Effect of mean radius to channel width ratio on the flow performance of Spiral Channel Viscous Micropump

A.T. AL-HALHOULI¹, M. I. KILANI², A. Al-SALAYMEH², and S. BÜTTGENBACH¹

¹Institute for Microtechnology Alte Salzdahlumer Str. 203, 38124 Braunschweig 38124 Braunschweig GERMANY

> ²Mechanical Engineering department University of Jordan Amman 11924 JORDAN

Abstract: - In this work, Analytical and numerical investigations for the effect of mean radius to channel width ratio (R_m/w) on the flow performance of a newly introduced spiral channel viscous micropump (Spiral CVMP) have been carried out. Drag and shape factors for the effect of R_m/w were also generated. For this purpose computational fluid dynamics (CFD) simulations using finite volume method (FVM) were performed, a number of 3D models for the pump geometry were built and analyzed at different boundary conditions. An analytical model estimating for the effect of mean radius to channel width ratio was also derived, and showed good agreement with the numerical simulations. It has been also found that the flow rate varies linearly with both the pressure difference and boundary velocity for a wide range of R_m/w , which supports the validity of the linear lubrication model for this problem for the full range of studied

parameter. Further, it is found that resulting error from ignoring $R_m / w \ge 1.0$ is less than 10%.

Key-Words: - micropumps, viscous flow, lubrication model

1 Introduction

The rate of progressing of the biology and medicine technologies is accompanied with further reductions in the systems size, which motivates efforts in developing miniaturized devices for pumping, controlling and/or manipulating small fluid volumes. Micropumps are one of the vital miniaturized devices have been used and implemented into the market and are also used in ink jet printers and fuel injection applications [1, 2, 3]. Recently, technology exists for the fabrication of devices for biological and medical applications on a scale level much smaller than that of the conventionally available technologies [4]. Efforts aiming at reaching an analytical understanding of the physical behavior of these devices are motivated by the opportunity for improving their performance, and their successful utilization in real-life applications. For devices with fluid flow functionality, the widespread use of computational fluid dynamics (CFD) simulation, and the availability of effective processing hardware have enabled an accurate solution of the governing nonlinear equations in many practical situations. This has lead to a substantial reduction in cost and in the development time for the proposed model [5].

Spiral CVMP idea depends on dragging the fluid along a spiral channel either by rotating the spiral channel disk against a stationary flat disk (moving condition) or rotating the flat disk against the spiral channel (stationary condition) as shown in Fig. 1. A net tangential viscous stress on the boundaries is built and produces a positive pressure gradient in the direction of flow [6].

In this work, the effect of mean radius to channel width ratio on the flow performance of a spiral CVMP were investigated analytically and numerically. 3D numerical models have been built and analyzed using FVM. Drag and pressure shape factors were also generated for a wide range of R_m/w .



Fig.1 illustration of spiral pump design

2 Analytical formulation

To study the effect of mean radius to channel width ratio (R_m/w) on the flow performance of a spiral CVMP, the flow is assumed to be dragged in a circular channel with the origin of the polar axis system is assigned to the center of the spiral coordinate system and a mean radius of R_m as shown in Fig. 2. The channel centerline of the spiral can be represented in polar coordinates by the linear Archemidean spiral as

$$r = k\theta + r_i, \ 0 \le \theta \le \Delta\theta \tag{1}$$

and the mean radius is defined as

$$R_m = \left(\frac{r_i + r_o}{2}\right) \tag{2}$$

where r_i , r_o are the initial and final spiral radiuses simultaneously, and $\Delta \theta$ is the angular span.

To solve for R_m/w , the flow field is assumed to be laminar, steady, and incompressible, with constant fluid properties. Navier-Stokes equation in cylindrical coordinates along the channel can be written as [7]:

$$\frac{1}{r}\frac{\partial p}{\partial \theta} = \mu \left\{ \frac{\partial}{\partial r} \left(\frac{1}{r}\frac{\partial}{\partial r} \left[rv_{\theta} \right] \right) + \frac{1}{r^2}\frac{\partial^2 v_{\theta}}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_{\theta}}{\partial z^2} \right\}$$
(3)

Where *P* is the pressure, μ is the dynamic viscosity, *r*, θ are the radial and tangential directions, and v_r, v_{θ} are the radial and tangential velocity components.

