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wheels of Wheeled Mobile Robots

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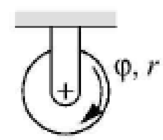
***Abstract* :** this paper presents a description and the kinematics of different wheels of mobile robots. Each type of existing wheels of wheeled mobile robots is mentioned in this paper notably spherical wheel.
Key words: wheels, robot, constraint.

1 Introduction

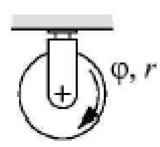
Autonomous Wheeled mobile robots (AMRs) have brought many advances to robotic applications specially in the following domains of robotics : 1- explorations of unstructured or / and dangerous terrains 2- robotic transportations 3-robotic manipulations with mobility . In each case, AMRs require mobility to achieve its purpose and the mobility is obtained by the use of wheels , legs or wheels and legs. Thus the AMRs can be classified into three main categories: 1-Wheeled mobile robots (WMRs) 2-legged mobile robots (LMRs) 3-Wheellegged mobile robots (WLMRs).
This paper concerns only the different types of wheels for WMRs. Mainly there exists two groups: conventional (or standard) wheels and special wheels. The conventional wheels are : conventional fixed wheel (or fixed standard wheel), steered standard wheel, castor wheel (or conventional off-centered wheel). The special wheels are: universal wheels (or Swedish wheel) and spherical wheel(or ball wheel). Their geometry and description are given in section 2.The principle of Swedish and spherical wheels are given in section 3.

2 Descriptions

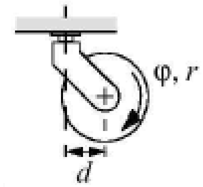
The schemas of different types of wheels generally used with WMR are represented in Fig.1. Each of them is able to rotate about an axle passing by the wheel center A (see fig4,5,7 and 8) and B for the castor wheel (see fig.6). This axle is perpendicular to the wheel plane. This rotation is represented by the driving angle φ .



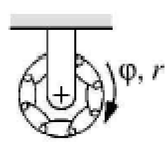
(a): fixed standard wheel



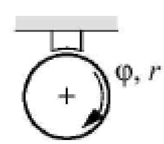
(b):steered standard wheel



(c):castor wheel



(d): Swedish wheel



(e): spherical wheel

Fig.1: schema of different wheels

with steered standard wheel (Fig.5) and castor wheel (Fig.6) wheels , the steering angle β represents the rotation of the wheel about a vertical axle passing by the wheel center A for the first one and passing by A and distant of d (eccentricity) to wheel center B for the second one (fig.1 c). The radius of the wheels are denoted by r.
Wheels (a), (b) and (c) in Figure.1 are commonly used in simple rolling mechanisms, but (d) and (e) are unusual that is why their photos are reported in

Fig.2. Their principles are explained in the next section.

3 Spherical and Swedish wheels principle

3.1 Spherical wheel
The spherical wheel is not driven by a shaft but by friction transmission. It is held to robot chassis only by the contact points of rollers and receives the traction forces from them (Fig2 (a)). In this case all of the rollers are connected by two different rings. Both of them hold the sphere and one of them actuates it [4]. Fig.2 (d) is a photo of a spherical wheel actuated by a Swedish wheel [3].

3.2 Swedish wheel
Two types of Swedish wheels are represented in Fig2 (b) and (c). Their wheel treads consist of at least 3 rollers (b) whose axes are tangent to the wheel circumference and free about rotation. As the driving shaft turns, the wheel is driven in a normal fashion in a direction perpendicular to the axis of the shaft. At the same time, the rollers can rotate allowing a free motion perpendicular to the rollers axis.

