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Conservative Averaging Method and its Application for One Heat Conduction Problem

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Abstract: - In this paper the description of the method of conservative averaging for partial differential equations in two-layer domain is given. As numerical example the non-linear heat conduction in electrical wire with insulation is considered. This problem is solved numerically.

Key-Words: - partial differential equations, discontinuous coefficients, conservative averaging, non-classical boundary conditions, heat conduction, electrical wire, insulation temperature.

1 Introduction

Conservative averaging method was developed for partial differential equations with discontinuous coefficients. Because of its publication in non-English language and hard accessible papers [1] [2], we give here a short general conception of the method. Application of this method to heat conduction in wire with insulation is demonstrated.

2 Description of Conservative Averaging Method for Rectangular Two-layer Domain in Cartesian coordinates

Size of this paper allow us to describe this method for second type boundary condition (5) only.

2.1 Initial Problem

Let us look at domain D where

$(x, y) \in \underline{D} \subset R^{n+1}$, $x \in R$, $y = (y_1, ..., y_n) \in R^n$.

Domain D consists of several sub domains (for simplicity we can image $(n+1)$ – dimensional parallelepiped, see Fig.1):

$$D = G_0 \cup G \cup H$$

$$G_0 = \{(x, y) \mid x \in (-\delta, 0), y \in D\}$$

$$G = \{(x, y) \mid x > 0, y \in D\}$$

With notation $x > 0$ we understand that domain G is located to the right from domain G_0. Shared border-hyper plane of domains G_0 and G we denote as H:

$$H \equiv \underline{G}_0 \cap \underline{G} = \{(x, y) \mid x = 0, y \in D\}.$$

Right border of the domain G_0 we denote as H_0:

$$H_0 = \{(x, y) \mid x = -\delta, y \in G_0\}.$$

We will use notation $x = 0$ and $x = -\delta$ for hyper planes H and H_0.

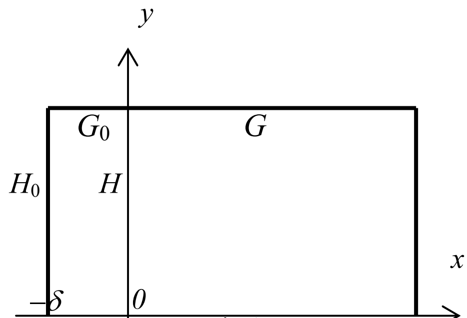

Fig.1

Objective is to find function $U_0(x,y)$ (continuous in domain $\underline{G}_0$) and function $U(x,y)$ (continuous in domain $\underline{G}$) that fulfills following equations:

a) differential equations:

$$\frac{\partial}{\partial x}\left(k_0 \frac{\partial U_0}{\partial x}\right) + L^0(U_0) = -F_0(x, y), \tag{1}$$

$$\frac{\partial}{\partial x}\left(k \frac{\partial U}{\partial x}\right) + L(U) = -F(x, y), \tag{2}$$

where L and L_0 are differential operators;

b) conjugation conditions:

$$U_0\Big|_{x=0} = U\Big|_{x=0}, \tag{3}$$

$$k_0 \frac{\partial U_0}{\partial x}\Big|_{x=-0} = k \frac{\partial U}{\partial x}\Big|_{x=+0}; \tag{4}$$

c) boundary condition on border H_0:

$$-k_0 \frac{\partial U_0}{\partial x}\Big|_{x=-\delta} = \varphi^0(y); \tag{5}$$

d) boundary conditions on the rest borders that are not necessary to concretize at this moment:

$$\tilde{l}(\tilde{U}) = \tilde{\Psi}(x,y),\ (x,y) \in \partial\tilde{D} = \partial D \setminus H_0. \quad (6)$$

Here continuous on $\bar{D}$ (see (3)) function $\tilde{U}(x,y)$ is defined to be equal to function $U_0(x,y)$ in domain $\overline{G_0}$ and – to be equal to $U(x,y)$ in domain $\overline{G}$.

