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Analytical Approximate Method for Modelling of Three-Dimensional Transport Processes in Layered Media

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Abstract: - In this paper new normalized form of the integral parabolic spline needed for the original method of conservative averaging is introduced. The three-dimensional system of two partial differential equations with discontinuous coefficients describing transport processes in porous layered stratum is transformed into two 2-D systems (the order of each system is equal to the number of layers) by means of special integral spline. This common system of 2-D partial differential equations with continuous coefficients fulfills all conservation laws of initial problem in averaged sense. The reduction of 2-D advection-dispersion equation to 1-D equation for layered stratum is analyzed.
Key-Words: - integral spline, transient process, three-dimensional, layered media, conservative averaging, advection-dispersion, dimensions' diminishing.

1 Introduction

Real processes take place in natural or technical systems with complicated structure. Very often such systems consist of separate layers with different thickness and different physical properties. It means that on the surfaces between two adjacent layers we have jump in coefficients of differential equations mathematically describing correspondent physical process. This makes additional difficulties for applications of traditional mathematical methods. One of the authors has developed a special method – conservative averaging method [3]-[6] - and he has introduced a special new type of spline [1]-[3], [5], [6] used for solving such problems for wide class of direct [5]-[7] and inverse [8] problems for partial differential equations with discontinuous coefficients.

In this paper we give modified (normalized) description of integral parabolic spline and employ this spline for some groundwater (or other fluids) flows and pollution problems in layered stratum. Proposed method differs from methods traditionally used by mathematical modelling of groundwater pollution [9], [10] or other transport processes in natural or artificial porous media. Our method as outcome gives little bit more complicated mathematical model, but it allows to describe broader spectrum of physical phenomena and wider variety of geometrical and physical parameters.

2 Mathematical Background

We will start with the formulation of the interpolation of the integral values of piecewise-smooth function by new type of polynomial spline.

2.1 Interpolation of Averaged Integral Values of Piecewise-Smooth Function

Let it be given a continuous, piecewise-smooth function $U(x), x \in [a,b]$. Further, let it be given, that in the different inner points $x_i, i = 1,...,N$ the first derivative $U'(x)$ of the function has a finite jump:

$$k_{i-1}U'(x_i - 0) = k_i U'(x_i + 0). \tag{1}$$

Here $k_i, i = 0,...,N$ are known (given) strongly positive coefficients. Since the function $U(x)$ is continuous on the closed interval $[a,b]$, we additionally have following continuity equalities in these points:

$$U(x_i - 0) = U(x_i + 0), i = 1,..., N. \tag{2}$$

Lastly, let additionally give the average integral values u_i of the function $U(x)$ over the all sub-segments $[x_i, x_{i+1}]$, $i = 0,..., N$, $x_0 = a, x_{N+1} = b$:

$$u_i = H_i^{-1} \int_{x_i}^{x_{i+1}} U(x)dx, H_i = x_{i+1} - x_i, i = 0,...,N. \quad (3)$$

The goal of the interpolation problem is to (approximately) reconstruct the function $U(x)$, be based on conditions (1)-(3) and following general boundary conditions (BC) on the interval end points $x = a$ and $x = b$:

$$-\nu_0 k_0 U'(a) + \lambda_0 U(a) = \Phi_0, \quad (4)$$

$$\nu_1 k_N U'(b) + \lambda_1 U(b) = \Phi_1. \quad (5)$$

Such form of BC is typical for the ordinary or partial differential equations. Indeed, if $\nu_0 = 0$ $(\nu_1 = 0)$ and $\lambda_0 = 1 (\lambda_1 = 1)$, we obtain Dirichlet BC, if $\nu_0 = 1 (\nu_1 = 1)$ it gives Neumann BC (for $\lambda_0 = 0$ $(\lambda_1 = 0)$) and Robin BC (for $\lambda_0 > 0$ $(\lambda_1 > 0)$).

