, represent in the long term the expectation for the future prices. In the economic equilibrium, the marginal cost is equal to the marginal demand, and thus, the water-values can be used as the marginal cost for the hydro power producer. Since the resource is limited over the year, the water-values give guiding signals for when to use the water, by using the water-values as the marginal cost.

This paper presents the results of a study using advanced computer programs for optimizing the short term operation of complex hydro power generation systems that is the detailed planning for the next 24 hours ahead to 2 weeks. Different modelling and implementation issues are discussed, and experiences and benefits from practical use are presented.

# **2** OPTIMIZATION IN AN OPEN MARKET

Statkraft is the largest power producer in Norway, with a total installed capacity of about 8700 MW in more than 55 power plants. Except for 40 MW of wind power, all of the installed capacity is hydro power. Today, the short term generation planning is done manually in two steps; a central production distribution by the dispatch centre, and a local production distribution within each of the four regions. The dispatch centre makes a distribution between the hydro power systems, while the regions make a distribution between the aggregates within their region. Both of these distributions are made manually according to subjective considerations based on results from the long term generation planning. The basis for these considerations is the water-values and the efficiency curves.

It is expected that there exists a potential for economic profit by implementing an optimization tool for the short term generation planning. The reason for this is the complexity represented by the short term optimization problem, and how this complexity increases by the number of plants and generators. There are many variables influencing the total efficiency of the production, and a marginal improvement of the efficiency can make a rather large cost reduction.

Use of discrete variables in short term hydropower scheduling has been investigated for some time. But only recently the use of discrete variables has become an alternative to heuristics and other methods like Dynamic Programming and Lagrange Relaxation. There are several reasons to include the unit commitment in the hydro power scheduling:

- It is a trend for increasing variation in spot price for electric power in the Nordic market. The power producers need tools for scheduling that account for the costs of starting and stopping units. The need arise when producers want to optimize income and to obtain coverage for costs from granted system services.
- A program used in a close to real-time environment needs to take into account all modeling options the user feel relevant, unless the prepared plans will be of minor value.

Short-term scheduling of hydropower is a challenging task in cascaded reservoir systems. The scheduling must interface with the boundary conditions received from the mid-term scheduling, fulfill discharge constraints within each time interval as well as maintain couplings as ramping constraints and reservoir balances between the intervals. The output of the scheduling problem should be a proper unit commitment sequence together with a power dispatch. This results in a large-scale optimization problem with a mix of discrete and continuous decision variables. Appropriate system models and solution strategies are important to be able to solve such a problem. This paper describes our modeling approach, the implementation and discusses the performance on a hydro system.

The program used is based on linear programming and branch and bound optimization methods where discrete variables are taken into account. It has linear programming as kernel, where the nonlinearity's in the hydro system are taken into account by successive linearizations and by multiple iterations with model refinement. This opens for the possibility to explicitly account for start-up cost of the units.

Plans suggested by the program have been compared with the equivalent plans made manually for the same scenarios, with the same assumptions. Three hydro power systems have been used, having a total installed capacity of 3330 MW divided on 9 power plants, and 20 generators.

# **3** SYSTEM MODELING

# 3.1 System topology

In a cascaded reservoir system, there is a need to model all the individual reservoirs as well as the different discharge paths between the reservoirs. Such discharge elements may be power production plants, bypass gates and spill paths. This is illustrated in Figure 1.



## Figure 1: Example topology

Each reservoir will normally have several discharge options and each path may have its separate downstream destination. Several plants can be defined for each upstream reservoir. In Figure 1 the configurations denoted A and B complicates the solution process. In the tunnel between the two reservoirs (A), the flow is highly a function of the reservoir levels in the two reservoirs. How the reservoir level develops over the study period is unknown when the model in built so this is source to strong non-linearity. In the configuration denoted B, the same plant discharges water from two reservoirs and the distribution between them depends on the plant discharge and the difference in reservoir levels. This is a source to strong nonlinearity.