We require that all derivatives of equation (1) are continuous in domain G_0 and derivatives of equation (2) – in domain G. Derivative $\partial\tilde{U}/\partial x$ is continuous function in domain D except hyper plane H where it has the first kind discontinuity and condition (4) is fulfilled. Note that value δ is comparatively small but still finite – averaging will be not made by passage to the limit $\delta \to 0$.

We assume that coefficient k_0 and coefficients of the differential operator L^0 are not dependent on argument x but could be dependent on argument y. Besides, operator L^0 is linear and it do not contain derivatives regarding the argument x.

Let us see two examples of operators L and L^0:

1) operators for 1D heat transfer problem:

$$L^0(U_0) = -c_0\rho_0 \frac{\partial U_0}{\partial t}, \quad (7)$$

$$L(U) = -c\rho \frac{\partial U}{\partial t}; \quad (8)$$

2) operators for steady-state 3D heat transfer:

$$L^0(U_0) = \frac{\partial}{\partial y}\left(k_1^0 \frac{\partial U_0}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_2^0 \frac{\partial U_0}{\partial z}\right), \quad (9)$$

$$L(U) = \frac{\partial}{\partial y}\left(k_1 \frac{\partial U}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_2 \frac{\partial U}{\partial z}\right). \quad (10)$$

Such estimate is valid:

$$\max_{x\in[-\delta,0]} \left|\frac{\partial U_0}{\partial x}\right| \le \frac{k}{k_0} \left|\frac{\partial U}{\partial x}\right|_{x=+0} + \delta \max_{x\in[-\delta,0]} \left|\frac{\partial^2 U_0}{\partial x^2}\right|$$

It shows that values of the solution U_0 changes comparatively little in the direction of argument x if coefficient k_0 is significantly larger than k or if δ is small (or both conditions take place). That means that function $U_0(x,y)$ could be approximately substituted by constant regarding argument x with accuracy $O\left(\frac{k}{k_0}\delta + \delta^2\right)$. Otherwise substitution by constant in the x –direction could be inappropriate – polynomial or exponential approximation should be used.

2.2 Transformed problem with non-classical boundary condition

We will convert differential equation (1) to non-classical boundary condition for the equation (2). It means that we will have other one problem instead of the problem (1)-(6). To make difference between these two problems clearer, new solution of the equation (2) we denote as $u(x,y)$ instead of the function $U(x,y)$.

We rewrite equation (2):

$$\frac{\partial}{\partial x}\left(k\frac{\partial u}{\partial x}\right) + L(u) = -F(x,y),\ (x,y) \in G \quad (11)$$

and introduce the integral averaged value function:

$$u_0(y) = \frac{1}{\delta}\int_{-\delta}^{0} U_0(x,y)dx. \quad (12)$$

After integrating equation (1) over the segment $[-\delta,0]$ and taking into account conjugation condition (4), we get:

$$k\left.\frac{\partial u}{\partial x}\right|_{x=+0} - k_0\left.\frac{\partial U_0}{\partial x}\right|_{x=-\delta} + \delta L^0(u_0) = -\delta f_0, \quad (13)$$

where $f_0(y) = \frac{1}{\delta}\int_{-\delta}^{0} F_0(x,y)dx$. (14)

Equation (13) is not a valid boundary condition yet because it has two unknown functions: $U_0(-\delta,y)$ and $u_0(y)$. We use boundary condition (5) to exclude function U_0 from the equation (13):

$$k\left.\frac{\partial u}{\partial x}\right|_{x=+0} + \delta L^0(u_0) = -(\varphi^0 + \delta f_0). \quad (15)$$

2.2.1 Approximation with constant

To move forward, we need to approximate function $U_0(x,y)$ in respect to argument x. Accuracy of this depends on particular task. We start with simplest one – approximation with constant. Taking into account expressions (3) and (12), we get:

$$U_0(x,y) = u_0(y) = u(0,y). \quad (16)$$

New boundary condition is found if we put this equality into equation (15):

$$k\left.\frac{\partial u}{\partial x}\right|_{x=+0} + \delta L^0\left(u|_{x=0}\right) = -(\varphi^0 + \delta f_0). \quad (17)$$

So the transformed problem consists of differential equation (11), non-classical boundary condition (17) on the border H and original boundary conditions (6) on the other borders. New problem is defined in the sub domain G of the initial domain D.