In our papers [1], [2] it was proved, that this interpolation problem can be solved by second order polynomial spline of following form:

$$S(x) = u_i + m_i(x - \bar{x}_i) + e_i \left[\frac{(x - \bar{x}_i)^2}{k_i H_i} - \frac{G_i}{12} \right], \quad (6)$$

$$\bar{x}_i = (x_i + x_{i+1})/2, G_i = H_i / k_i > 0.$$

This form of spline exactly fulfills the integral equalities (3) for all real values of unknown coefficients m_i, e_i. For the determination of $2(N+1)$ free coefficients, we have exactly the same number of equations (1),(2),(4) and (5). In [1] it was shown, that all coefficients m_i can be represented through coefficients e_i. Equalities (1), (2) allow us to express unknown coefficients m_i through unknown coefficients e_i in two forms:

a) for $i = 0,...,N-1$

$$\begin{aligned} &k_i m_i (G_i + G_{i+1}) = 2(u_{i+1} - u_i) - \\ &e_i (G_i / 3 + G_{i+1}) - 2/3 e_{i+1} G_{i+1}; \end{aligned} \quad (7_a)$$

b) for $i = 1,...,N$

$$\begin{aligned} &k_i m_i (G_i + G_{i-1}) = 2(u_i - u_{i-1}) + \\ &e_i (G_i / 3 + G_{i-1}) + 2/3 e_{i-1} G_{i-1}. \end{aligned} \quad (7_b)$$

Elimination the coefficients m_i from the expressions (7_a), (7_b) gives us for $i = 1,...,N-1$ the system of linear algebraic equations regarding the coefficients e_i (see [1], [2], [6]):

$$A_i e_{i-1} + C_i e_i + B_i e_{i+1} = F_i^- u_{i-1} - F_i u_i + F_i^+ u_{i+1}. \quad (8)$$

Here for $i = 1,...,N-1$:

$$\begin{aligned} &A_i = G_{i-1}(G_i + G_{i+1}), \ B_i = G_{i+1}(G_i + G_{i-1}), \\ &C_i = A_i + B_i + D_i, \ D_i = (G_i + G_{i+1})(G_i + G_{i-1}), \quad (9) \\ &F_i^- = 3(G_i + G_{i+1}), \ F_i^+ = F_{i-1}^-, \ F_i = F_i^+ + F_i^-. \end{aligned}$$

The approximation of BC (4), (5) looks as

$$m_0 k_0 \left(\nu_0 + \lambda_0 \frac{G_0}{2} \right) - e_0 \left(\nu_0 + \lambda_0 \frac{G_0}{6} \right) = \lambda_0 u_0 - \Phi_0,$$

$$m_N k_N \left(\nu_1 + \lambda_1 \frac{G_N}{2} \right) + e_N \left(\nu_1 + \lambda_1 \frac{G_N}{6} \right) = \Phi_1 - \lambda_1 u_N.$$

In papers [1], [2] they were writing in identical with (8) form by introducing two additional degenerate fictive coefficients: $e_{-1} = e_{N+1} = 0$.

For this goal in [1],[2] some additional notations was used and was distinguished two different cases:

1) $\lambda_0 \neq 0$ (and $\lambda_1 \neq 0$). Then

$$G_{-1} = 2\nu_0 / \lambda_0, G_{N+1} = 2\nu_1 / \lambda_1,$$
$$u_{-1} = \Phi_0 / \lambda_0, \ u_{N+1} = \Phi_1 / \lambda_1;$$

2) $\lambda_0 = 0$ (and $\lambda_1 = 0$). Then

$$G_{-1} = 2\nu_0 - G_0, G_{N+1} = 2\nu_1 - G_N,$$
$$u_{-1} = \Phi_0 + u_0, \ u_{N+1} = \Phi_1 + u_N.$$

In second case some coefficients different from general case (given by expressions (9)) must be calculated additionally:

$$A_0 = D_0, \ B_N = D_N.$$

Coefficients e_i of integral parabolic spline can be represented explicit through all averaged integral values ([2], [5], [6]):

$$e_i = \sum_{j=0}^{N+1} \alpha_{ij} (u_{j-1} - u_j) \operatorname{sgn}(i - j + 0.5), \quad i = \overline{-1, N+1}. \quad (10)$$