## 3.2 Reservoirs

The reservoirs are the main connecting nodes of the watercourse. All plants and discharge gates must be associated with a reservoir. For each reservoir and time increment of the study period, a reservoir balance equations as given below is set up:

$$-X_{i}(t-1) + X_{i}(t) - \sum_{j=1}^{n_{u}} q_{j}^{u}(t-\tau_{j}) + \sum_{j=1}^{n_{d}} q_{j}^{d}(t) = 0 \quad (1)$$

where:

- X<sub>i</sub>(t-1) Reservoir content of reservoir 'i' in the end of time interval 't-1'. (Start volume of 't')
- X<sub>i</sub>(t) Reservoir content in the end of interval 't'
- $q_j^u(t-\tau_j)$  Inflow from upstream sources in time interval 't- $\tau_j$ ' where ' $\tau_j$ ' is the time delay. These sources can be regular inflows, discharge through plants or through gates.
- Q<sub>j</sub><sup>d</sup>(t) Discharge from the reservoir. This can include a hydropower production plant, and/or a number of bypass gates and a reservoir overflow.
  - N<sub>d</sub> Number of downstream elements
  - n<sub>u</sub> Number of upstream elements

## Reservoir endpoint description

All reservoirs must be given some guidelines about feasible range of reservoir levels at end of the optimization period. Such a description is necessary since the optimization is based on a multi-stage formulation where the use of the available resources over the study period will be optimized.

These options for defining the endpoint description can be given:

- Specified reservoir volume in the end of the study period
- Range of feasible reservoir volumes
- An incremental cost of water as a function of the endpoint reservoir volume (water value)

In the current implementation we have used the water values for each reservoir representing the fuel cost of the water. Hence the decision variable is the fuel cost represented by the water-value. What we achieve by using water-values as the decision variable in the short term optimization is a close and direct connection between the planning periods to make sure of a direct connection between the forecast of the price and the marginal cost. The boundary conditions obtained from the medium term generation planning are taken explicitly into the short term optimization.

# 3.3 Power Plants

A hydro power plant is a main element in the formulation of the optimization problem. The reason for using the plant as a main element rather than the individual units is the hydraulic coupling within the plant. The production on each unit can influence on each other due to common tunnels/penstocks or because of the tailrace effects.



Outlet line/Downstream reservoir level

Figure 2: Possible plant topology

The figure above illustrates the internal plant configuration.

## 3.3.1 Waterways

The plant is always connected upstream to one or two reservoirs. The waterways description consists of the main tunnel and/or a number of pressure shafts and is used to calculate the losses from the intake reservoir to the turbines.

There can be several pressure shafts with individual loss factors. Units, like in figure 2, can share common penstocks or be connected to individual penstocks.

This detailed description of the waterways enables an accurate head loss calculation.

Figure 2 also illustrates the problem of including explicit modeling of start/stop in hydropower optimization. The decisions about which unit to run become more difficult when the units influence on each other, as is the case here. The unit 2 would normally not be used together with unit 1 if unit 3 or unit 4 were available because of the quadratic loss in the penstock tunnel.

In this implementation, these aspects are solved in a two-step approach. The description is simplified in the initial iterations when the unit commitments are made. The simplification is to assume individual tunnels and penstock for all units. After the unit commitment is made, the accurate modeling of the loss can be included.

# 3.3.2 Units

Each unit has it own set of efficiency curves for the turbine so that the connection between head and efficiency can be modeled for each individual unit. In this way it is possible to obtain an accurate modeling of the characteristic diagram for the hydropower turbine. Individual generator efficiency curves can also be applied. Minimum and maximum of the turbine can either be fixed values, or be functions of the plant head.