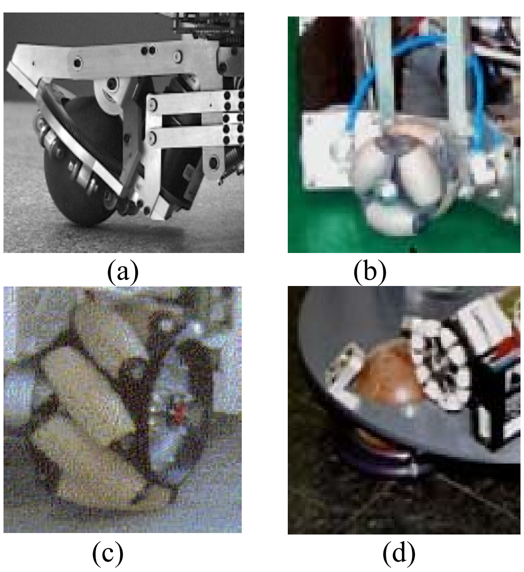


Fig.2: Photos of spherical and Swedish wheels
(a): spherical , (b) and (c): Swedish , (d): hybrid of spherical and Swedish wheels.

In Fig.2 (b), the roller axis and the wheel circumference are in the same plane. The type in Fig.2 (c) is called “mecanum wheel”. Its rollers axis are in a plane inclined with respect to the wheel circumference.

4 Kinematics
In this section, the kinematics constraints of each type of wheel are given. This purpose was obtained by the projections in the wheel plane and in a plane perpendicular to the wheel plane of the following vectors: the robot speed and the driving speed of the wheel for non-steered wheels; the robot speed, the driving speed and the steering speed of the wheel for the steered wheels.

OX_IY_I and OX_RY_R represent the world coordinate and the robot coordinate (Fig. 3). The robot posture ξ is defined in the world coordinate by x, y and θ . x and y are the coordinate of the point P and θ is the robot orientation .

$$\xi = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \qquad (1)$$

The rotation matrix expressing the orientation of OX_IY_I with respect to the robot frame OX_RY_R is given by:

$$R(\theta)=\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (2)$$

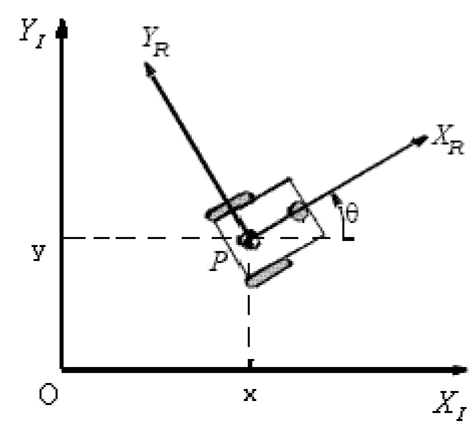


Fig.3: robot posture

5.1 Assumptions
-The wheels move on a horizontal plane.

-The wheels are not deformable and their contacts with the ground is a point .
-wheel's motions are pure rolling leading to a null velocity at the contact point.
-No slipping, skidding, sliding or friction for rotation around the contact point.
-The steering axes are orthogonal to a plan surface of the ground.
-The wheels are connected by rigid bodies to a rigid chassis of the robot.

5.2 Fixed standard wheel:

A is a fixed point of the robot frame and the center of the wheel (Fig.4). Its position is characterized by PA=l and the angle α . The constant angle β represents the orientation of the wheel plane with respect to PA and $\varphi(t)$ represents the rotation angle of the wheel.

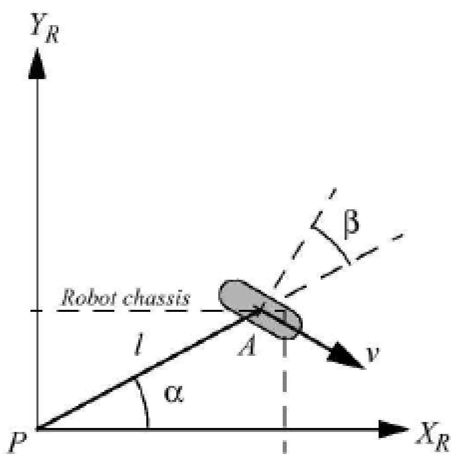


Figure 4: parameters of fixed standard wheel

When the components of velocity of the contact point are projected on the wheel plane we obtain the following constraint:

$$[-\sin(\alpha+\beta) \quad \cos(\alpha+\beta) \quad l\cos\beta]R(\theta)\dot{\xi} + r\dot{\varphi} = 0 \quad (3)$$

The projection on a plane orthogonal to the plane gives:

$$[\cos(\alpha+\beta) \quad \sin(\alpha+\beta) \quad l\sin\beta]R(\theta)\dot{\xi} = 0 \quad (4)$$

5.3 Steered standard wheel

Here the orientation angle β (t) of the wheel is a time varying variable (Fig.5).