In [2] the matrix $\{\alpha_{ij}\}$ was calculated sequential for all $j = \overline{0, N}$:

$$\begin{cases} \alpha_{-1,j} = \alpha_{N+1,j} = 0, \\ A_i \alpha_{i-1,j} + C_i \alpha_{ij} + B_i \alpha_{i+1,j} = 0, i \neq j, i \neq j-1, \\ A_i \alpha_{i-1,j} + C_i \alpha_{ij} - B_i \alpha_{i+1,j} = F_i^+, \quad i = j-1, \\ -A_i \alpha_{i-1,j} + C_i \alpha_{ij} + B_i \alpha_{i+1,j} = F_i^-, \quad i = j. \end{cases} \quad (11)$$

It is very easy to see that the system of linear algebraic equations (8) (and also the system (11)) is a system with strongly dominated main diagonal. It means that each of these systems have exactly one solution, where the errors of calculations can be easy estimated through the right hand side of the system. The approximation error can be estimated as follows: $\left|U^{(p)}(x)-S^{(p)}(x)\right| \le C_p\alpha_N \left\|\Delta_N\right\|^{2-p}$,

$p=0,1,2;\ \Delta_N = \max_i G_i, \alpha_N = \omega\left(U'', \left\|\Delta_N\right\|\right)$.

Here ω is continuity modulus of corresponding function on the given grid.

2.2 Normalized Form of Spline's Coefficients and its New Representation

We propose in this paper a different ("normalized") form for the calculation of the spline "crucial" coefficients e_i :

$$a_i e_{i-1} + (1+a_i+b_i)e_i + b_i e_{i+1} = f_i^- u_{i-1} - f_i u_i + f_i^+ u_{i+1}, i=1,\dots,N-1 \tag{12}$$

and we use other form for the first and last equations of the system of linear algebraic equations:

$$\begin{aligned}&(1+a_0+b_0)e_0 + b_0 e_1 = f_0^- u_{-1} - f_0 u_0 + f_0^+ u_1,\\ &a_N e_{N-1} + (1+a_N+b_N)e_N = \\ &f_N^- u_{N-1} - f_N u_N + f_N^+ u_{N+1}.\end{aligned} \tag{13}$$

Here $f_i = f_i^- + f_i^+$ and

$a_i = G_{i-1}/\left(G_i+G_{i-1}\right), b_i = G_{i+1}/\left(G_i+G_{i+1}\right)$,

$f_i^- = 3/\left(G_i+G_{i-1}\right), f_i^+ = 3/\left(G_i+G_{i+1}\right)$.

Similarly as earlier in the case $\lambda_0 = 0$ ($\lambda_1 = 0$) we have special formulas for coefficients a_0 and b_N : $a_0 = b_N = 1$. Instead of (10) we here propose also other form of the explicit representation for coefficients e_i of integral parabolic spline through all averaged integral values. This representation shows in explicit form the influence of the BC type and its right hand side on the spline:

$$e_i = \gamma_i^{(0)} f_0^- u_{-1} + \gamma_i^{(1)} f_N^+ u_{N+1} + \sum_{j=0}^{N} \beta_{ij} u_j,\ i=\overline{0,N}. \tag{14}$$

The coefficients in the representation (14) are determinate from following systems of linear algebraic equations:

a) the system for $\gamma_i^{(0)}$:

$$\begin{aligned}&(1+a_0+b_0)\gamma_0^{(0)} + b_0\gamma_1^{(0)} = 1,\\ &a_i\gamma_{i-1}^{(0)} + (1+a_i+b_i)\gamma_i^{(0)} + b_i\gamma_{i+1}^{(0)} = 0,\\ &i=1,\dots,N-1,\\ &a_N\gamma_{N-1}^{(0)} + (1+a_N+b_N)\gamma_N^{(0)} = 0;\end{aligned} \tag{15}$$

b) the system for $\gamma_i^{(1)}$:

