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Multiscale Synthetic Computational Modeling of Turbulent Fluid Interfaces *

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Abstract: - We develop a methodology for the synthetic computational multiscale modeling of turbulent interfaces by successive geometrical deformations. Modeling of such interfaces is useful in engineering applications and scientific phenomena where physical processes occur on or across fluid interfaces, such as molecular diffusion or electromagnetic wave propagation. The proposed method involves successively deforming an interface, that initially has purely large scale structure, with successive smaller-scale deformations that ensure smoothness of the interface and introduce scale dependent interfacial features according to a prescribed distribution of scales. We demonstrate and validate the method for two-dimensional interfaces with an exponential distribution of scales and azimuthal symmetry. Results are presented on the scale dependence of the fractal dimension for these interfaces. General implications of the proposed multiscale computational synthetic modeling approach are discussed.

Keywords: - Synthetic Turbulence, Fractals, Generalized Fractal Dimensions, Interfaces.

1. Introduction

Computational models of complex interfaces are useful in various engineering applications and scientific phenomena where physical or chemical processes occur on or across fluid interfaces, such as molecular diffusion or electromagnetic wave propagation, etc. A particularly challenging case of interfaces is the case of turbulent interfaces because they are highly multiscale [1, 2]. The usual approach to simulations of turbulent flows and interfaces is either through full-resolution direct numerical simulations [3] or reduced-resolution simulations such as large-eddy simulations [4], where in both cases the governing equations of flow motion are utilized.

An alternative approach, which we pursue in this work, is to generate synthetic turbulence [5, 6], i.e. to conduct synthetic computational modeling in which the multiscale flow feature of interest, e.g. a fluid interface, is constructed by starting with a purely large-scale structure and subsequently introducing deformations at successively smaller scales. This is also an approach that has been used to create fractal objects [1], i.e. objects with self-similarity. However, in turbulence, physical features can exhibit scale dependence in addition to self-similarity and, moreover, physical objects such as fluid interfaces must be continuous and differentiable in continuum-mechanics descriptions. For these reasons, the present work is aimed toward the development of a computational synthetic modeling method for multiscale fluid interfaces with scale dependence.

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2. Synthetic Turbulent Interface Generation Technique

In the present work, a Monte Carlo type algorithm is developed and employed to simulate the deformation pattern of turbulent fluid interfaces. A similar type of analysis was done [7] for 1D point sets where the spacing between points placed on the line was determined by a prescribed statistical distribution. In this work an effort is made to extend such analyses to a case of the constructed simple closed curve with given statistical distribution of scales and produce the best possible correspondence between the geometrical characteristics that are theoretically expected and the ones present in the generated image.

Consider a circle, i.e. the initial generator, with radius R lying in the plane XoY, as in figure 1 (top left). The iteration algorithm developed randomly distorts this curve transforming it into a new curve described by:

$$f_m(x,y) \;=\; 0\,, \tag{1}$$

where m is the number of distortions made and each transformation

$$\Phi_j(f_{j-1}) \;=\; f_j\,, \tag{2}$$

with $1 \leq j \leq m$ represents the additional distortion introduced after which a total of j distortions are present.

For each j^{th} transformation a length of each distortion interval is obtained from a prescribed statistical distribution. The amplitude factor is taken, as an example, to be $\sqrt{3}/2$ to draw an analogy with the Von Koch snowflake. After the center point of the distortion is randomly picked the unit tangent vector A_j is defined there and the length of the distortion interval is converted in to a total number of points defining this interval. To have a better physical representation of a real fluid boundary the curve should maintain its smoothness, i.e. one must avoid sharp corners, and all the points in the distortion interval are to be moved along an outward normal vector. A linear transformation $\Phi_j : R^2 \rightarrow R^2$ maps the points on the curve $f_{j-1} = 0$ on to the curve $f_j = 0$. Since all points except the ones on the distortion interval remain invariant under transformation, let n be the number of points defining the distortion interval, $\vec{x_o}$ be an array of point coordinated on the distortion interval before deformation is made, then $\vec{x_f} = \Phi(\vec{x_0})$ defines the points on the newly made deformation. Then for each particular k^{th} point on the distortion interval ($k \in [1,n]$):

