Proceedings of the 4th WSEAS International Conference on Fluid Mechanics and Aerodynamics, Elounda, Greece, August 21-23, 2006 (pp63-67)

Consideration of 2D Unsteady Boundary Layer Over Oscillating Flat Plate

N.M. NOURI, H.R. AHMADI FAKHR, H. Madadi, R. Abdollahpour
Applied Hydrodynamics Lab., Mechanical Engineering Dept.
Iran University of Science and Tech.
Narmak, 16846-13114, Tehran
Iran

Abstract: - The unsteady boundary layer due to small amplitude sinusoidal oscillation of a plate in viscous incompressible fluid is investigated here using Random Vortex Method. While the plate oscillates in its own plane. The unsteady boundary layer causes the unsteady velocity profile and shear waves propagation. The numerical result is compared with analytical solution for the case that the oscillation amplitude is small enough to neglect nonlinear convectional term. The results of RVM for unsteady boundary layer show good similarity confirming the ability of the proposed method. The nonlinear convectional term can also be taken in to account in RVM, in the cases that they can not be neglected.

Key-Words: - Viscous fluid; Oscillating wall; Unsteady flow; Transient flow

1 Introduction

The motion of viscous fluid caused by sinusoidal oscillation of a flat plate is termed as stocks second problem by schlichting [1]. It is not only of fundamental theoretical interest but it also occurs in many applied problems; Such as acoustic streaming around oscillating body [2]. As early as 2000, M. Emin Erdogan has considered the flow of an incompressible viscous fluid caused by the small amplitude oscillation of the plane wall [3, 7].

This motion will produce, far from the body, acoustic wave of small amplitude. the flow near the body will, in general, have normal and tangential velocity component relative to the body. On the body's surface the normal velocity component is fixed by the requirement that there be no flow through the boundary. And also when the viscosity effects are taken into account the fluid in contact with the body can no longer slip over the body; Instead, it adheres to it. This is not the only effect of viscosity, for in the same way that is precludes slip between fluid and solid, it also prevents complete slippage between contiguous layers of fluid. Therefore, no slip condition at a boundary will make the whole tangential velocity profile significantly different from which would exist if the fluid were inviscid.

The propagation of shear waves and unsteady boundary layer are analyzed here via Random Vortex Method, In RVM, the Navier–Stokes equations, in the form of vorticity, is split into diffusion and convection parts, according to the fractional step method. A random Walk method is used to solve the diffusion equation. So unlike the analytical method using RVM, the nonlinear convectional term is also taken into account.

2 Problem Formulation

To study the motion of infinite plate a rectangular system of coordinate is attached to the plate is such a manner that the plane wall is chosen as x-axis and it oscillates in its own plane, as sketched in fig. 1.because of viscosity the fluid above the plane is also move, but it is clear that the fluid velocity will have only one component, and this will be parallel to the velocity of the plane. Further, this velocity component can not depend on distance along the plane so that $\mathbf{u}=\left[u(y,t),0,0\right]$. Therefore $\nabla.\mathbf{u}=0$, so the momentum equation yields:

$$\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u}.\nabla)\mathbf{u}=\nu_0\frac{\partial^2 \mathbf{u}}{\partial y^2} \tag{1}$$

2.1 Analytical solution [8]

When the fluid is initially at rest and oscillation amplitude is small, the nonlinear term is assumed to be in small order. Neglecting the convection term, linear form of equation is obtained:

$$\frac{\partial \mathbf{u}}{\partial t}=\nu_0\frac{\partial^2 \mathbf{u}}{\partial y^2} \tag{2}$$