$$\begin{aligned}&(1+a_0+b_0)\gamma_0^{(1)} + b_0\gamma_1^{(1)} = 0,\\ &a_i\gamma_{i-1}^{(1)} + (1+a_i+b_i)\gamma_i^{(1)} + b_i\gamma_{i+1}^{(1)} = 0,\\ &i=1,\dots,N-1,\\ &a_N\gamma_{N-1}^{(1)} + (1+a_N+b_N)\gamma_N^{(1)} = 1\end{aligned} \tag{16}$$

c) and $N+1$ systems ($j=0,\dots,N$) for β_{ij} :

$$\begin{aligned}&(1+a_0+b_0)\beta_{0,j} + b_0\beta_{1,j} = 0,\\ &a_i\beta_{i-1,j} + (1+a_i+b_i)\beta_{ij} + b_i\beta_{i+1,j} = \\ &f_j^-\delta_{i-1,j} - f_j\delta_{i,j} + f_j^+\delta_{i+1,j}, i=1,\dots,N-1,\\ &a_N\beta_{N-1,j} + (1+a_N+b_N)\beta_{N,j} = 0.\end{aligned} \tag{17}$$

We would like to draw reader's attention to some important aspects of this new type of (integral) spline. Firstly, this spline interpolates exactly the average integral value (3) of the function $U(x)$. Secondly, it fulfills exactly both conjugations conditions (1), (2). Thirdly, the new type of representation (10) (and especially (14)) has interesting and very important property in application to differential equations. As reader can see, the components of the vector $\gamma^{(k)} = (\gamma_i^{(k)})_{i=0}^N, k=\{0,1\}$ and of the matrix $\beta = (\beta_{ij})_{i,j=0}^N$ depend on the location of grid points x_i , coefficients k_i and type of BC (4), (5), but they are independent from averaged integral values u_i and right hand sides' values Φ_0, Φ_1 of BC. This property implies that for fixed grid points and coefficients k_i we need to calculate the components of the two vectors $\gamma^{(k)}$ and the matrix β only once. After this calculation, for the construction of the integral parabolic spline we need only to compute the finite sum (14). As the reader will see further, the representation (14) is very important by utilizing this spline for the differential equations with discontinuous coefficients.

We finish our remarks with notation that representation as (14) can be constructed for all classical splines too (see [11]).

3 Transforming the 3-D Formulation to 2-D System

We will start with the description of well known difficulties for layered stratum by substituting 1-D formulation instead of 2-D problem. Then we propose our vision how to overcome this complication by the usage of spline introduced here for the interpolation of the integral values of piecewise-smooth function.

In monograph [9] (see subsection 7.2.3) is analyzed the 2-D advection-dispersion equation for horizontal layered stratum

$$\frac{\partial C}{\partial t}+V(y)\frac{\partial C}{\partial x}=D_{xx}(y)\frac{\partial^2 C}{\partial x^2}+\frac{\partial}{\partial y}\left[D_{yy}(y)\frac{\partial C}{\partial y}\right]$$

and there is shown that this 2-D equation can't be replaced by following 1-D equation with some averaged constant velocity V and dispersion coefficient D:

$$\frac{\partial C}{\partial t}+V\frac{\partial C}{\partial x}=D\frac{\partial^2 C}{\partial x^2}.$$

We will examine here more general 2-D equation

$$\frac{\partial C}{\partial t}+V(y,t)\frac{\partial C}{\partial x}=\frac{\partial}{\partial x}\left[D_{xx}(x,y,t)\frac{\partial C}{\partial x}\right]+$$
$$\frac{\partial}{\partial y}\left[D_{yy}(x,y,t)\frac{\partial C}{\partial y}\right]+F(x,y,t)$$

and we will show how our integral parabolic spline allows to reduce this 2-D equation to system of coupled 1-D differential equations in the way where all the conservations laws of initial problem are fulfilled (no additional "apparent source/sink term" appears). But we start with general 3-D problem statement and its reducing to 2-D system.