# 3.3.3 Head

Calculation of head includes the up- as well as downstream reservoir levels. Especially for run of river systems, the tailrace is modeled. This means that rising of the backwater can be accounted for at high water flows. Plant head optimization is important when the effective plant head is low and the variation of the reservoir level is high. This program uses a local approach to plant head optimization.

$$P = k Q H \tag{2}$$

$$\Delta P = k Q_0 \Delta H + k H_0 \Delta Q \tag{3}$$

$$\Delta H = \Delta H_1 - \Delta H_2 = \Delta V_1 / A_1 - \Delta V_2 / A_2 \qquad (4)$$

When the production-discharge equation is linearized, the contribution from a perturbation in plant head is included. Based on the discharge from the previous iteration, a correction term is included in the load balance equation. This method gives improved reservoir level profile on small reservoirs and improves the convergence since a more active search for the best solution is used.

# 3.3.4 Constraints

In hydro power scheduling it must be possible to define different types of constraints. Examples are:

- Schedules on discharges, power production plants and units
- Time dependent limits on reservoir levels

- Minimum and maximum power production on units and plants.
- Maximum rate of change on production discharges and reservoir levels

The complexity of including these constraints depends on the type. Schedules and simple limits on variables are uncomplicated to include.

Special schemes for processing ramping constraints have been developed. This is discussed in the section of solving strategy. A major challenge with many and possibly conflicting constraints is the possibility of non-existing solution. Special care is needed in the design to identify the source of infeasibility.

## 3.3.5 Modeling with discrete variables

Using a discrete model for the plant is optional. This make it is possible to model some hydropower plants with continuous and some with discrete variables in the same case. The use of discrete variables is most helpful in the case of strong hydraulic connections in the watercourse. Hydropower plants connected to reservoir with some storage capacity can be handled by other means. This makes it possible to model realistic systems using discrete variables only on the critical hydropower plants. Another consequence of this flexibility is the possibility to use iteration logic where discrete variables are added for one and one plant in each iteration.

The following equations for the hydropower plants are used in the model. Similar modeling is used for starting and stopping pumps. The equations are valid for the one unit case.

First the startup costs are subtracted from the objective function by adding the following term:

$$\sum_{t} c_{SU} s u_t \tag{5}$$

Calculate the production on the plant:

$$\sum_{j} \alpha_{j,t} q_{j,t} + \delta_t \gamma_t - p_t = 0 \tag{6}$$

Ensure  $\delta = 1$  when p > 0:

$$p_t - m_t \cdot \delta_t \le 0$$
, and  $m \ge p_{\max}$  (7)

Ensure 
$$p \ge p_{\min}$$
, when  $\delta = 1$ :  
 $p_t - p_{\min} \cdot \delta_t \ge 0$  (8)

Calculate start variable su

$$\delta_t - \delta_{t-1} - su_t \le 0 \tag{9}$$

Constraints on discharge per segment

$$0 \le q_{j,t} \le q_{\max j,t} \tag{10}$$

Definition of  $\delta$ :

$$\delta \in \{0,1\} \tag{11}$$

su start up variable

C<sub>su</sub> is the start up cost j is the segment index

t is the time index

 $\alpha_{i}$  is the incline of the segment

 $\delta_{t}$  is the plant running indicator:

 $\delta_{t} = 0$ , plant is standing

 $\delta_{t} = 1$ , plant is running

 $\gamma_{\rm t}$  is the constant term on the PQ curve

p<sub>t</sub> is sum production

p<sub>min</sub> is minimum production

 $p_{max}$  is maximum production

q<sub>j,t</sub> is discharge per segment

 $q_{\text{max},j}$  is maximum discharge per segment

# 3.4 Gates/Tunnels

Gates can be defined connecting any two reservoirs in the watercourse. The downstream destination for the gate does not have to be the same as the downstream destination for the plants connected to the same reservoir. A special feature for gates is implemented to represent a tunnel between two reservoirs. The flow in this tunnel may depend on both reservoir levels as indicated in the equations below.

$$Q = f(H_1, H_2, \alpha)$$
(12)  

$$\Delta Q = f'(\Delta H_1 - \Delta H_2)$$
(13)  

$$\Delta H = \Delta H_1 - \Delta H_2 = \Delta V_1 / A_1 - \Delta V_2 / A_2 (14)$$

The reservoir volumes are explicitly represented in the optimization problem. To get an appropriate approximation to the flow over the study period, the constraints must be expressed as a function of the reservoir volumes. The relation between the change in flow for an incremental change in the reservoir volumes is achieved by combining the equations (13) and (14). The non-linear relation of flow between the reservoirs as a function of difference in reservoir level as is defined as a number of tables where each table is referred to a reference level of a reservoir.