For low Reynolds number flows, at small spiral curvature and aspect ratios, (i.e. $k/r_o = 1.0$, and h/w = 1.0), the inertia and centrifugal forces can be neglected compared to the viscous stresses and the Navier-Stokes equation can be simplified as:

$$\frac{1}{r}\frac{\partial p}{\partial \theta} = \mu \left\{ \frac{\partial^2 v_\theta}{\partial z^2} \right\}$$
(4)



Fig. 2 Spiral CVMP model in cylindrical coordinates

Since the spiral channel is long compared to its width and height, and by considering the velocity along the channel as the channel boundary mean velocity, a fully developed flow in the θ -direction can be assumed, and the continuity equation can be written as:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) = 0.0 \tag{5}$$

the pressure gradient along the θ -direction also becomes

$$\frac{\partial p}{\partial \theta} \approx \frac{\Delta p}{\Delta \theta} \tag{6}$$

Introducing the non-dimensional parameters

$$z^* = \frac{z}{h}$$
, $r^* = \frac{r}{w}$ and define the non-dimensional

numbers: reduced Reynolds number: $\overline{\text{Re}} = \frac{\rho \overline{U} h^2}{l \mu}$, and

Euler number:
$$Eu = \frac{\Delta p}{\overline{U}^2 \rho}$$
.

Eq. 4. can be written as

$$\frac{d^2 v_{\theta}}{dz^{*2}} = \frac{1}{r^*} \frac{R_m}{w} \overline{\text{Re}}.Eu$$
(7)

This equation will be solved at the following boundary conditions:

$$v_{\theta}(0) = 0, v_{\theta}(1) = 1$$
 (8)

Solving for v_{θ} yields

$$v_{\theta} = \frac{1}{r^{*}} \frac{R_{m}}{w} \overline{\text{Re}} Eu(\frac{z^{*2}}{2} - z^{*}) - z^{*} + 1$$
(9)

The non-dimensional volumetric flow rate can be also calculated as

$$q^{*} = \frac{q}{\overline{U}.w.h} = \int_{0}^{1} \int_{r_{1}/w}^{r_{2}/w} v_{\theta}^{*} dr^{*} dz^{*}$$
(10)

This yield

$$q^* = \left[0.5 F_{dr} - \frac{1}{12} F_{pr} \overline{\mathrm{Re}} . E u \right]$$
(11)

Where F_{dr} and F_{pr} are the drag and pressure mean radius to channel width ratio shape factors and are defined as

$$F_{dr} = 1.0$$
 (12)

and
$$F_{pr} = \frac{R_m}{w} \ln \left(\frac{2 \left(\begin{array}{c} R_m \\ w \end{array} \right) - 1}{2 \left(\begin{array}{c} R_m \\ w \end{array} \right) + 1} \right)$$
 (13)

3 Numerical simulations

Three dimensional CFD numerical simulations have been carried out using FVM. Different discretization schemes were analyzed to determine the appropriate grid for simulating the flow field. The structured grid scheme with hexahedral elements was identified to be the most suitable for the present study, as shown in Fig. 3. The process of meshing began by dividing the height edges to be two times the number of their units, while the width edges were divided into divisions equal to the number of width units with 1.206 double sided ratios. The edges of the circumferential lines were divided to units equal to the angular span of the channel, these units

were scaled later such that 1 unit length scaled to $1 \ \mu m$. The spiral edge of the moving wall was then divided into units equal to the angular span angle, and the volume then meshed with the hexahedral elements.



Fig. 3 Numerical flow model at $R_m / w = 5.0$

Grid independent solution was assured by observing the outlet mass fluxes and the velocity components through the micropump channel. The obtained numerical models were exported to the CFD program, where flow analysis was performed and results were reported.

Laminar, steady, viscous flow model was used for this analysis. SAE10W30 motor oil with 825 kg/m^3 density and 0.0901 kg/m.s viscosity were defined as the interior fluid. In the boundary conditions menu the values for the outlet pressure and moving walls were assigned for each case. The Simple-Consistent algorithm was used for the pressure velocity coupling, a second order upwind scheme was used for the momentum equations while a second order pressure interpolation scheme was used for pressure.

At the beginning, several runs for the moving and stationary wall conditions were performed, the same slope for the change of the flow rate against pressure drop were obtained, and the simulations were considered for the stationary condition.

4 **Results and discussion**

Analytical and numerical flow rates in dimensionless forms for the spiral CVMP at different R_m/w , constant aspect ratio h/w = 0.1, where the drag and pressure aspect ratio shape factors are $F_{da} = 0.95$, and $F_{pa} = 0.9568$ simultaneously [8] and at small spiral curvature ratios $k/r_o < 0.16$ are shown in Fig. 4.