This is a diffusion equation. Therefore if the fluid starts at $t=0$ and imparts some momentum to the fluid in contact with it, would expect this momentum to be diffused slowly into the fluid. The motions of fluid after all transient effects have disappeared; since the plane is oscillating as:

$$U_P = U_0 \operatorname{Re}(e^{-i\omega t}) \tag{3}$$

The fluid velocity is also expected to depend harmonically on time. Therefore:

$$u(y,t) = \operatorname{Re}\left[\tilde{U}(y)e^{-i\omega t}\right] \tag{5}$$

So it yields:

$$\tilde{U}''(y) + K^2\tilde{U}(y) = 0 \tag{6}$$

$$K = (1+i)\sqrt{\omega / 2\nu_0} \tag{7}$$

The solution is:

$$u(y,t) = e^{-i\omega t}\left(Ae^{i(1+i)y/\delta_v} + Be^{-i(1+i)y/\delta_v}\right) \tag{8}$$

where:

$$\delta_v = \sqrt{2\nu_0 / \omega} \tag{9}$$

Since for $y \to \infty$ the velocity must be small, we must set $B=0$. Also at $y=0$ the fluid velocity is equal to that of the plane, so that $A = U_0$ and:

$$u(y,t) = U_0 e^{-(y/\delta_v)} \cos\left(\omega t - y/\delta_v\right) \tag{10}$$

The fluid therefore, also oscillates harmonically in time, but the oscillation lag those of the plane, and has very small amplitude far from the plane. fig. 3 depicts relative-velocity profiles at various times during one oscillation. In the figure time is measured from the point during a cycle when $u = U_0$ at $y=0$. It is seen that for $\omega t \le \frac{\pi}{4}$ the maximum fluid velocity amplitude is at the plane $y=0$. However, for $y>0$ in fact its location in the fluid is given by:

$$y_{\max}(t) = \left(\omega t - \frac{\pi}{4}\right)\delta_v \tag{11}$$

Thus one if the feature of the oscillation, namely, the point of maximum fluid velocity is seen to be moving into the fluid with velocity $\omega\delta_v = \sqrt{2\omega\nu_0}$.

2.2 The Numerical method

Equation (1) can be written in the form of vorticity:

$$\frac{\partial \omega}{\partial t} + (u.\nabla)\omega = \nu_0 \frac{\partial^2 \omega}{\partial y^2} \tag{12}$$

This equation called vorticity transport equation and may be split into linear diffusion and nonlinear convection equations according to the fractional step method of Chorin [9,13], giving.

$$\frac{\partial \omega}{\partial t} = \nu_0 \frac{\partial^2 \omega}{\partial y^2} \tag{13}$$

$$\frac{\partial \omega}{\partial t} = -(u.\nabla)\omega \tag{14}$$

where ω is vorticity vector. The idea of the fractional step method is to solve these equations sequentially rather than simultaneously. The sequential solution means that at each time step the diffusion equation is solved using the state of the flow at the end of the previous time step as the new initial condition. Then the convection part is solved using, as the initial conditions, the solution of the diffusion equation for the current time step. By taking the convective term into account the nonlinear problems with large amplitude oscillation can also be solved.

The transport of vorticity due to diffusion in random vortex method is implemented by dispersion of a finite number of vortex elements with finite and constant vorticity according to a 2-dimensional Gaussian statistics. This based on the fact that the green functions of 1-dimentional form of equation (13) is: [14]

$$G(y,t) = \sqrt{\operatorname{Re}/4\pi t}\ \exp\left(-\frac{R}{4t}y^2\right) \tag{15}$$

In identical to the probability density function of Gaussian random variable η with a zero mean and a standard deviation σ :

$$P(\eta,t) = \sqrt{1/2\pi\sigma^2}\ \exp\left(-\frac{1}{2\sigma^2}\eta^2\right) \tag{16}$$

If $\sigma = \sqrt{\frac{2t}{\operatorname{Re}}}$ The green function of diffusion equation in 2-dimension is:

$$G(x,y,t) = \sqrt{\operatorname{Re}/4\pi t}\ \exp\left(-\frac{R}{4t}(x^2+y^2)\right) \tag{17}$$

which is equivalent to:

$$G(x,y,t) = G(x,t)G(y,t) \tag{18}$$

where $G(x,t)$ and $G(y,t)$ have the same form as in equation (15).then the corresponding probability density function is the product of two 1-dimensional probability density functions:

$$P(\eta_1,\eta_2,t) = P_1(\eta_1,t)P_2(\eta_2,t) \tag{19}$$

So the solution of equation (13) is simulated stochastically by a 2-dimensional displacement of vortex elements in two perpendicular directions using two sets of independent Gaussian random numbers, each have a zero mean and standard deviation of $\sigma = \sqrt{\frac{2Dt}{\operatorname{Re}}}$.

