Calculation of 2D Velocity Field in Supercavitating Flow by Combination of Slender Body Theory and BEM

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Abstract: - In this paper, 2D potential flow field caused by a supercavitating slender body has been studied using Boundary Element Method. The profile of the supercavity interface determines the boundary which separates liquid phase from gas phase, has been obtained from Pellone et al's study. Achieving to a constant pressure on the supercavity using BEM shows that, the supposed cavity profile can stand for a real supercavity interface. Calculated cavitation numbers corresponding with non-dimensional length of cavity shows that the closure point's length for cavity differs a little from Pellone et al's results.

Key-Words: - Two Dimensional, Potential Flow, Boundary Element Method, Supercavitating flow, Slender Body Theory.

1 Introduction

Cavity flows are characterized by the coexistence of two phases, liquid and vapor. At sufficiently highspeeds, cavitation will occur on the surface of submerged bodies at the point where the local pressure drops below the value of vapor pressure of the fluid. Supercavitation is a level of cavitation where cavitation number reduces and big bubble appears behind the cavitator. Initial Modeling of supercavity based on free streamline theory was done by Riabouchinsky [1], Plesset [2] and Wu [3]. Brennen [4] employed a relaxation method in a transformed velocity potential-stream function plane for analyzing axisymmetric cavitating flows behind a disk and also a sphere between solid walls. Slender body theory is an approach to study this phenomenon. Varghas et al [5], Pellone et al [6] and Tulin [7] can be mentioned to have worked in this field. In recent years the most researches on supercavitating flows are done using Boundary Elements Method (BEM). Nonlinear boundary element models were developed for cavitating flows about hydrofoils by Uhlman [8,9], Kinnas, Fine, Lee and Young [10,11,12,13], among others. They distributed sources and normal dipoles along the body-cavity surface. The unknown values of these sources and dipoles were determined by imposing the dynamic condition on a presumed cavity boundary. The kinematic boundary condition was then used to update the cavity shape. J. H. Chen [14,15] introduced a semi-direct method to find the cavity shape caused by a two-dimensional airfoil in

which the cavitation number is prescribed and then cavity length is the output. Varghese [16] studied partially cavitating axisymmetric flow behind a disk in the case that cavity closure was located in different part of solid body using BEM. Haese [17] used interior source methods for modeling planer and axisymmetric supercavitating flows. Vaz [18] worked on modeling two-dimensional partial cavities using BEM. Their work begins with an initial guess for cavities boundary and then continues by updating this boundary in each iteration and regrinding the new cavity.

2 **Problem Formulation**

In this part, first the mathematical formulations which determines interface of supercavity caused by a slender body are presented. Also having cavity boundary's profile, bases of BEM formulation are described.

2.1 Evolution of a cavity interface

The model presented below also requires that the slenderness of the cavity is small enough. This is generally true for supercavitation since the cavity length increases more rapidly than its thickness as the cavitation number is decreased.

As mentioned above we are going to model a supercavity caused by a slender body in an infinite liquid medium. According to existence of a slender body we can consider that perturbation of velocities in x and y directions are small comparing with the free stream velocity. So as in Fig 1., for simplicity, we consider a symmetric configuration with respect to the plane y = 0.

The *v*-component on the cavity interface (denoted v_c) is deduced from the kinematic condition on the interface. It can be inferred that, if a fluid particle is on the interface at a given time, it will remain on it at any subsequent time until it reaches the closure region where it will separate from the cavity. Hence, v_c is given by:

$$v_c = \frac{\P y_c}{\P t} + U_{\Upsilon} \frac{\P y_c}{\P x} \tag{1}$$

where $y = y_c(x, t)$ is the equation of the cavity interface at time *t*. For a spherical bubble, the radial velocity behaves like $1/r^2$. Pellone et al [6] assumed that the *v*-component behaves like a given power $1/y^n$ of the distance *y* to the plane of symmetry. Considering this assumption, *v*-component will be

$$v(x,y,t) = v_c \frac{\overset{\circ}{e} y_c}{\overset{\circ}{e} y} \overset{\circ}{\mathbf{u}} = \overset{\circ}{e} \frac{\overset{\circ}{e} \mathbb{I} y_c}{\overset{\circ}{e} \mathbb{I} t} + U_{\underbrace{*}} \frac{\mathbb{I} y_c}{\mathbb{I} x} \overset{\circ}{\underline{u} \overset{\circ}{e} y} \overset{\circ}{\mathbf{u}}$$
(2)

They have shown that n is a function of cavitation number and it is always grater than 1. In a potential flow with assumption of slender body approximation, the momentum equation in ydirection is:

$$\frac{Dv}{Dt} = -\frac{1}{r} \frac{\P p}{\P y}$$
(3)

where p is the pressure, ρ the liquid density and D/Dt the material derivative given by:

$$D/Dt = \frac{\P}{\P t} + U_{\Upsilon} \frac{\P}{\P x}$$
(4)

Substituting Eq. 2 in momentum balance equation and after some mathematical manipulations, the cavity interface shape in the steady form is given by:

$$\frac{d^2(y_c^2)}{dx^2} = -(n-1)s$$
(5)

Where σ is the cavitation number defined by:

$$\boldsymbol{S} = \frac{\boldsymbol{p}_{\boldsymbol{\chi}} - \boldsymbol{p}_c}{\frac{1}{2}\boldsymbol{r}\boldsymbol{U}_{\boldsymbol{\chi}}^2} \tag{6}$$

Equation (5) has the following elliptic solution for the shape of the cavity interface:

$$\frac{y_c^2}{(n-1)(s_2)(l_4^2)} + \frac{(x-l_2)^2}{l_4^2} = 1$$
(7)