## 3.5 Spinning Reserves

Spinning reserves can be defined for each time interval 't'. The principal formulation is given by:

$$\sum_{i} (P_{\max,i,t} - P_{i,t}) \ge R_t$$
(15)

'i' All online units

We have chosen to let the maximum power production be the upper production limit in the equation (15). The explicit processing of start/stop is important for the spinning reserve modeling. Without an explicit representation of on-line units, it is very difficult to model the reserves without using heuristics.

# 4 SOLVING STRATEGY

The optimization approach is based on successive linear programming. A branch and bound technique is used for handling integer variables. The commercial optimization package CPLEX from the company ILOG CPLEX Optimization Inc. is used as a kernel in the calculations.

The overall solution consists of iterations (main iterations) using one of two modeling modes. The goal for the first mode (full description) is to find an initial solution that can act as a linearization point for a detailed description. In the second mode (incremental description) a more detailed modeling is used line arising around the efficiency curves for the committed units.

# 4.2 Main iterations

The main iterations make it possible to use the results from the last iteration to perform a refinement in modeling from iteration to iteration. The main motivation for using an iterative approach is that some of the nonlinearities and constraints will depend on the power production/discharge decisions. These decisions are unknown and cannot be taken into account in the first main iteration. An optimization model of the system is built for the entire study period based on the available information. This model couples all time intervals and takes into account the endpoint reservoir constraints. The solution of this problem is the optimal decisions based on the current approximation of the system description.

The sequence of main iterations refines the system description based on the discharge profiles and the reservoir trajectories from the previous iteration. The refinement involves new linearized descriptions of:

- The unit efficiency curves
- The reservoir level volume relation
- The gate discharges

Major nonlinearities are represented by more than one segment in the linearization process.

The constraint set is also updated in the outer loop. This involves constraints as ramping of reservoir volumes and releases.

After each main iteration that consists of the model building and an optimization, a feasible and close to optimal solution will be available unless there are any constraints that are excluded due to the strategy of adding the constraints.

Each main iteration can either be based on a full or an incremental description.

#### 4.3 Full description

In this mode the optimal decisions for production level, gate discharge are computed taking head optimization and constraints into account. Normally two main full iterations are used before it is changed to the incremental. The second full iteration is needed for the reservoir level/plant head optimization. When solving the full description the entire valid production area of the hydropower plants is used. As linear programming require convexity in the modeling the whole working area of the plant or unit cannot be modeled in detail. Losses in the tunnels and penstocks are accounted for in a simplified way. In the full description the discrete nature of the start/stop is accounted for by introducing binary variables for the unit commitment. This model needs to be solved by a Branch and Bound technique.

# 4.4 Incremental description

After the unit commitment is made, an incremental formulation is used. Based on the online units and the actual discharge, incremental formulations are established around the current operating point. In these incremental descriptions, the accurate loss models are included. This is possible because the committed units and the tentative production levels already are determined. The number of incremental iterations is normally two to three to obtain satisfactory convergence.

# **5** APPLICATIONS

The program can in principle be used in two major modes:

- Decision support for bidding in a spot market. The challenge is then to find appropriate amount of power production in the plants for forecasted market price profiles.
- To decide an optimal fulfillment of a given load obligation and to adapt changed conditions. This is normally the case when the program is used in on-line environments.

The flexibility to combine schedules, load obligations and market prices within the same optimization model makes it possible to use the program within a wide range of short-term scheduling activities.

# 6 TEST CASE MARKET BIDDING

In order to demonstrate the effect from including start-stop costs in the market bidding, the program was used to optimize the hydro production in Figure 1 versus a 24 hour price profile from the Nordic electricity market as given in Figure 3. Figure 3: Price profile from the Nordic market.