$$\vec{x_{f_k}} \;=\; \Phi_k(\vec{x_{o_k}}) \;=\; \vec{x_{0_k}} + \Xi(k)\,\vec{e}\,, \tag{3}$$

where $\vec{e}$ is the outward normal vector to the curve at center point which can easily be determined from 90 degree rotation of A_j and $\Xi(k)$ determines the amount by which the k^{th} point is moved along a unit normal. One of the possible $\Xi(k)$ that satisfy the smoothness criteria is:

$$\Xi(k) \;=\; \frac{1}{2}\left[1 - \cos\left(\frac{2\pi k}{n}\right)\right] * A_k\,, \tag{4}$$

where each amplitude A_k is proportional to the length of the interval and $\Xi(k)$ will be maximum when k corresponds to the centerpoint of the distortion.

Before each distortion is made, a specially-developed non-intersection algorithm developed below is utilized in order to prevent any possible self intersections of the curve. This is important in order to construct a simple closed curve that could be viewed as a physical representation of the turbulent interface.

Let $\vec{x_{o_k}} =< x_{o_k}, y_{o_k} >$, $k \in (0,n)$ be the points on the distortion interval before the distortion was made and $\vec{x_{f_k}} =< x_{f_k}, y_{f_k} >$, $k \in (0,n)$ be the points on the same interval defining the shape of the distortion in question. Also, let $\vec{V_k}$ be a vector in R^2 such that $\vec{V_k} = \vec{x_{f_k}} - \vec{x_{o_k}}$ (each $\vec{V_k}$ is parallel to the normal at the centerpoint vector $\vec{e}$) and let $\vec{M_i}$ be a vector in R^2 such that $\vec{M_i} = \vec{x_i} - \vec{x_{o_k}}$ where $\vec{x_i} =< x_i, y_i >$is the coordinate vector of the remaining points on the curve (points outside the distortion interval) $i = 1,...,T-n$, where T is the total number of points on the curve.