To be correct we need also to add boundary condition on the border ∂H:

$$l^0(u) = \psi_0(y),\ y \in \partial H, \tag{18}$$

where operator l^0 is defined in domain $\partial\widetilde{D} \cap G_0$ and is equal to operator $\widetilde{l}$ here; ψ_0 is analogue of averaged value integral (12) for the function Ψ_0. Ψ_0 in its turn is G_0 part of the function $\tilde{\Psi}$.

If φ^0 is not identical to zero then boundary condition (5) and assumption of independency of x are true only for $k_0 = \infty$. That means that approximation of argument x of function U_0 by constant is applicable when values of coefficient k_0 is relevantly large.

2.2.2 Linear Approximation

If we apply linear approximation to the function $U_0(x,y)$ in direction of x axis

$$U_0(x,y) = u_1(y) + xu_2(y) \tag{19}$$

and use conjugation condition (3) and boundary condition (5), we get

$$U_0(x,y) = u(0,y) + \frac{x}{k_0}\varphi^0(y). \tag{20}$$

Now we apply integral (12) to the expression (20) and put the result into equation (15) to get boundary condition on the border H:

$$k\left.\frac{\partial u}{\partial x}\right|_{x=+0} + \delta L^0\left(u|_{x=0}\right) = -\varphi_1, \tag{21}$$

where $\varphi_1 = \varphi^0 + \delta f_0 + \frac{1}{2}\delta^2 L^0\left(\varphi^0/k_0\right)$. (22)

After finding solution $u(x,y)$ we can use formula (20) to restore function $U_0(x,y)$ approximately.

2.2.3 Approximation with polynomial of the second order

We assume such representation for the function U_0:

$$U_0(x,y) = u(0,y) + \frac{x}{\delta}u_1(y) + \left(\frac{x}{\delta}\right)^2 u_2(y). \tag{23}$$

Conjugation condition (4) then gives:

$$k\left.\frac{\partial u}{\partial x}\right|_{x=+0} = \frac{k_0}{\delta}u_1(y) \tag{24}$$

From the condition (5) and integral (12), we get:

$$\frac{k^0}{\delta}\left(2u_2 - u_1\right) = \varphi^0, \tag{25}$$

$$u_0 = u|_{x=0} - \tfrac{1}{2}u_1 + \tfrac{1}{3}u_2. \tag{26}$$

After writing down u_1 and u_2 in terms of u_0, we can transform equation (24) in the following form:

$$k\left.\frac{\partial u}{\partial x}\right|_{x=+0} = \frac{3k_0}{\delta}\left(u|_{x=0} - u_0\right) + \frac{\varphi^0}{2}. \tag{27}$$

Now we need only to express u_0 from this equation and put it into (15) to get boundary condition on the border H:

$$k\frac{\partial u}{\partial x} + \delta L^0\left(u - \frac{\delta k}{3k_0}\frac{\partial u}{\partial x}\right) = -\varphi_2 \tag{28}$$

where $\varphi_2 = \varphi^0 + \delta f_0 + \frac{1}{6}\delta^2 L^0\left(\varphi^0/k_0\right)$. (29)

For the calculating purposes it could be useful to leave boundary condition in the form of system. Then it would contain equations (27) and (15). If we put formula (27) into (15), we could use following form for the second equation of the system as well:

$$\delta L^0\left(u_0\right) + \frac{3k_0}{\delta}\left(u|_{x=0} - u_0\right) = -\left(\tfrac{3}{2}\varphi^0 + \delta f_0\right). \tag{30}$$

We can also use formulas (27) and (30) together as the system of equation on the boundary $x = 0$.