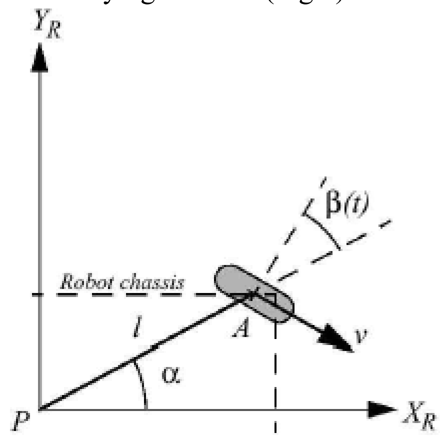


Figure.5: Parameters of steered standard wheel

That is the difference between fixed and steered standard wheels. So the constraint forms remain the same:

$$[-\sin(\alpha+\beta) \quad \cos(\alpha+\beta) \quad l\cos\beta]R(\theta)\dot{\xi} + r\dot{\varphi} = 0 \quad (5)$$

$$[\cos(\alpha+\beta) \quad \sin(\alpha+\beta) \quad l\sin\beta]R(\theta)\dot{\xi} = 0 \, (6)$$

5.4 Castor wheel

The center B of the Castor wheel is connected to a fixed point A of the robot frame by a rigid body (offset link) which has $\varphi(t)$ as time varying rotation angle (Fig.6). The distance between A and B is denoted by d.

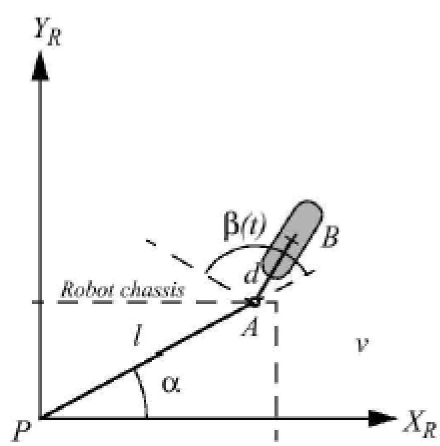


Figure 6: castor wheel's parameters

Along the Castor wheel plane and then along an orthogonal plane to the wheel plane, the kinematics constraints has the following forms:

$$\left[-\sin(\alpha+\beta)\quad \cos(\alpha+\beta)\quad l\cos\beta\right]R(\theta)\dot{\xi}+r\dot{\varphi}=0 \quad (7)$$

$$\left[\cos(\alpha+\beta)\quad \sin(\alpha+\beta)\quad d+l\sin\beta\right]R(\theta)\dot{\xi}+d\dot{\beta}=0 \quad (8)$$

5.5 Swedish wheel

The position of the wheel with respect to the robot frame is described by 3 constant parameters, α,β,l plus an additional parameter γ to characterize the direction, with respect to the wheel plane, of the zero component of the velocity of the contact point (Fig.7)

γ should not be equal to $\frac{\pi}{2}$ otherwise the direction of the zero component of the velocity will be orthogonal to the wheel plane. This condition will be like a non-slipping constraint of a standard fixed wheel, hence loosing the use of Swedish wheel. The constrain form is:

$$\left[-\sin(\alpha+\beta+\gamma)\quad \cos(\alpha+\beta+\gamma)\quad l\cos(\beta+\gamma)\right]R(\theta)\dot{\xi}$$

$$+r\cos\gamma.\dot{\varphi}=0 \quad (9)$$

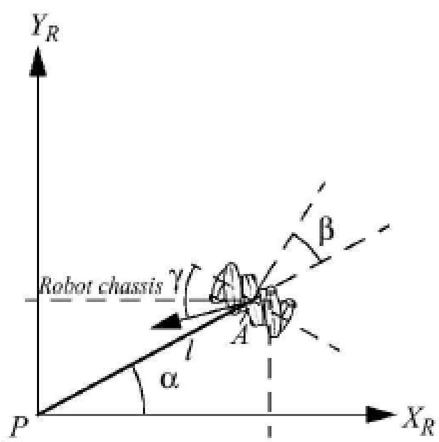


Fig.7: Swedish wheel's parameters

5.6 Spherical wheel

The wheel radius r is the radius of the sphere and its rotation angle is denoted by $\varphi(t)$. The wheel position in robot frame is described as for the standard fixed wheel by α,β and l (Fig.8).