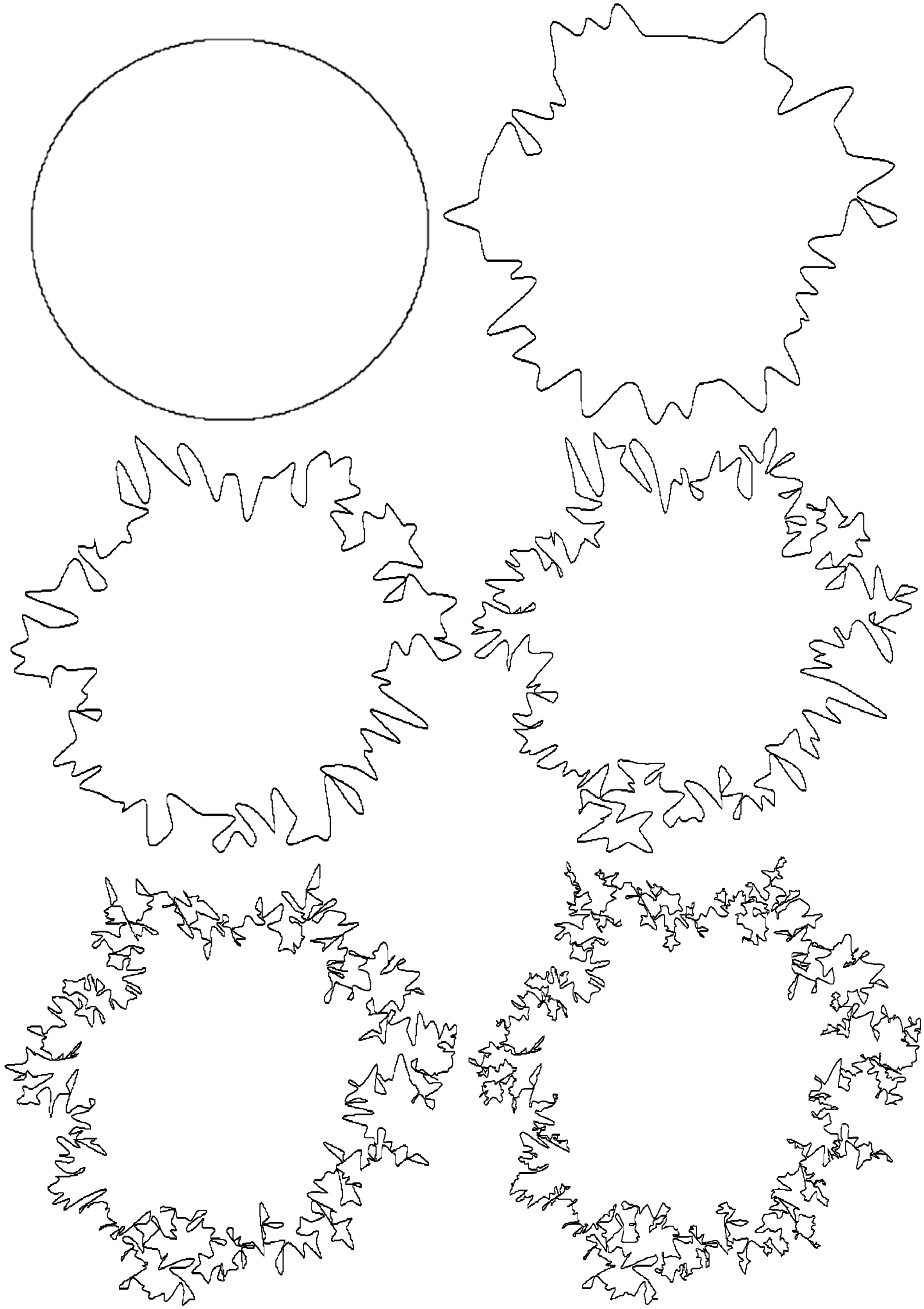

Figure 1: Generation of synthetic turbulent interface by successive multiscale scale-dependent deformations, using the present technique.

Similarly $\vec{M}_{i+1} = \vec{x_{i+1}} - \vec{x_k}$ such that the line connecting the tips of the vectors $\vec{M}_i$ and $\vec{M}_{i+1}$ represents a segment of the curve between two consecutive points. Knowing the coordinates of the distortion in question and the coordinates of all the points defining the curve: all n vectors $\vec{V}_k$ and all vectors $\vec{M}_i$ can be determined. This algorithm is based on the assumption that if the vector $\vec{V}_k$ intersects any one segment of the curve for some k then this distortion will cause self intersection of the curve and should be canceled or replaced.

The first step in the algorithm is to check if $\vec{V}_k$ lies between some vectors $\vec{M}_i$ and $\vec{M}_{i+1}$. This can be done by vector product computation with the following two criteria:

1. If both $\vec{M}_i \times \vec{V}_k$ and $\vec{M}_{i+1} \times \vec{V}_k$ are pointed the same way (same sign) then the vector $\vec{V}_k$ does not intersect the segment of the curve connecting the tips of Mi and $Mi + 1$.

2. If $\vec{M}_i \times \vec{V}_k$ and $\vec{M}_{i+1} \times \vec{V}_k$ are pointed in opposite directions (opposite in sign) then the vector $\vec{V}_k$ lies between $\vec{M}_i$ and $\vec{M}_{i+1}$ which can result in intersection and further criteria below are needed.