2.2.4 Exponential Approximation

We could assume another representation for function U_0:

$$\begin{aligned} U_0(x,y) &= u(0,y) + (e^{-qx/\delta} - 1)u_1(y) + \\ &+ (1 - e^{qx/\delta})u_2(y) \end{aligned} \tag{31}$$

where parameter $q > 0$ is arbitrary (positive) constant. It could be freely chosen taking into account physical and geometrical properties of particular problem. Simplest way is to take $q = 1$.

We get boundary condition on the border H using similar mathematical modification as in previous case:

$$k\left.\frac{\partial u}{\partial x}\right|_{x=+0} + \delta L^0\left(u|_{x=0} - C_1\frac{\delta k}{k_0}\left.\frac{\partial u}{\partial x}\right|_{x=+0}\right) = -\varphi_3 \tag{32}$$

where $\varphi_3 = \varphi^0 + \delta f_0 + C_2\delta^2 L^0\left(\dfrac{\varphi^0}{k_0}\right)$. (33)

Constants C_1, C_2 depend on choice of parameter q:

$$C_1(q) = \frac{e^{2q}(q-1) + q + 1}{q^2\left(e^{2q} - 1\right)}, \tag{34}$$

$$C_2(q) = \frac{e^{2q} - 2qe^q - 1}{q^2\left(e^{2q} - 1\right)}. \tag{35}$$

Again, we can use system on the boundary $x = 0$. First equation of the system is (15) and the second is

$$k\left.\frac{\partial u}{\partial x}\right|_{x=+0} = \frac{k_0}{C_1\delta}\left(u|_{x=0} - u_0\right) + \frac{C_2}{C_1}\varphi^0, \tag{36}$$

or

$$\begin{aligned} &\delta L^0\left(u_0\right) + \frac{k_0}{C_1\delta}\left(u|_{x=0} - u_0\right) = \\ &= -\left(\left(1 + \frac{C_2}{C_1}\right)\varphi^0 + \delta f_0\right) \end{aligned} \tag{37}$$

3 Conservative Averaging Method in Cylindrical Coordinates

3.1 Initial Problem

Let us take domain $D = G_0 \cup G \cup H$ with points $(r, y) \in \overline{D} \subset R^4$, $r \in R$, $y = (\varphi, z, t) \in R^3$.

$$G_0 = \{(r, y) \mid r \in (\tilde{r}_0, r_0), y \in D\},$$
$$G = \{(r, y) \mid r \in (r_0, r_1), y \in D\},$$
$$H \equiv \overline{G}_0 \cap \overline{G} = \{(r, y) \mid r = r_0, y \in D\},$$
$$H_0 = \{(r, y) \mid r = \tilde{r}_0, y \in G_0\}.$$

Argument r represent radius in this case. Similarly as in previous section, we denote domain (surface) H_0 as $r = \tilde{r}_0$ and domain H – as $r = r_0$.

We have

a) differential equations for domains G_0 and G:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk_0\frac{\partial U_0}{\partial r}\right) + L^0(U_0) = -F_0(r, y), \quad (38)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial U}{\partial r}\right) + L(U) = -F(r, y); \quad (39)$$

where L^0 is linear differential operator that is not dependent on argument r;

b) conjugation conditions:

$$U_0\big|_{r=r_0} = U\big|_{r=r_0}, \quad (40)$$

$$k_0\frac{\partial U_0}{\partial r}\bigg|_{r=r_0-0} = k\frac{\partial U}{\partial r}\bigg|_{r=r_0+0}; \quad (41)$$

c) boundary condition at $r = \tilde{r}_0$:

$$\frac{\partial U_0}{\partial r}\bigg|_{r=\tilde{r}^0} = \varphi_0(y). \quad (42)$$

