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Dynamic Excitation Analysis of Laminar Boundary Layer via extended RVM by vortex filament of Finite Length

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Abstract: - Random Vortex Method (RVM) is a powerful method to simulate viscous incompressible flow fields. In this study, RVM is extended and use was made of vortex filament model in order to solve flow field over a flat plate. Then, a new algorithm is proposed to investigate the effects of dynamical perturbation in laminar boundary layer theory. Contact layer method is used to model the diffusion mechanism. Contact layer model is based on satisfaction of shear stress in a layer near the wall in contrast with satisfying the zero-velocity on walls. The vortex filaments are discretized and each element perturbed by changing its angle and employing step function at the beginning of the plate. The stability of laminar boundary layer is investigated and the obtained results were in a good agreement with available results in Re=10^3 and Re=10^4.

Key-Words: Random Vortex Method, Vortex Filament, Contact Layer

1 Introduction

Vortex filament method is based on Kelvin and Helmholtz theorem which assumes the vorticity of vortex tubes are constant. Vortex filaments can move in inviscid incompressible flows. The three dimensional vortex theorem is first proposed by Leonard and he used it in 3D vortex elements[1,2].Chorin [3] used this method in a simpler way and the continuity of the vortex were preserved. Leonard [2] approximated the geometrical distribution of vortex filaments. Couet [4]was used 3D vortex filament and in his study ,he utilized second order approximation for integrals. Leonard [5] used 3D vortex filament method with homogeny core structure thin filaments, which is the benchmark of the following works. Knio and Ghoniem [6,7] used a "Thin tube " method to study the stability of vortex rings. Leonard [5] shows that good results will be obtained if the radius of the vortex core increases monotonically and the volume of the vortex filament remain constant. Pothou [8] uses a vortex filament method to predict the acoustic field resulting from the impact of two vortex rings.The convergence of the vortex filament method is presented in Greengard [9].In this study, we used RVM with vortex filament model of finite length in order to study the flow field over a flat plate and its laminar boundary layer stability.

2 Problem Definition

The flow is assumed to be Newtonian viscous and incompressible one. The governing equations in two-dimensional coordinate are:

$$\nabla g u = 0 \tag{1}$$

$$\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U}.\nabla)\mathbf{U} = -\nabla P + \frac{1}{\mathrm{Re}}\nabla^2\mathbf{U} + \mathbf{f} \tag{2}$$

$$\mathbf{U} = 0 \qquad \textit{on the walls} \tag{3}$$

$$\mathbf{U} = (u,v) = (1,0) \qquad \textit{far from the walls} \tag{4}$$

3 Method of solution

The vorticity transport equation will be obtained by applying curl operator over two sides of equation (2).

$$\frac{\partial \omega}{\partial t} + (\mathbf{U}.\nabla)\omega = \frac{1}{\mathrm{Re}}\nabla^2\omega + \nabla\times\mathbf{f} \tag{5}$$

Equation (5) may be split into linear diffusion and nonlinear convection equations according to the fractional step of chorine [3] ,giving:

$$\frac{\partial \omega}{\partial t} + (\mathbf{U}.\nabla)\omega = 0 \tag{6}$$

$$\frac{\partial \omega}{\partial t} = \frac{1}{\mathrm{Re}}\nabla^2\omega + \nabla\times\mathbf{f} \tag{7}$$

where ω is vorticity vector. The idea of the fractional step method is to solve these equations sequentially rather simultaneously. The sequential solution means that at each time step the diffusion equation is solved using the state of the flow at the end of the previous time step as the new initial condition. Then the convection part is solved using, as the initial condition, the solution of the diffusion equation for the current time step. Use was made of contact layer model to generate vorticity near the wall. In this region, diffusion terms are more efficient in comparison with convection terms. Also, $\partial v/\partial x$ is much less than $\partial u/\partial y$ and therefore the vorticity can be calculated as

$$\omega \approx -\partial u/\partial y \tag{8}$$

Assume that the vorticity has a continuous distribution near the wall with thickness of α (fig.15). According to Eq.(8), the vorticity has a asymptotic expansion in a contact layer,

$$\omega = \omega_0 + \omega_1 y + \omega_2 y^2 + \cdots + \omega_n y^n + \cdots \tag{9}$$