If case 2 is true for some $\vec{V}_k$ and $\vec{M}_i$ the intersection will occur due to the amplitude and orientation of the vector $\vec{V}_k$. These two additional criteria need to be determined in order to conclude intersection or non intersection:

i. If $\left|\vec{V}_k\right| < \left|\vec{M}_i\right|$ and $\left|\vec{V}_k\right| < \left|\vec{M}_{i+1}\right|$ then the vector is too short and the intersection will not occur.

ii. If $\left|\vec{V}_k\right| \geq \left|\vec{M}_i\right|$ or $\left|\vec{V}_k\right| \geq \left|\vec{M}_{i+1}\right|$ then the intersection will occur unless the orientation condition below holds.

The orientation condition is as follows. Let Θ be the angle between $\vec{V}_k$ and $\vec{M}_i$, i.e.

$$\Theta = \cos^{-1}\left(\frac{\vec{V}_k \bullet \vec{M}_i}{\left|\vec{V}_k\right|\left|\vec{M}_i\right|}\right) \qquad (5)$$

The coordinates of $\vec{V}_k$ and $\vec{M}_i$ are known and Θ can easily be calculated.

a. If $|\Theta| < \frac{\pi}{2}$ radians then the vector $\vec{V}_k$ is oriented undesirably and intersection occurs.

b. If $|\Theta| \geq \frac{\pi}{2}$ radians then the vector $\vec{V}_k$ is pointed in the direction away from the possible intersection region and the intersection will not occur.

It is noted that the choice of $\frac{\pi}{2}$ radians for a critical angle is flexible since for the orientation resulting in intersection Θ will be very small $|\Theta| << \frac{\pi}{2}$ and for the orientation that does not cause intersection $|\Theta| \approx \pi > \frac{\pi}{2}$.

The turbulent interface modeling program employs the non-intersection algorithm outlined above to achieve the desired simple (non-intersecting) closed curve. If a particular distortion does in fact cause intersection it is canceled and a new centerpoint is randomly picked to create another distortion on the same length scale in order to preserve the prescribed statistical characteristics.

3. Results on Generalized Fractal Dimensions of Synthetic Turbulent Interfaces

Different prescribed statistical distributions of scales, i.e. probability density functions $p(l)$ of scales l, can be studied such as exponential:

$$p(l) \ = \ \frac{1}{l_m}\exp(-l/l_m)\,, \qquad (6)$$

where l_m is the mean length, or lognormal:

$$p(l) \ = \ \frac{1}{\sqrt{2\pi}\sigma l}\exp\left\{-\left[\frac{ln(l/l_m)}{\sigma}+\frac{\sigma}{2}\right]^2/2\right\}\,, \qquad (7)$$

where σ is the variance, or other distributions of scales for turbulent interface modeling.