3.1 Mathematical Description of Transport Processes in Orthotropic Layered Media by 3-D Model

The equation for solid matrix characteristic $U_i(x,y,z,t)$ (concentration, temperature etc.) in the $i-th$ layer we write in following form:

$$c_1^i\frac{\partial U_i}{\partial t}=\frac{\partial}{\partial x}\left(k_{11}^i\frac{\partial U_i}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_{22}^i\frac{\partial U_i}{\partial y}\right)$$
$$+\frac{\partial}{\partial z}\left(k_{33}^i\frac{\partial U_i}{\partial z}\right)+F_i(x,y,z,t). \tag{18}$$

We assume that the source term consists of two parts. Firstly, the linear interaction with fluid (gas) phase characteristic $V_i(x,y,z,t)$ and, secondly, the internal source $\Phi_i(x,y,z,t)$:

$$F_i(x,y,z,t)=\alpha_{i,1}(V_i-U_i)/\left(1-m_i\right)+\Phi_i(x,y,z,t).$$

Here $0<m_i<1$ is $i-th$ layer porosity and $\alpha_{i,1}$ is known interaction coefficient. The equation for moving phase we write in a similar form:

$$c_2^i\frac{\partial V_i}{\partial t}=\frac{\partial}{\partial x}\left(\kappa_{11}^i\frac{\partial V_i}{\partial x}\right)+\frac{\partial}{\partial y}\left(\kappa_{22}^i\frac{\partial V_i}{\partial y}\right)$$
$$+\frac{\partial}{\partial z}\left(\kappa_{33}^i\frac{\partial V_i}{\partial z}\right)+G_i(x,y,z,t). \tag{19}$$

The source term for moving phase has more complicated form in relation with its possible movement with velocity $\vec{w}_i=(w_{i,1},w_{i,2},0)$:

$$G_i(x,y,z,t)=\alpha_{i,2}(U_i-V_i)/m_i+$$
$$\Gamma_i(x,y,z,t)-\frac{\partial}{\partial x}(w_{i,1}V_i)-\frac{\partial}{\partial y}(w_{i,2}V_i). \tag{20}$$

As reader can see, we exclude the possible motion of fluid in the $z-th$ direction: orthogonal to the layer plane. Additionally we allow the dependence the velocity components $w_{i,1},w_{i,2}$ and coefficients k_{jj}^i,κ_{jj}^i of the arguments x,y,t only.

3.2 Transformation to 2-D Problem

We will use our original method of conservative averaging and for this goal we introduce averaged integral values:

$$u_i(x,y,t)=H_i^{-1}\int_{z_i}^{z_{i+1}}U_i(x,y,z,t)dz,$$
$$v_i(x,y,t)=H_i^{-1}\int_{z_i}^{z_{i+1}}V_i(x,y,z,t)dz. \tag{21}$$

We integrate now the differential equation (18) in the $z-$ direction:

$$c_1^i\frac{\partial u_i}{\partial t}=\frac{\partial}{\partial x}\left(k_{11}^i\frac{\partial u_i}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_{22}^i\frac{\partial u_i}{\partial y}\right)$$
$$+H_i^{-1}k_{33}^i\left.\frac{\partial U_i}{\partial z}\right|_{z=z_i}^{z=z_{i+1}}+f_i(x,y,t), \tag{22}$$

$$f_i(x,y,t)=\alpha_{i,1}(v_i-u_i)/\left(1-m_i\right)+\varphi_i(x,y,t),$$
$$\varphi_i(x,y,t)=H_i^{-1}\int_{z_i}^{z_{i+1}}\Phi_i(x,y,z,t)dz.$$

Next step of our conservative averaging method is the approximation of the functions $U_i(x,y,z,t)$ and $V_i(x,y,z,t)$ by the spline (6) in the $z-$ direction. As it was shown in the section 2.1 the construction of spline reduces to the calculation of its coefficients $e_i^{(u)}, i = 0,...,N$ (the top index (u) indicates the association of the corresponding spline coefficients with the function $U_i(x,y,z,t)$). The following crucial step is the approximation by spline's derivative the fluxes difference in integrated differential equation (22):

$$k_{33}^i \frac{\partial U_i}{\partial z}\bigg|_{z=z_i}^{z=z_{i+1}} \approx \frac{dS^{(u)}}{dz} = 2e_i^{(u)}. \tag{23}$$

It must be underlined that this is exclusive step in which the approximate substitution in the conservative averaging method is made.