Condition in such form is used when domain is a ring or cored cylinder. Boundary condition is homogeneous for $\tilde{r}_0 = 0$:

$$\frac{\partial U_0}{\partial r}\bigg|_{r=0} = 0; \quad (43)$$

d) boundary conditions on other borders:

$$\tilde{l}(\tilde{U}) = \tilde{\Psi}(r, y), \ (r, y) \in \partial\tilde{D} = \partial D \setminus H_0 \quad (44)$$

where function $\tilde{U}(r, y)$ is defined similarly as in previous section.

3.2 Transformed problem

Let us apply conservative averaging method to the subdomain G_0 that is located closer to the center of the domain D. Solution of transformed problem we denote as $u(r, y)$. It is defined in the domain G. We rewrite equation (40) with new notation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial u}{\partial r}\right) + L(u) = -F(r, y). \quad (45)$$

Again, we introduce integral averaged value function that is different from equation (12) because of cylindrical coordinates:

$$u_0(y) = \frac{2}{r_0^2 - \tilde{r}_0^2}\int_{\tilde{r}_0}^{r_0} rU_0(r, y)dr. \quad (46)$$

For simplicity we will consider only the case $\tilde{r}_0 = 0$ in further transformations.

We integrate equation (38) over the segment [0, r_0] and take into account conjugation condition (41):

$$\frac{2k}{r_0}\frac{\partial u}{\partial r}\bigg|_{r=r_0+0} + L^0(u_0) = -f_0(y), \quad (47)$$

where $f_0(y) = \dfrac{2}{r_0^2}\displaystyle\int_0^{r_0} rF_0(r, y)dr$. (48)

We will use equality (47) to obtain non-classical boundary condition for the equation (45) on the border $r = r_0$.

3.2.1 Approximation with constant

Let us assume that function U_0 is constant over argument r. Taking into account conjugation condition (40), we find boundary condition on the border $r = r_0$:

$$\frac{2k}{r_0}\frac{\partial u}{\partial r}\bigg|_{r=r_0+0} + L^0(u\big|_{r=r_0}) = -f_0(y). \quad (49)$$

Approximation with linear function leads to the same equation because of boundary condition (43) at the point $r = 0$.

3.2.2 Approximation with polynomial

We assume such representation for the function U_0:

$$U_0(r, y) = u_1(y) + ru_2(y) + r^2u_3(y). \quad (50)$$

Conjugation conditions (40), (41) and boundary condition (43) allow us to find unknown functions u_1, u_2, u_3. This allows to rewrite the expression for the temperature U_0 as follows:

$$U_0(r, y) = u_0 + \left(\frac{2r^2}{r_0^2} - 1\right)\left(u\big|_{r=r_0} - u_0\right). \quad (51)$$

Here $u_0(y) = u\big|_{r=r_0} - \dfrac{r_0k}{4k_0}\dfrac{\partial u}{\partial r}\bigg|_{r=r_0+0}$. (52)

The equality (47) gives the first boundary equation:

$$\frac{8k_0}{r_0^2}\left(u\big|_{r=r_0} - u_0\right) + L^0(u_0) = -f_0. \quad (53)$$

Derivation of function (51) by argument r and putting the result into conjugation condition (41) gives the second equation on the boundary $r = r_0$:

$$k\left.\frac{\partial u}{\partial r}\right|_{r=r_0+0} = \frac{4k_0}{r_0}\left(u|_{r=r_0} - u_0\right). \tag{54}$$

The new formulation of the problem consists of differential equation (45), boundary conditions (53), (54) on the border $r = r_0$ and original boundary conditions on the other borders of the domain G. After solving this problem we can find approximation for the function $U_0(r, t)$ from (51).