There are two high price periods during the day. The last peak has shorter duration and makes the focus on start/stop costs important. The highest price is 193 NOK/MWh (hour 9) and the lowest is 163 NOK/MWh (during the night).

Initially an optimization was performed where costs associated with start/stop was ignored. Each reservoir was given an individual resource cost description (water value). Using water values adds flexibility to the optimization and the discharge from each reservoir will be a function of price level and profile.

In order to compare the importance of including the start-stop cost, two different optimizations were performed:

- The first optimization: start-stop costs were *not* included. Optimal bidding of the plants was calculated by using the tool.
- Second optimization: start-stop costs (NOK 3000) were included in the optimal bidding

The results for power plants 6 and 7 for the two optimizations are shown in Figure 4 and 5 respectively.

For power plant 6, the first optimization resulted in a production profile where it was in operation from hour 8 to 13 and for a single hour in the afternoon when the price is high. Ignoring the costs involved with start/stop, short periods of operation for marginal changes in market price can occur.

When start-stop costs were included, the operation plan changed by removing the startup at the last price spike. The revenue from the market is not compensating the start-up cost.

Figure 4: Production plan power plant 6.





Figure 5: Production plan plant 7.

The results for power plant 7 are given in Figure 5. A significant change is observed with start costs included. Now, the plant also runs from 3 pm to 5 pm. The margin between the water value and the market price is making this profitable. For the periods with high prices, all the units run at maximum output. For the other periods, the plants run according to its marginal cost. The marginal cost is defined by the incremental unit efficiency and the water values. This plant has four units. There are two units on each penstock. In some cases when the difference between water value and price is low, the unit head loss may make it profitable to only run one unit on each penstock. In this case though, all four units were used in the same time intervals.

# 7 TEST CASE LOAD COVERING

In this test case we have changed from a marked bidding issue to a load covering case. The load profile is specified for a 24 hour period as illustrated in Figure 6. The hydro system performance is optimized in order to cover the load at minimum







Figure 6: Load profile for 24 hour



As we can see from Figure 7, The number of units started and stopped are following a nice sequencedue to the given load profile in Figure 6.

# 8 LARGE SCALE STUDY

In order to demonstrate the benefits from using such a tool on large scale systems, it was applied to three different hydro systems located in three different water courses. The optimization period is now increased to 168 hours. Each hydro system comprises of multiple reservoirs and power plants as illustrated in Figure 1, but the structure and complexity varies from one system to the next.

# System 1:

The hydro power system has five reservoirs in parallel with a total reservoir capacity of 1536 GWh. The mean annual inflow to the five reservoirs is 2978 GWh. The generation is performed in two power plants with a total installed capacity of 1120 MW. The hydro system is a pure parallel system with no serial coupling, meaning that the water from plant 1 can not be utilized in plant 2 and vice versa.

## System 2:

This hydro power system comprises of the nine reservoir system in Figure 1 with a total reservoir capacity of 8371 GWh. The mean annual inflow is 5032 GWh. The generation system comprises of five power plants with a total installed capacity of 2055 MW. The system is a combined parallel and series system as illustrated in Figure 1.

#### System 3:

The system comprises of ten reservoirs with a total reservoir capacity of 412 GWh. The mean annual inflow is 897 GWh. The power is generated by three power plants with a total installed capacity of 136 MW. This is a pure series system, meaning that the water from plant 1 can be utilized in plant 2, and water from plant 2 can be utilized in plant 3.

The load profiles for the different systems are given in Figure 8. The total load for all three systems, which is found by adding the three local system loads, is given in Figure 9. The case represents a high load level with a large volatility from peak to off-peak hours.



Figure 8: Local System loads in MW as function of hours.



Figure 9: Total load all three systems in MW as function of hours.

In order to find the economic benefit, the optimization was carried out in three steps:

- First step: Optimal schedules were found by 'manual' procedures by covering the local loads without using the tool. We can name this as manual local scheduling.
- Second step: each system was optimized separately with respect to the local system load by using the optimization tool. We can name this as local optimization.
- Third step: all three systems were optimized simultaneously as one joint large system covering the total load in Figure 9. We can name this as total optimization.