The next and in the same time the last step of the conservative averaging method is to use the representation (14) in (23) and to substitute this approximate equality in the integrated differential equation (22). We obtain:

$$c_1^i \frac{\partial u_i}{\partial t} = \frac{\partial}{\partial x}\left(k_{11}^i \frac{\partial u_i}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_{22}^i \frac{\partial u_i}{\partial y}\right) + f_i(x,y,t)$$

$$+\frac{2}{H_i}\left[\sum_{j=0}^{N} \beta_{ij}^{(u)} u_j + \gamma_i^{(0,u)} f_0^- u_{-1} + \gamma_i^{(1,u)} f_N^+ u_{N+1}\right],$$

$$f_i(x,y,t) = \alpha_{i,1}(v_i - u_i)/(1-m_i) + \varphi_i(x,y,t). \tag{24}$$

In similar way we transform the partial differential equation (19) and finally instead of two 3-D problem statement we obtain the system of 2-D differential equations with continuous coefficients:

$$c_2^i \frac{\partial v_i}{\partial t} = \frac{\partial}{\partial x}\left(\kappa_{11}^i \frac{\partial v_i}{\partial x}\right) + \frac{\partial}{\partial y}\left(\kappa_{22}^i \frac{\partial v_i}{\partial y}\right) +$$

$$\frac{2}{H_i}\left[\sum_{j=0}^{N} \beta_{ij}^{(v)} v_j + \gamma_i^{(0,v)} f_0^- v_{-1} + \gamma_i^{(1,v)} f_N^+ v_{N+1}\right] \tag{25}$$

$$+g_i(x,y,t),\ g_i(x,y,t) = \alpha_{i,2}(u_i - v_i)/m_i +$$

$$\gamma_i(x,y,t) - \frac{\partial}{\partial x}(w_{i,1} v_i) - \frac{\partial}{\partial y}(w_{i,2} v_i),$$

where

$$\gamma_i(x,y,t) = H_i^{-1} \int_{z_i}^{z_{i+1}} \Gamma_i(x,y,z,t)\,dz.$$

3.3 Conservative Averaging for 2-D Advection-Dispersion Equation in Layered Media

We will show now the application of the conservative averaging method for the advection-dispersion equation in layered stratum, mentioned at the introduction of section 3. It can be written in following form for all $i = 0,...,N$ (here $N+1$ is the number of layers with thickness $H_i = y_{i+1} - y_i$):

$$\frac{\partial C_i}{\partial t} + V_i(t)\frac{\partial C_i}{\partial x} = \frac{\partial}{\partial x}\left[D_{i,xx}(x,t)\frac{\partial C_i}{\partial x}\right]$$

$$+D_{i,yy}(x,t)\frac{\partial^2 C_i}{\partial y^2},\ y_i < y < y_{i+1} \tag{26}$$

together with conjugations conditions

$$C_{i-1}\big|_{y=y_i-0} = C_i\big|_{y=y_i+0},$$

$$D_{i-1,yy}\frac{\partial C_{i-1}}{\partial y}\bigg|_{y=y_i-0} = D_{i,yy}\frac{\partial C_i}{\partial y}\bigg|_{y=y_i+0} \tag{27}$$

and BC of general form (4),(5):

$$\left[-\nu_0 D_{0,yy}\frac{\partial C_0}{\partial y} + \lambda_0 C_0\right]\bigg|_{y=y_0} = \Phi_0(x,t), \tag{28}$$

$$\left[\nu_1 D_{N,yy}\frac{\partial C_N}{\partial y} + \lambda_1 C_N\right]\bigg|_{y=y_{N+1}} = \Phi_1(x,t). \tag{29}$$