To construct an algorithm the vorticity in the flow is represented by a number of discrete vortices, which are given a random Gaussian motion, or random walk with zero mean and variance of $\sqrt{\frac{2Dt}{\text{Re}}}$ where Dt is the time step. These vortices are generated on the surface to satisfy the no–slip boundary condition. Such that the surface of the body is represented by m panels. Each of which is allocated a vortex distribution of Γ_i per unit length, This vortex distribution is then discretized into a number of point vortices, such that the circulation of each vortex being less than some prescribed maximum $\Gamma_{\max}$ and the distribution Is such that made linear velocity profile on the panels with respect to y and zero resultant tangential Velocity at the central collocation point. The panels' height is chosen small enough to place under the laminar sub-layer so their linear velocity profile role as a boundary condition and force the velocity profile to be linear near the body.

The convection term is then taken into account with moving the vortices with their inviscid velocities in the Lagrangian scheme. So the new position of the vortices due to the convection and diffusion is given by:

$$\begin{aligned} x_i^{t+\Delta t} &= x_i^t + u_i^t Dt + \eta_{ix} \\ y_i^{t+\Delta t} &= y_i^t + v_i^t Dt + \eta_{iy} \end{aligned} \tag{20}$$

where (x_i^t, y_i^t) and (u_i^t, v_i^t) represented the position and velocity vector of $i'th$ vortex at time t, and (η_{ix}, η_{iy}) is Gaussian random translation vector. The vortices velocity (u_i^t, v_i^t) is calculated using potential velocity around desired geometry and velocity induced by other vortices.

3 Conclusion

The problem under consideration is a sinusoidal in plane oscillation of infinite plate with velocity amplitude $U_0 = 0.001$ and oscillation frequency $\omega = 10\pi$, the considered fluid is water with $\mu = 0.000894$ and $\rho = 1000$. The plate oscillation is considered in two cases:

The RVM and analytical results are compared for small amplitude wall oscillation in stationary fluid, so as assumed in analytical solution the convection term is small enough to be neglected; Comparisons between analytical and numerical result for this case in fig. 4, fig. 5 and fig. 7 show the capability of RVM for unsteady boundary layer consideration,.

References:

[1] H. Schlichting, *Boundary Layer Theory*, McGraw-Hill, New York, 1968

[2] N. Tokuda, on the impulsive motion of a flat plate in a viscous fluid, *J. Fluid Mech.,* Vol. 33, 1968, pp. 657-672.

[3] M. Emin Erdogan, A note on an unsteady flow of a viscous fluid due to an oscillating plane wall, *International Journal of Non-Linear Mechanics,* Vol. 35, 2000, pp. 1-6.

[4] Dennis SCR., The motion of a viscous fluid past an impulsively started semi-infinite flat plate., *IMA Journal of Applied Mathematics,* Vol. 10, 1972, pp. 105–117.

[5] K. Stewartson, On the impulsive motion of a flat plate in a viscous fluid (Part I). *Quarterly Jnl. of Mechanics & App. Maths.,* Vol. 4, 1951, pp.182–198.

[6] K. Stewartson On the impulsive motion of a flat plate in a viscous fluid(Part II). *Quarterly Jnl. of Mechanics & App. Maths.,* Vol. 26, 1973, pp. 143–152.

[7] MG Hall., The boundary layer over an impulsively started flat plate, *Proceedings of the Royal Society of London,* 1969, pp. 401–414.

[8] S. Temkin, *Elements of Acoustics*, John Wiley & Sons, 1981

[9] A.J. Chorin, Numerical Study Slightly Viscous Flow, *J. Fluid Mach.*, Vol. 57, 1973, pp.785-796

[10] G. H. Cottet, P. Koumoutsakos, M. L. Ould.Salihi, Vortex Methods with Spatially Varying Cores, *Journal Of Computational Physics*, Vol.162, 2000, pp. 164–185

[11] N. R. Clarke, O. R. Tutty, Construction and validation of a discrete vortex method for the two-dimensional incompressible Navier-Stokes equations, *Journal of Computational Fluids*, Vol. 23 (6) , 1994, pp. 751-783.