The economic results were calculated for all three steps, and the benefits from using the optimization

tool was found by using the economic criteria from step 1 as the reference. The benefit for step 2 compared to step 1 is found as the difference between the respective cost functions, while the benefit between step 3 and step 1 is found as the difference between the cost functions of step 1 and step 3 respectively. The benefit using the tool for optimizing three small scale systems compared to optimizing one large scale system was also found as the cost difference between the step 2 and step 3 optimization.

The economic benefits are shown in Figure 10.





From Figure 10, it appears that the local optimization gives a production plan with about \$104 000 in cost reduction, while the total optimization gives a production plan with about \$256 000 in cost reduction - corresponding to 2,2% and 5,6% of the total cost respectively. It clearly demonstrates that the largest benefit is found when optimizing the three systems as a whole for covering the lumped total load. We can also see that when performing the total optimization, there has been a shift in generation from system 1 to system 2 and 3 (seen as reduced operation cost in system 1 and increased generation cost in system 2 and 3) due to an improved distribution in relation to the water-values in addition to improved total efficiency.

The results reveal a large potential for cost reduction by using an optimization tool in the short term generation planning. The start-up cost represents about 1.5% of the total cost. Although this cost is a rather small share of the total cost, it is important to take the start-up cost into account as the production plan will be more realistic and better adapted to the overall schedule. By using the optimization tool, the start-up cost is reduced with an average of about \$2 500 per week.

The cost calculations in SHOP are made by multiplying used water with the corresponding watervalue. This involves both the water-values, the efficiency curves for both generator and turbine, and loss in the water courses. The efficiency for the generators depends on the production level, and the efficiency for the turbines depends on both reservoir level, and the amount of water into the turbine.

Also, by optimizing a larger part of a system, we will obtain a better total efficiency for the whole system, as well as a better distribution according to the water-values. Finally, this will result in a better overall plan in relation to the operation costs.

# 9 COMPUTATIONAL ASPECTS

This work has shown that Branch and Bound techniques can be useful in short-term scheduling of hydropower systems. However, such techniques can be time consuming if the number of discrete variables becomes too large. The time horizon of the study and the number of hydro units with explicit modeling of start/stop will be critical for the performance. Most river systems in Norway have less that 15 production units. For such systems it is no problem to study periods of 2-3 days with hourly time resolution without more than a few minutes computation time.

When using Branch and Bound, the algorithm may find the optimum rather fast, only to spend a long time verifying that the solution is indeed the optimal one. Especially when there are many different solutions with almost the same value of the objective function. This is the case when the price profile is flat, or almost flat.

It is also found that the solution time spent on a specified load profile is higher than to solve a case giving the same production profile against a market price profile. The computation time, on the system and with a market price profile used in this paper, is very low.

## 10 DISCUSSION/CONCLUSION

Introduction of discrete variables in the hydro scheduling is an important extension to the methods based on traditional linear programming. This capability will reduce the operation cost and give unit bility will reduce the operation cost and give unit plans that are more practical to implement. However, since the techniques at this stage are infeasible for really large system, it is important to apply a hierarchy for the decisions. For coordination of multiple river systems with a week's time horizon, it is necessary to use the traditional techniques based on successive linear programming with only continuous variables. The main result from this stage will be how much the different river systems should contribute to the overall plan. The next stage is then the detailed study of how to implement each sub system requirement. Other principles have also been tested for the hydro unit commitment decisions. Dynamic programming (DP) has been used in combination with the successive linear techniques. The DP was then used for the postprocessing of the plant productions with the sensitivity signals/marginal costs from the linear programming as decision support. This technique was fast and gave almost the same results as the Branch and Bound in cases with some flexibility of the reservoir storages. However, for small buffer reservoirs it was difficult to get satisfactory results by DP.

The Branch and Bound technique was found to be superior on small/medium systems due to the robustness and systematic approach for finding the optimal solution while taking all flow constraints into account.

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