3.2.3 Exponential Approximation

We assume following representation for function U_0:

$$U_0(r, y) = u_1(y) + (e^{-r/r_0} - 1)u_2(y) + \\ + (1 - e^{r/r_0})u_3(y) \tag{55}$$

We can obtain unknown functions u_1, u_2, u_3 and write down function U_0 after using boundary condition (43) and both conjugation conditions:

$$U_0(r, y) = u|_{r=r_0} + \frac{r_0 k}{\sinh(1)k_0}\left.\frac{\partial u}{\partial r}\right|_{r=r_0+0} \times \\ \times\left(\cosh(r/r_0) - \cosh(1)\right) \tag{56}$$

Integral averaged value function in this case is given by expression

$$u_0(y) = u|_{r=r_0} - \frac{r_0 k}{\alpha_0 k_0}\left.\frac{\partial u}{\partial r}\right|_{r=r_0+0}, \tag{57}$$

where constant $\alpha_0 = \left(e^2 - 1\right)/\left(e^2 - 4e + 5\right)$. (58)

This is the first equation on the border $r = r_0$. We find the second one by using equality (47):

$$\frac{2\alpha_0 k_0}{r_0^2}\left(u|_{r=r_0} - u_0\right) + L^0(u_0) = -f_0. \tag{59}$$

After finding solution we can approximately reconstruct values in the domain G_0 by using formula (56). Function U_0 could be expressed also in terms of u_0 that is more convenient for calculation purposes:

$$U_0(r, y) = u|_{r=r_0} + \beta_0(r)\left(u|_{r=r_0} - u_0\right) \tag{60}$$

where $\beta_0(r) = \dfrac{2e}{e^2 - 4e + 5}\left(\cosh\dfrac{r}{r_0} - \cosh(1)\right)$. (61)

4 Computation of Heat Transfer in Cylindrical Wire with Insulation

Calculation of heat transfer and heat emission in the electrical wires is a vital issue in many industries. Electric circuitry is used in cars, houses, consumer electronics, etc. Damage and accidents are possible if insulation melts because of heightened temperature in the wires. Thermal properties of conductor and insulation significantly differ that makes calculation of critical temperatures more difficult. Besides, wires frequently are combined in bundles. In this section, a single round wire is considered. Since length of the wire is much larger than diameter, 1D model is used as in [3] and [4].

4.1 Statement of the Problem

We have two-layer domain that consists of conductor and insulation. With r_0 we denote radius of metallic wire, with r_1 – radius of the wire with insulation. We have

a) heat conduction equation for conductor $r \in (0, r_0)$:

$$c_0\frac{\partial U_0}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rk_0\frac{\partial U_0}{\partial r}\right) + F_0(r,t), \tag{62}$$

and homogeneous non-linear heat conduction equation for the insulation $r \in (r_0, r_1)$:

$$c\frac{\partial U}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial U}{\partial r}\right); \tag{63}$$

b) conjugation conditions at $r = r_0$:

$$U_0|_{r=r_0} = U|_{r=r_0}, \tag{64}$$

$$k_0\left.\frac{\partial U_0}{\partial r}\right|_{r=r_0-0} = k\left.\frac{\partial U}{\partial r}\right|_{r=r_0+0}; \tag{65}$$

c) boundary conditions:

$$r = 0:\ \left.\frac{\partial U_0}{\partial r}\right|_{r=0} = 0; \tag{66}$$

$$r = r_1:\ k\left.\frac{\partial U}{\partial r}\right|_{r=r_1} + h(U|_{r=r_1} - \theta) = 0 \tag{67}$$

d) initial conditions:

$$t = 0:\ U_0 = \theta,\ U = \theta. \tag{68}$$

Here c_0, c, k_0, k, h – coefficients which correspond to heat capacity, heat conductivity and heat exchange; θ – temperature of environment.