where $y = O(\alpha)$ and ω_i is the different magnitude of vorticity in the contact layer. Therefore, Eq.(7) can be solved in two small time steps. In the first time step, the vorticity generates due to external body force and in the second time step, these vortex filaments of finite length diffuse into the flow field. Continous vorticity in $0 \le y \le \alpha$ discretized into N elements. So, the circulation for each element is

$$\Gamma_i = \omega_i . dA_i \quad (i = 1,2,...N) \tag{10}$$

Γ_i Can be divided in each time step into some vortex filaments of finite length. Number of vortex filaments is dependent on the vorticity in an element in contact layer. The vortex filament in each contact layer element should be distributed in a way that the velocity profile becomes linear (Fig.16).The generated vorticity moves to the field by Random walk. The diffusion equation of vorticity is:

$$\frac{\partial \omega}{\partial t} = \nu \nabla^2 \omega \tag{11}$$

where ν is the kinematic viscosity of the fluid. The Green function of one-dimensional form of Eq.(11) is:

$$Gr(y,t) = \sqrt{1/4\pi\nu t}.\exp\left(\frac{-1}{4\nu t} y^2\right) \tag{12}$$

In identical to the probability density function of Gaussian random variable η with a zero mean and a standard deviation $\sigma = \sqrt{2\nu t}$:

$$P(\eta;t) = \sqrt{1/2\pi\sigma^2}.\exp\left(-\frac{1}{2\sigma^2}\eta^2\right) \tag{13}$$

The Green function of diffusion equation in two-dimension is:

$$P(\eta;t) = \sqrt{1/2\pi\sigma^2}.\exp\left(-\frac{1}{2\sigma^2}\eta^2\right) \tag{14}$$

Random walk of vortex filaments of finite length in contact layer is the same as their random walk in the flow field. Convection terms can be neglected in contact layer. If $y_i < \alpha$, the vortex filament remains in contact layer and will be eliminated and in the next step will be regenerated in a new position depending on new conditions. If $y_i > \alpha$, the vortex filament moves toward the flow field. Therefore, the total displacement can be written as:

$$\mathbf{x}_j(t+\Delta t) = \mathbf{x}_j(t) + \sum_k \mathbf{U}(\mathbf{x}_{jk}).\Delta t + \boldsymbol{\eta}_j \tag{15}$$

where $\boldsymbol{\eta}_j = (\eta_{xj}, \eta_{yj})$ is a 2D Gaussian number and $\mathbf{U}(\mathbf{x}_{jk})$ is the velocity field due to potential background flow and vortex effects. α , in contact layer model, should be chosen enough small to have a linear velocity profile. Considering $\sigma = \sqrt{2\nu\Delta t}$, if α is assumed to be equal to σ then Δt can be calculated ,and the other parameters will be specified.

The induced velocity of the infinite length vortex filament at the middle of it according to Biot-Savart law is $\Gamma/2\pi r$. Which is the same as the velocity induction of a point vortex. Therefore in the first time step, the middle section of vortex filament is considered. The gravity center of vortex filament is also considered in convection and diffusion mechanisms, in which in the second time step two random number η_1, η_2 is employed for the gravity center of vortex filament. Random vortex method with vortex filament of finite length is a time-consuming algorithm, so we change its parameters in order to increase it efficiency. We choose the length of vortex filaments 0.9 and the very good results obtained.

3.1 Discretization of vortex filament

The calculation of distribution of vorticity in flow field needs a big memory and also a time-consuming job. Chorin [3] in his study of analysis of vortex method and its instabilities shows that each vortex element moves in the field under the induced velocity of other vortex element. In this study, each vortex filament of finite length discretized to some elements for example a vortex filament of 0.9 length is divided to 9 elements and also vorticity generates in the middle section, z=0.45. As in Fig 17, each element is described by the position of its both ends. Moreover, the strength of each vortex filament remains constant along its length.

3.2 Introducing perturbation on vortex elements

Two random angles are added to each element along x-axis and y-axis. Therefore each element has four random number, two random number for the position of gravity center and two for its rotation along x and y axes. Thus the flow field stability can be studied by considering perturbed θ along x-axis and ϕ along y-axis.

$$\theta = \eta_1 * \frac{\pi}{2} \tag{16}$$

$$\phi = \eta_2 * \frac{\pi}{2} \tag{17}$$

$$\psi = 0 \tag{18}$$

Where ψ is the angle along z-axis and η_1, η_2 are two random numbers of normal distribution. A step function is applied at the beginning of the plate.