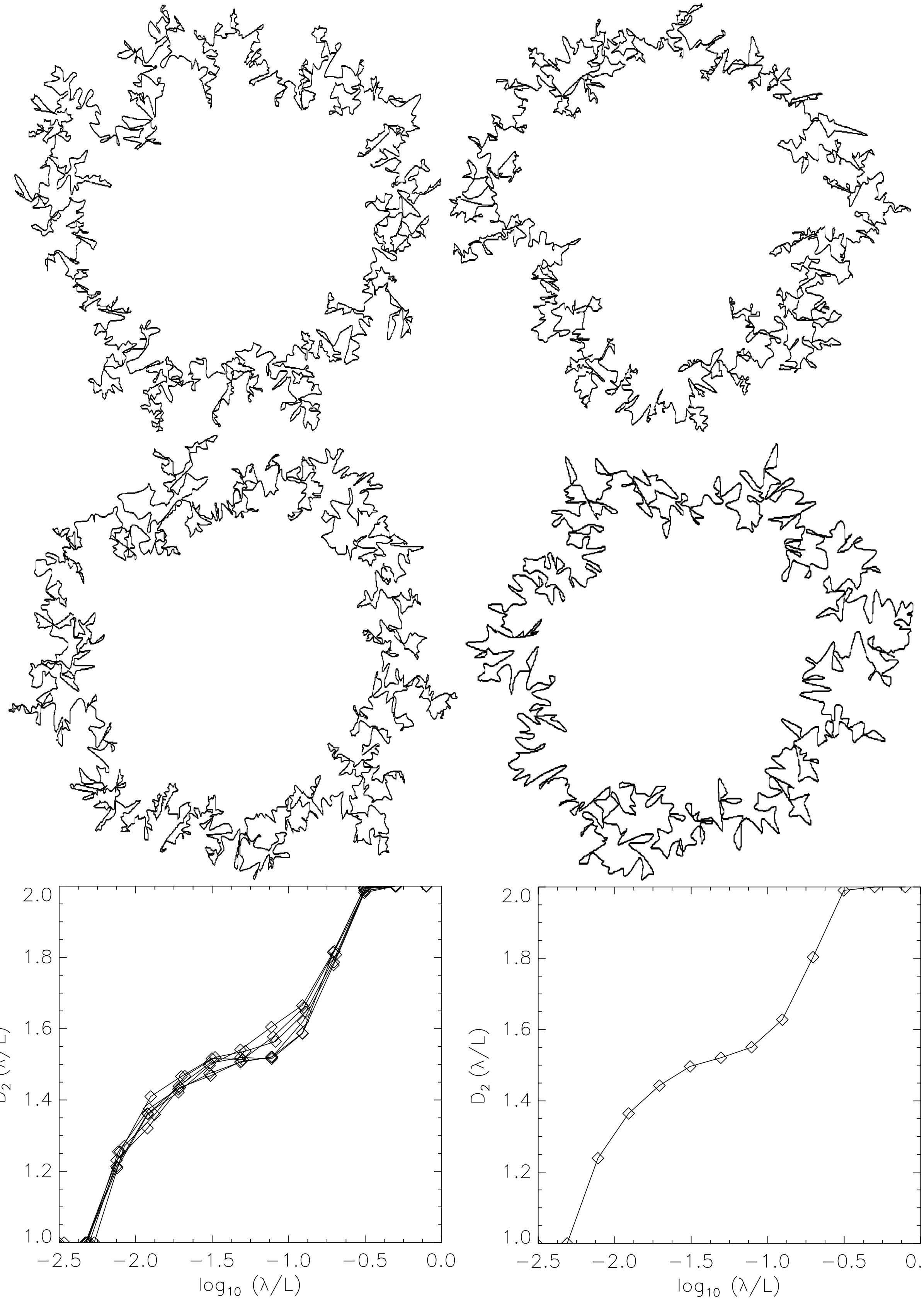

Figure 2: Top and middle: Full-generation synthetic interfaces. Bottom: Generalized fractal dimension vs. scale for several interfaces (left) and ensemble average (right).

Using the synthetic generation technique developed above, multiscale interface modeling and generalized fractal computational analysis were conducted for an exponential statistical distribution. A meshless algorithm, based on the method pioneered by Catrakis [8], was utilized in order to accurately and meshlessly compute the generalized fractal dimension as a function of scale. The results are shown in figure 2 which shows several examples of full-generation synthetic interfaces (top and middle) as well as the generalized fractal dimension as a function of scale for individual synthetic interfaces (bottom left) as well as for the ensemble-averaged behavior (bottom right). The behavior of the generalized fractal dimension is very similar to experimental results [2]. This supports the viability of the present computational modeling technique for generating synthetic turbulent interfaces with multiscale physical properties that simulate the observed behavior.

4. Conclusions

We have develop a methodology for the synthetic computational multiscale modeling of turbulent interfaces by successive geometrical deformations, starting with a purely large scale structure and generating smaller-scale deformations that ensure smoothness of the interface with scale dependence. We have demonstrated and validated the method for two-dimensional interfaces with an exponential distribution of scales and azimuthal symmetry. Modeling of such interfaces is useful in engineering applications and scientific phenomena where physical processes occur on or across fluid interfaces, such as molecular diffusion or electromagnetic wave propagation.

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