4.2 Transformed problem

We use conservative averaging method to exclude wire from definition domain – we inspect heat transfer only in the insulation. Calculation of temperature in the wire was discussed also in paper [3], where authors used finite volume method for both regions – conductor (wire) and insulation.

The new solution transformed by conservative averaging method we denote as $u(r, t)$. This statement of problem consists of differential equation (63) (replacing U by u), boundary condition (67), initial condition (68) and boundary condition on the border $r = r_0$ that we obtain from previous section – formula (49) if we use approximation by constant, formulas (53)(54) for polynomial approximation and formulas (57)(59) – for exponential one.

4.3 Numerical solution

Since it is not possible to obtain analytical solution for the stated problem because of non-linearity, we solve it in numerical way. We constructed standard difference scheme with second order approximation regarding to r and first order approximation for t.

As an example we take wire of copper with poly-vinylchloride (PVC) insulation. Coefficients in this case are following:

a) heat conductivity coefficient k_0 and k (W/m K)

$$k_0 = 401, \ \ k = 0.2;$$

b) specific heat capacity coefficient c_0, c (W/kg K)

$$c_0(T) = 381 + 0.17T,$$

$$c(T) = 920 - 1.3T + 0.074T^2;$$

c) heat generation is achieved by electric current; only direct current and ohm resistance is considered:

$$f_0(T) = \frac{I^2 \rho_{env}}{A^2}\left(1 + \alpha_\rho (T - \theta)\right)$$

where I – electric current, A – cross sectional area of the metallic conductor, α_ρ – copper resistance

$\alpha_\rho = \alpha_{20} / (1 + \alpha_{20}(\theta - 20))$, $\alpha_{20} = 3.9 \cdot 10^{-3}$,

$\rho_{env} = \rho_{20}\alpha_{20} / \alpha_\rho$, $\rho_{20} = 1.75 \cdot 10^{-8}$,

ρ_{20} – specific resistance of copper at reference temperature 20°C;

d) heat convection coefficient h(T) was calculated using given computer subroutine for laminar unforced convection to air of horizontal cylindrical surface. Range of coefficient's values are 3–20 if temperature range is 0–100°C and temperature of environment θ = 0°C. All mentioned coefficients are valid in the temperature range of our interest. Temperature is given in centigrade degrees.

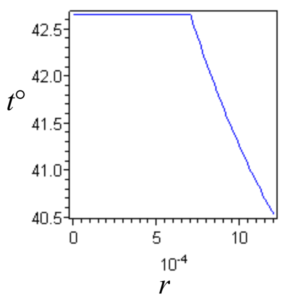

Fig. 1

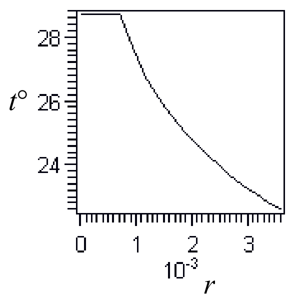

Fig. 2

We can observe temperature distribution after it become stable in Fig. 1 if we use following values for parameters: $r_0 = 0.7 \cdot 10^{-3}$ m, $r_1 = 1.2 \cdot 10^{-3}$ m, $\theta = 0$ °C, $I = 20$ A.

Graphics of solutions matches even if we use different approximations for function U_0. Difference starts only by fifth significant digit (see Table).

	$r = 0$
Constant approx.	42.6638
Polynomial approx.	42.6648
Exponential approx.	42.6647

Note that conservative averaging method does not make solution U linear as it is visually observed in Fig. 2. As an example, we can take outer radius 3 times larger: $r_1 = 3.6 \cdot 10^{-3}$ (Fig. 2). We also numerically verified that we could choose such coefficients for stated problem that choice of approximation of U_0 by polynomial or exponential instead of constant is significant. But such parameters are useless from practical point of view.

5 Conclusions

Conservative averaging method can be applied to stationary and non-stationary problems. Constant approximation is sufficient in many practical heat transfer problems for the one material with good heat conductivity.

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