The spherical wheel does not have fixed constraints as for the other. Its kinematics depends on the way it is assembled to a robot. To illustrate that, let us refer again to Fig.2(d) where the sphere is actuated through a universal wheel[3]. The sphere motion is constrained by the constraints of the roller of the universal wheel so that it does not have any extra free motion. This robot (ROLLMOBS) has three of such a type of wheel. Its kinematics is just like a WMR with three universal wheels. The spherical wheel in Fig.2(a) has different constraints (see[4]). That is why spherical have not been mentioned in wheel classification [1].

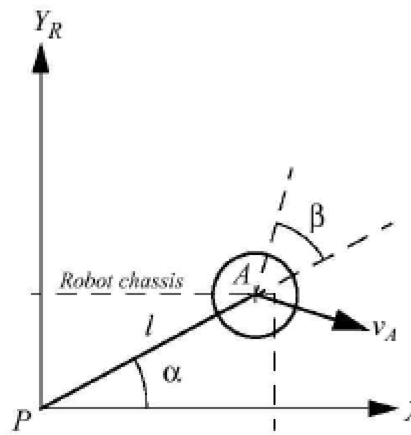


Fig.8: parameters of spherical wheel

6 Robot Mobility

The 4 followings subscripts are used to identify quantities relative to the 4 above described categories of wheels: f for fixed standard wheel, c for steered standard wheel, oc for castor wheel, sw for Swedish wheel . Spherical wheel will not be taken in to account for the above reason.

We consider a general WMR with N wheels, The number of wheels of each type is denoted by:

N_f, N_c, N_{oc}, and N_{sw} with

$$N_f+N_c+N_{oc}+N_{sw}=N$$

The configuration of the robot is fully described by the following vectors of coordinates:

- Posture coordinates:

$\xi(t) = \begin{bmatrix} x(t) \\ y(t) \\ \theta(t) \end{bmatrix}$ for the position coordinates of the robot in $OX_I Y_I$.

-Angular coordinates: $\beta_c(t)$ for the orientation angles of the steered standard wheels and $\beta_{oc}(t)$ for the castor wheels.

-And the Rotation coordinates:

$$\varphi(t) = \begin{bmatrix} \varphi_f(t) \\ \varphi_c(t) \\ \varphi_{oc}(t) \\ \varphi_{sw}(t) \end{bmatrix}$$

The whole set of posture (ξ), angular ($\beta_{c,}\beta_{oc}$), and rotation (φ) is called the set of configuration coordinates in the sequel. The total number of configuration coordinates is clearly $N_f + 2N_c + 2N_{oc} + N_{sw} + 3$

With these notation the constraints can be written under the general matrix form

$$J_1(\beta_c, \beta_{oc}) R(\theta)\ \dot{\xi} + J_2\ \dot{\varphi} = 0 \quad (10)$$

$$C_1(\beta_c, \beta_{oc}) R(\theta)\ \dot{\xi} + C_2\ \dot{\beta}_{oc} = 0 \quad (11)$$

With the following definition:

$$-J_1(\beta_c, \beta_{oc}) = \begin{bmatrix} J_{1f} \\ J_{1c}(\beta_c) \\ J_{1oc}(\beta_{oc}) \\ J_{1sw} \end{bmatrix}$$

where J_{1f}, J_{1c}, J_{1oc}, J_{1sw} are respectively an($N_f \times 3$),an ($N_c \times 3$), an ($N_{oc} \times 3$), and an ($N_{sw} \times 3$) matrix those forms derive readily from the constraint (3),(5),(7) and (9). J_{1f} and J_{1sw} are constant while J_{1c} and J_{1c} are time varying respectively trough $\beta_c(t)$ and $\beta_{oc}(t)$. J_2 is a constant ($N \times N$) matrix whose diagonal entries are the radius of the wheels, except for the Swedish wheels which are multiplied by $\cos\gamma$.