To determine n Pellone et al [7] compared this elliptic solution with Tulin's classical solution for the cavity flow around a symmetric body in an infinite flow field [8]. They found out that n should

be as follows.

$$n = 1 + \frac{\frac{s_2}{\left(1 + \frac{s_2}{2}\right)^2}}{\left(1 + \frac{s_2}{2}\right)^2}$$
(8)

2.2 BEM formulation

To find the velocity field caused by existence of a slender plus a supercavity behind that in an infinite fluid medium, Boundary Element Method has been used. In order to do this, first the potential on the boundary of solid body and supercavity is calculated. Having the potentials on the boundary, the velocity for any arbitrary point in the flow field can be found by integrating the effects of potential on boundary nodes and free stream velocity. The governing equation for planer potential flow is:

$$\hat{\mathbf{N}}\boldsymbol{j} = -U_{\boldsymbol{\chi}\boldsymbol{\chi}}.\hat{\boldsymbol{n}} \tag{9}$$

where φ is perturbation potential. U_{*} is the free stream velocity and \hat{n} is a unit normal vector outward to the boundary. This equation means that the normal flux through the boundary is zero. Therefore, total potential for any point in the flow field can be expressed as follows

$$\nabla f = U_{\infty} + \nabla j \tag{10}$$

Using the Green's identity and the two dimensional Green's function, the solution for the perturbation potential on a boundary point x satisfies the equation:

$$pj(x) = \int_{s} \left[j(x) \frac{\partial}{\partial n} \left[\ln r(x, \tilde{x}) \right] + \ln r(x, \tilde{x}) \frac{\partial j(x)}{\partial n} \right] ds$$
(11)

where $\frac{\partial}{\partial n}$ denotes differention along the outward direct normal to the boundary. In this equation the potential is considered as induced by source and dipoles distributions, with strengths $\frac{\partial j}{\partial n}$ and $-\varphi$, respectively [19].

3 Problem Solution

To solve this integral equation numerically we have to discrete the boundary to N panels. Then there will be a $N \times N$ system of equations. In this work the potential on the boundary elements has been considered to have constant value. The Gaussian elimination method was used to solve the system of equations. To find the pressure field, Bernoulli equation can be used.

$$P_{\infty} + \frac{1}{2} r U_{\infty}^{2} = P + \frac{1}{2} r U^{2}$$
 (12)

Where P_{∞} and U_{∞} are pressure and velocity at infinity and ρ is the density of fluid. U is the amount of velocity in the flow field which it's pressure is needed. According to Bernoulli's equation C_p can be expressed as follow:

$$C_p = 1 - \left(\frac{q}{U_{\infty}}\right)^2 \tag{13}$$

Where q is the magnitude of tangential velocity. Fig. 2 shows the C_p with respect to Non-Dimensional length of cavity in s = 0.1. One way to determine that if the profile used for the supercavity's boundary can be a real boundary for a cavity or not, is controlling the pressure on the boundary. In Fig. 2, it can be seen that C_p on the supercavity's boundary has a constant amount. Therefore the calculated profile for cavity's boundary from slender body theory can stand for a real cavity boundary.

The peak appeared in Fig. 2 around detachment point of cavity from solid body is because of BEM's numerical fault at singularities. In this point, slop is changed suddenly and therefore here is a singular point.

Comparing the results of this work in terms of the evolution of the cavity length with the cavitation number along Tulin's and Pellone et al's in Fig. 3, it is obvious that cavitation number which is calculated from BEM for a constant cavity length is lower than the two other results. In other view, results of this work comparing with the two available results, describe that, there will be a shorter length for a cavity in a constant cavitation number.

Fig. 4 indicates that the consideration of slender body for Supercavity flow was not wrong, because it is obvious that the perturbation velocity is small comparing to U_{∞} . The maximum velocity occurs at detachment point and after this point it remains constant till vicinity of the closures point of cavity.

4 Results and Discussion

There were some hypothesizes and simplifications to find cavity's interface profile using slender body theory, but according to the results depicted in Fig. 2, it is clear that there is not so much deviation from a constant pressure on cavity. So it is a logical reason to call the obtained profile, a cavity interface. When applying slender body theory, it should be considered that all the suppositions be satisfied. For example if the half vertex angle of symmetric wedge increase to more than 25 degrees, results for C_p will show much more deviation from being constant, and it is all because the cavitator's geometry is not a slender body so far.

According to Fig. 4, obtained velocity field shows that effects of body and cavity in flow field keeping out the way from them, decreases. Calculated flow field in closure of the cavity is not as same as what happens in real flow. In real physical supercavitating flow this part of cavity is located in a turbulent flow and is not stable. BEM manage this region as a part that, a stagnation point is located there; and so tending to this point velocity decrease to zero. As the closure of supercavity occurs in down stream of flow, the effects of turbulent flow existing in this region will not be so efficient on the flow field near the solid body. Hence, results taken near the cavitator are not far from the existing fact. Deviations between three different works in Fig. 3 can be explained for the same reason.

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Fig.1: Schematic of a supercavity caused by a slender body.



Fig. 2: C_p distribution on solid body and cavity.



Fig. 3: Comparison of the present work with Tulin's [7] solution and Pellone et al's [6] results for the evolution of the cavity length with the cavitation number. Case of a symmetrical wedge of chord length c and half vertex angle 8 degrees in an infinite flow field. The cavity length is non-dimensioned by the wedge chord length.



Fig. 4: Velocity field caused by a symmetric wedge with half vertex angle 8 degrees in an infinite flow field and a Supercavity behind it ($U_{x} = 5$).