4 Results and Conclusion

In this study, extended random vortex method with vortex filament of finite length along with contact layer model is used to investigate the flow field of an incompressible fluid over a flat plate and also the effects of dynamical excitation in laminar boundary layer is studied. 250 panels with thickness of α are considered in the middle section of vortex filament of finite length in contact layer model. The parameters of random vortex method are listed in table 1, where Γ_{max} is the maximum circulation of each vortex filament.[10] The turbulence intensity is assumed to be less than 0.2%. δ is the radius of vortex core of finite length. The velocity profile is calculated for 4 points of the length of the flat plate and the obtained results are in Fig(1-4) in comparison with Blasius analytical results. Because of extensive calculation works when the length of the filament is 1000, we revised the algorithm of vortex omission to obtain the optimum length of the filament. Finally, we arrive at the length of 0.9 and omission of the vortexes over x=10. Two fluids with different viscosities are chosen and the kinematic viscosities are assumed to be 10^{-3} and 10^{-4}. The length of the plate is 1 and also the mean flow velocity is assumed to be 1.

The response amplitude of step function with respect to the frequency for two fluids is shown in Figs. 5 and 6. Two points are considered in y=0.004 over plate and the velocity fluctuations are illustrated in Figs. 7-10. Figures 11-14 show the frequency amplitude fluctuations of velocity. The Re number is less than critical Re number, so as predicted before and according to the marginal stability curve [12], the amplitude of fluctuation is decreased and the flow field remains laminar.

References:

[1] A. Leonard, Numerical simulation of interacting, three-dimensional vortex _laments, in: Proceedings of the IV Intl. Conference on Numerical Methods of Fluid Dynamics, no. 35 in Lecture Notes in Physics, Springer-Verlag, 1975, pp. 245-250.

[2] A. Leonard, Vortex methods for flow simulation, J. Comput. Phys. No. 37, 1980, pp. 289-335.

[3] A. J. Chorin, Vortex models and boundary layer instability, SIAM J. Sci. Stat. Comput. No. 1, 1980, pp. 1-21.

[4] B. Couet, O. Buneman, A. Leonard, Simulation of three-dimensional incompressible flows with a vortex-in-cell method, J. Comput. Phys. No. 39 (2), 1981, pp.305-328.

[5] A. Leonard, Computing three-dimensional incompressible flows with vortex elements, Ann. Rev. Fluid Mech. No. 17, 1985, pp. 523-559.

[6] O. M. Knio, A. F. Ghoniem, Numerical study of a three-dimensional vortex method, J. Comput. Phys. No. 86, 1990, pp. 75-106.

[7] O. M. Knio, A. F. Ghoniem, Three-dimensional vortex simulation of rollup and entrainment in a shear layer, J. Comput. Phys. No. 97, 1991, pp. 172-223.

[8] K. P. Pothou, S. G. Voutsinas, S. G. Huberson, O. M. Knio, Application of three dimension particle method to the prediction of aerodynamic sound, ESAIM Proc.NO.1 ,1996, pp.349-362.

[9] C. Greengard, Convergence of the vortex filament method, Math. Comput, 47 (176), 1986, pp. 387-398.

[10] N.M. Nouri, S Eslami,. "Modeling of free-stream turbulence with random vortex method (RVM) in incompressible flow and solution for circular cylinder", 9th Fluid Dynamic Conference .Shiraz, Iran, 2004, (in persian).

[11] N.M. Nouri, M. Ebrahimi, "Contact Layer Vorticity Generation Modeling in Fluid Motion Near the Wall", 9th Fluid Dynamic Conference, Shiraz, Iran, 2004, (in persian).

[12] H. Schlichting, *Boundary Layer Theory*, McGraw-Hill, New York, 1968, pp. 437-450.