$$-C_1(\beta_c, \beta_{oc}) = \begin{bmatrix} C_{1f} \\ C_{1c}(\beta_c) \\ C_{1oc}(\beta_{oc}) \end{bmatrix},\ C_2 = \begin{bmatrix} 0 \\ 0 \\ C_{2oc} \end{bmatrix}$$

where C_{1f}, C_{1c}, and C_{1oc} are 3 matrices respectively of dimension ($N_f \times 3$), ($N_c \times 3$) and ($N_{oc} \times 3$) whose rows derive from the constraints (4),(6),(8). C_{1f} is constant while C_{1c} and C_{1oc} are time varying. C_{1oc} is a diagonal matrix whose diagonal entries are equal to d for N_{oc} castor wheels.

Consider now the ($N_f + N_c$) constraints from (11) and written explicitly as:

$$C_{1f} R(\theta)\ \dot{\xi} = 0 \qquad (12)$$

$$C_{1c}(\beta_c) R(\theta)\ \dot{\xi} = 0 \qquad (13)$$

These constraints imply that the vector $R(\theta)\ \dot{\xi}$ is belong to the null space of the following matrix $C_1^*(\beta_c)$:

$$C_1^*(\beta_c) = \begin{bmatrix} C_{1f} \\ C_{1c}(\beta_c) \end{bmatrix}$$

All analysis about the mobility of WMRs depends on $C_1^*(\beta_c)$ because the degree of mobility δ_m is defined as:

$\delta_m = \dim \mathcal{N}\left[C_1^*(\beta_c)\right] = 3 - rank\left[C_1^*(\beta_c)\right]$ and

the degree of steeribility δ_s is defined as :

$\delta_s = rank C_{1c}(\beta_c)$

Where $\mathcal{N}\left[C_1^*(\beta_c)\right]$ represent the null space of the matrix $C_1^*(\beta_c)$.

We will not redo this analysis here (see[1]) but it is important to know that it has shown that there exist only five types of WMRs (without take into account the spherical wheel) , corresponding to five pairs of values of δ_m and δ_s.They are represented in table1.

-Type(1,1): Car-like-a-robot structure, these robots have one or several fixed standard wheels on a single common axle. They have at least one steered standard wheel not located in this axle.

-Type(2,0): Wheel chair structure, these robots have one or more fixed standard wheels on a single common axle but no steered standard wheel.

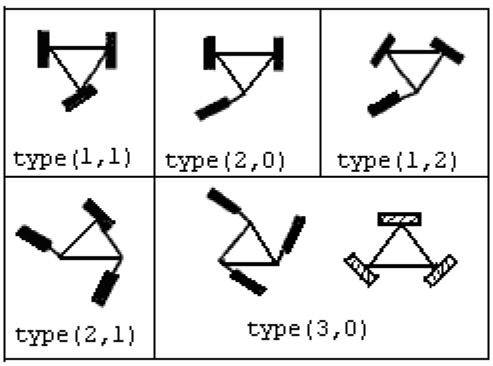


Table 1: robot types

-Type(1,2): these robots have at least two steered standard wheels but no fixed standard wheel.

-Type(2,1): They have at least one steered standard wheel but no fixed standard wheel.

-Type(3,0): they do not have fixed or steered standard wheel. They are omni-directional.

7 Conclusion

The spherical wheel is mentioned just in a few papers, also the above five classes of WMRs have been obtained without it . These facts do not have any relation with the role it can play in robotics.That is why we have paid attention to it in this paper. Among the five types of WMRs only the type(3,0) (i.e., robot equipped with three castor wheels or three universal wheels) are omni-directional but an adequate use of spherical wheels can also lead to an omni-directional and holonomic wheeled mobile robot.

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