Here, the non-dimensional flow rates have been estimated by considering the effect of channel aspect ratio through writing Eq.11 including the aspect ratio shape factors as in the following form:

$$q^{*} = \left[0.5 F_{da} F_{dr} - \frac{1}{12} F_{pa} F_{pr} \overline{\text{R e}} \cdot E u \right]$$
(14)

Results showed that the linear trend is repeated for all values R_m/w , indicating that the flow rate decreases linearly with increasing the pressure difference. The error in calculating the flow rates at different R_m/w are summarized in table 1. The error was found to be less than 10% for $R_m/w \ge 1.0$.

R_m / w	Re.Eu	$q^{*}_{numerical}$	$q^{*}_{\it analytical}$	% error
0.55	0.0	0.475	0.475	0.0
	1.50	0.296	0.275	7.6
	3.00	0.116	0.0745	56.1
0.6	0.0	0.475	0.475	0.0
	1.5	0.315	0.303	3.91
	3.0	0.154	0.131	18.1
0.75	0.0	0.473	0.475	0.4
	1.5	0.332	0.331	0.406
	3.0	0.191	0.186	2.46
	4.5	0.0497	0.0419	18.7
1.0	0.0	0.471	0.475	0.8
	1.5	0.341	0.344	0.806
	3.0	0.210	0.212	0.82
	4.5	0.0801	0.0808	0.878
2.0	0.0	0.472	0.475	0.6
	1.5	0.348	0.353	1.43
	3.0	0.223	0.231	3.14
	4.5	0.099	0.108	8.7
5.0	0.0	0.472	0.475	0.6
	1.5	0.349	0.355	1.7
	3.0	0.226	0.235	3.93

Table 1. Effect of R_m / w on flow rate

The effect of mean radius to channel width ratio on the pressure shape factors is plotted for the analytical solution and numerical results as shown in Fig. 5. The value of F_{pr} decreases with increasing R_m/w , and goes to unity at high values of R_m/w ratios. Numerical results showed good agreement with the analytical results. In case of the drag shape factor, where the theoretical results yields a constant value of unity, numerical results are also in good agreement with error less than 0.8%.



Fig. 5. Effect of R_m / w on the pressure shape factor

5 Conclusions

From above results, it can be concluded that the analytical model derived can be used to estimate for the flow performance of the spiral CVMP at a wide range of mean radius to channel width ratios with an error less than 10% for $R_m / w \ge 1.0$. This model showed also a very good agreement with the numerical results. This model can be extended to estimate for the effect of channel aspect ratios on the flow field.

Acknowledgment

The authors wish to acknowledge the support received from the Institute for Microtechnology (IMT), TU-Braunschweig, and the Deanship for Scientific Research in the University of Jordan. The scholarship for the first author by the German Academic Exchange Service (DAAD) is gratefully appreciated.

References:

- P. Woias, Micropumps-past, progress and future prospects, *Sensors and actuators B*, Vol.105, 2005, pp. 28-38.
- [2] D. J. Laser and J. G. Santiago, A review of micropumps, *J Micromech. Microeng*, Vol.14, 2004, pp. 35-64.
- [3] M. Abdelgawad, I. Hassan, and N. Esmail, Transient behavior of the viscous micropump, *Microscale Thermophysical Engineering*, Vol.8, 2004, pp. 361-381.
- [4] M. Gad-el-Hak. The fluid mechanics of microdevices- the freeman scholar lecture. *Journal of Fluids Engineering*, Vol.121, pp. 5-33.
- [5] A. Rasmussen, C. Mavriplis, M. Zaghloul, O. Mikulchenko, and K. Mayaram, Simulation and optimization of a microfluidic flow sensor. *Sensors* and Actuators. A, Vol. 88, 2001, pp. 121-132.
- [6] M. I. Kilani, P.C. Galambos, Y.S. Haik, and C.J. Chen, Design and analysis of a surface micromachined spiral-channel viscous pump. Journal of Fluids Engineering, Vol.125, 2003. pp. 339-344.
- [7] R. Fox, and A. McDonald, *Introduction to fluid mechanics*, John Wiley&Sons, inc.1994.
- [8] M.I. Kilani, A. Al-Salaymeh, and A. T. Al-Halhouli, Effect of channel aspect ratio on the flow performance of a spiral-channel viscous micropump. *Journal of Fluids Engineering*, Vol.128, 2006, pp. 618-627.