Table 1. Parameters in Random Vortex Method (RVM)

Computational Parameter	$\mathrm{Re}_L = 1000$	$\mathrm{Re}_L = 10000$
$\Gamma_{\max}\left(m^2/S\text{ec}\right)$	7.59×10^{-5}	7.59×10^{-5}
$\alpha(m)$	2×10^{-3}	5×10^{-4}
$\sigma(m)$	2×10^{-3}	5×10^{-4}
$\delta(m)$	2×10^{-3}	5×10^{-4}
$\Delta t\left(Sec\right)$	2×10^{-3}	1.25×10^{-3}

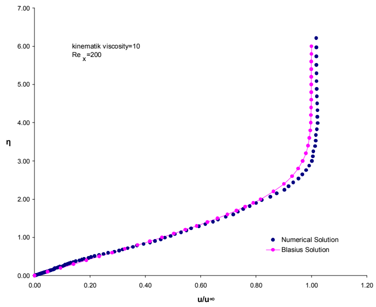

Fig1. Velocity profile for $\upsilon = 10^{-3}\, m^3/s$ at x=0.2

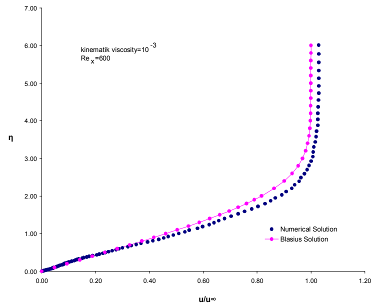

Fig2. Velocity profile for $\upsilon = 10^{-3}\, m^3/s$ at x=0.4

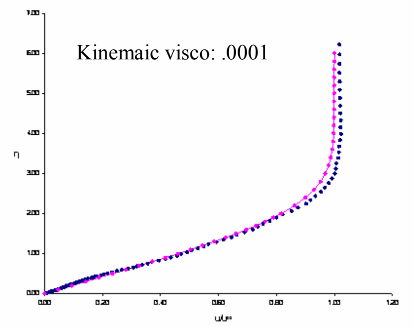

Fig3. Velocity profile for $\upsilon = 10^{-3}\, m^3/s$ at x=0.6

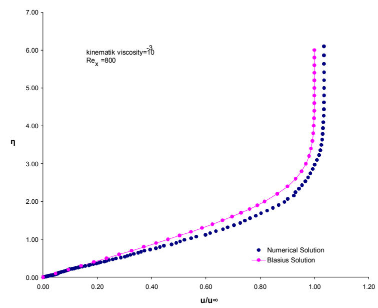

Fig4. Velocity profile for $\upsilon = 10^{-3}\, m^3/s$ at x=0.8

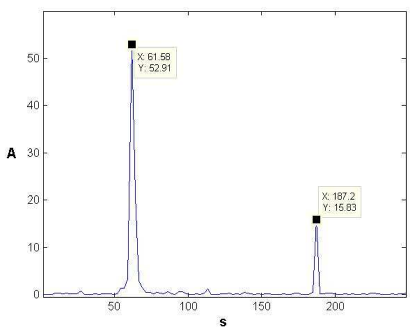

Fig5. Frequency amplitude of step function over frequencies, $\upsilon = 10^{-3}\, m^3/s$

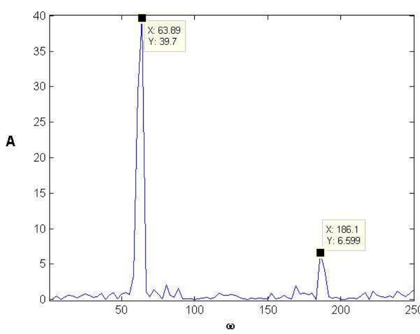

Fig6. Frequency amplitude of step function over frequencies, $\upsilon = 10^{-4}\, m^3/s$