We again introduce the integral averaged values

$$c_i(x,t) = \frac{1}{H_i}\int_{y_i}^{y_{i+1}} C_i(x,y,t)\,dy,$$

integrate equation (26) in the $y-$direction and repeat all the steps as in previous subsection 3.2. Finally we obtain following coupled system of total $N+1$ one space dimension partial differential equations of generalized advection-dispersion type:

$$\frac{\partial c_i}{\partial t} + V_i(t)\frac{\partial c_i}{\partial x} = \frac{\partial}{\partial x}\left[D_{i,xx}(x,t)\frac{\partial c_i}{\partial x}\right] + f_i(x,t)$$

$$+\frac{2}{H_i}\left[\sum_{j=0}^{N}\beta_{ij} c_j + \gamma_i^{(0)} f_0^- c_{-1} + \gamma_i^{(1)} f_N^+ c_{N+1}\right]. \tag{30}$$

Here parameters G_i for system of the equations (30) in comply with definition (6) are:

$$G_i = H_i / D_{i,yy}(x,t). \tag{31}$$

Further, if $\lambda_0 \neq 0$ and $\lambda_1 \neq 0$:

$$G_{-1} = 2\nu_0/\lambda_0, G_{N+1} = 2\nu_1/\lambda_1,$$

$$c_{-1} = \Phi_0/\lambda_0,\ c_{N+1} = \Phi_1/\lambda_1;$$

if $\lambda_0 = 0$ and $\lambda_1 = 0$:

$$G_{-1} = 2\nu_0 - G_0, G_{N+1} = 2\nu_1 - G_N,$$

$$c_{-1} = \Phi_0 + c_0,\ c_{N+1} = \Phi_1 + c_N.$$

Coefficients $\gamma_i^{(0)}, \gamma_i^{(1)}, \beta_{ij},\ i,j = 0,...,N$ are calculated from systems (15), (16) and (17) respectively. Formula (31) shows that in general case $\gamma_i^{(0)} = \gamma_i^{(0)}(x,t), \gamma_i^{(1)} = \gamma_i^{(1)}(x,t), \beta_{ij} = \beta_{ij}(x,t)$. As follows the systems of linear algebraic equations (15)-(17) must be solved for all values of the argument x. In contradistinction to this case the dependence the right hand sides $\Phi_0(x,t), \Phi_1(x,t)$ of BC (28), (29) don't to lead up to necessity to repeat the calculations of $\gamma_i^{(0)}$, $\gamma_i^{(1)}, \beta_{ij}$. These dependences are included in the right hand side of the equations (30) trough terms c_{-1}, c_{N+1} and $f_0^- = 3/\left(G_0 + G_{-1}\right), f_N^+ = 3/\left(G_N + G_{N+1}\right)$.

It is evident that in case of constant dispersion coefficients in the $i-th$ layer (however different for different layers!) we obtain system (30) of $N+1$ coupled one space dimension equations with constant coefficients. The attempt to reduce 2-D system to one 1-D equation leads to necessity to introduce time and space depending on advection-dispersion coefficients (see [9]).

We conclude this paper with remark that in the case of big difference of coefficients of adjacent layers ($k_i << k_{i+1}$) in $i-th$ layer the boundary layer arise. In this situation, instead of polynomial integral parabolic spline, rational integral spline is more appropriate [3]. One of two forms of such spline was investigated in [3] and looks as follows:

$$S_r\left(t\right) = \overline{v}_i + G_i[\mu_i\left(t-0.5\right) + \frac{d_i}{2}\left(\frac{t^2}{1+2p_i\left(1-t\right)} + \frac{\left(1-t\right)^2}{1+2q_i t}\right)].$$

Coefficients $\overline{v}_i, \mu_i, d_i$ are unknown and can be determined from equalities (1)-(3). In paper [7] this spline was used for modelling of one advection dominated industrial process.

4 Conclusion

The integral parabolic spline allows transforming 3-D problem for layered stratum (for partial differential equations with discontinuous coefficients) to 2-D system of partial differential equations with continuous coefficients with number of equations equal to the number of layers. The system of coupled 1-D equations instead of 2-D advection-dispersion equation for layered stratum is obtained.

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