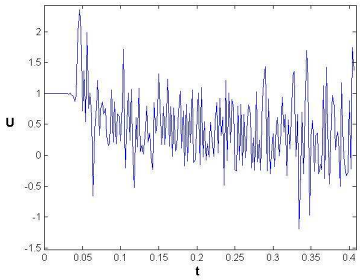

Fig7. Velocity fluctuation for $\upsilon = 10^{-3}\, m^3/s$, x=0.4

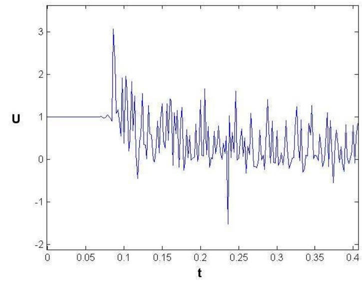

Fig8. Velocity fluctuation for $\upsilon = 10^{-3}\, m^3/s$, x=0.8

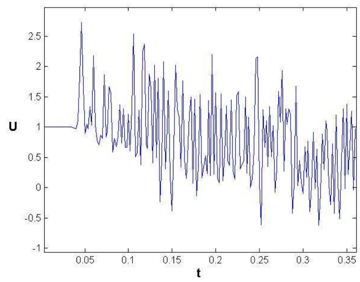

Fig9. Velocity fluctuation for $\upsilon = 10^{-4}\, m^3/s$, x=0.4

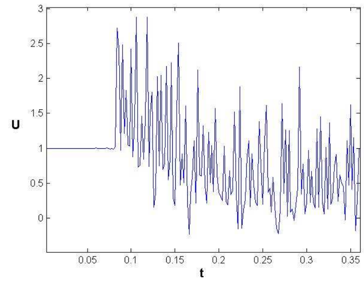

Fig10. Velocity fluctuation for $\upsilon = 10^{-4}\, m^3/s$, x=0.8

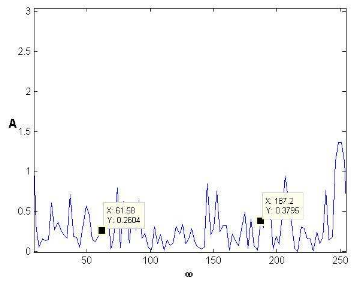

Fig11. Frequency amplitude over fluctuation frequency for $\upsilon = 10^{-3}\, m^3/s$, x=0.4

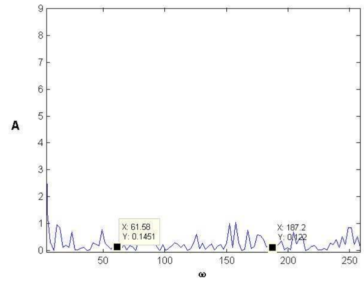

Fig12. Frequency amplitude over fluctuation frequency for $\upsilon = 10^{-3}\, m^3/s$, x=0.8

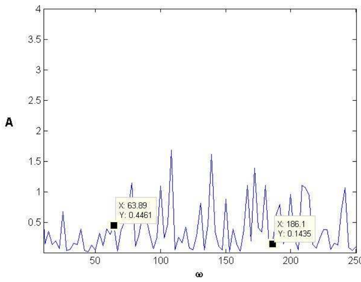

Fig13. Frequency amplitude over fluctuation frequency for $\upsilon = 10^{-4}\, m^3/s$, x=0.4

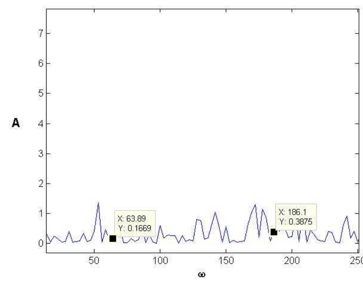

Fig14. Frequency amplitude over fluctuation frequency for $\upsilon = 10^{-4}\, m^3/s$, x=0.8

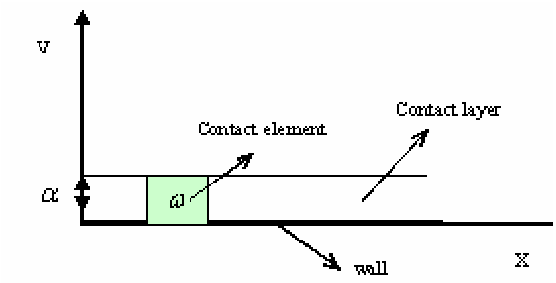

Fig15. Contact layer element

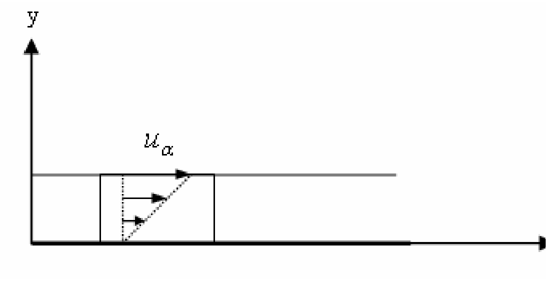

Fig16. Velocity profile in contact layer element

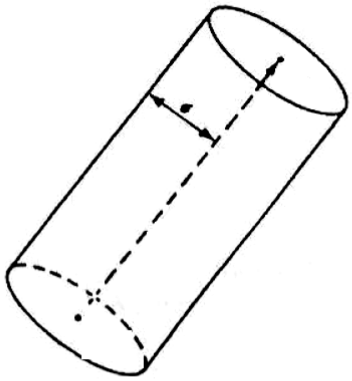

Fig17. schematic of vortex element