[12] J. S. Marshall, J. R. Grant, Penetration of a blade into a vortex core: vorticity response and unsteady blade forces, *Journal of Fluid Mechanics*,Vol. 306, 1996, pp. 83-109.

[13] N. M. Nouri, S. Eslami, Modeling of free-stream turbulence with random vortex method (RVM) in incompressible flow and solution for circular cylinder, 9th Conference of Fluid Mechanics, Shiraz, Iran.

[14] A.F Ghoniem and Y.Gagnon, vortex Simulation of Laminar Recalculating Flow, *J. Comp. Phys.*, Vol.68, 1987, pp.346-377

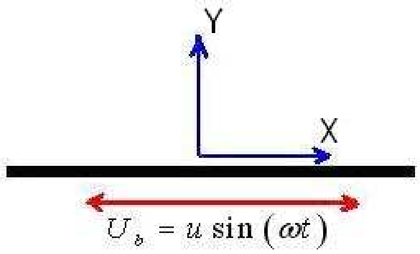

fig. 1 schematic of in plane oscillation of half plane wall (stocks second problem)
fig. 2

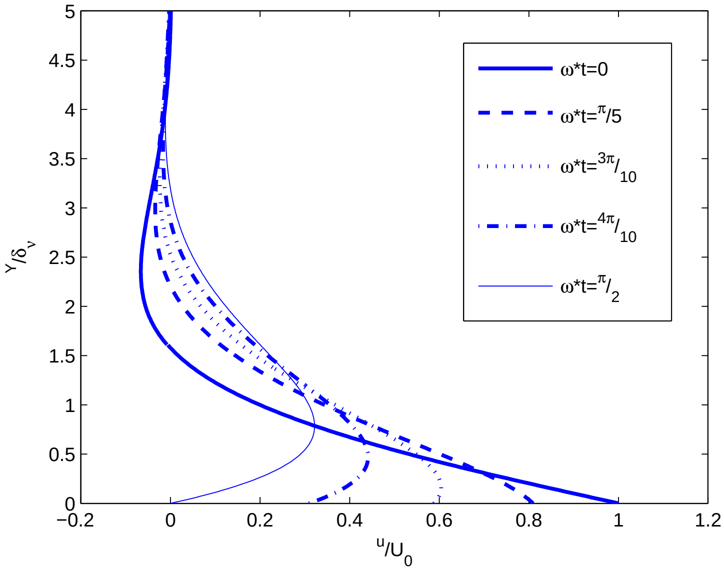

fig. 3 nondimensional-velocity profiles at various times during one oscillation

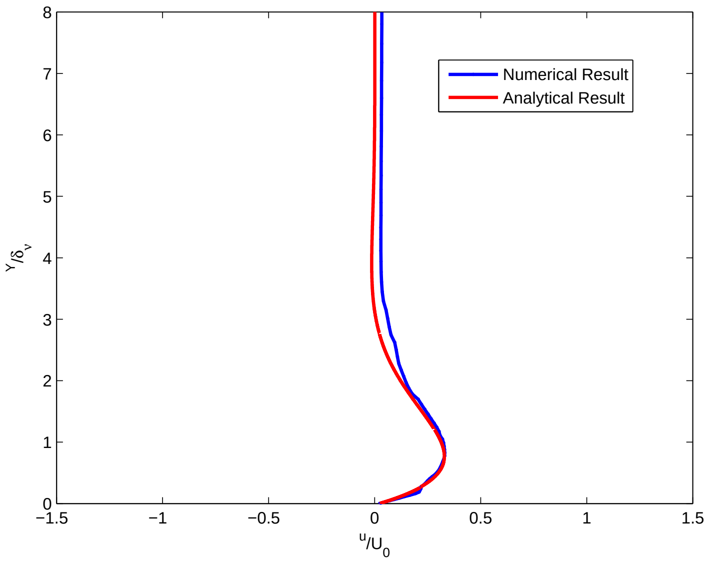

fig. 4 Analytical and Numerical results of velocity profile in $\omega t = 2\pi$

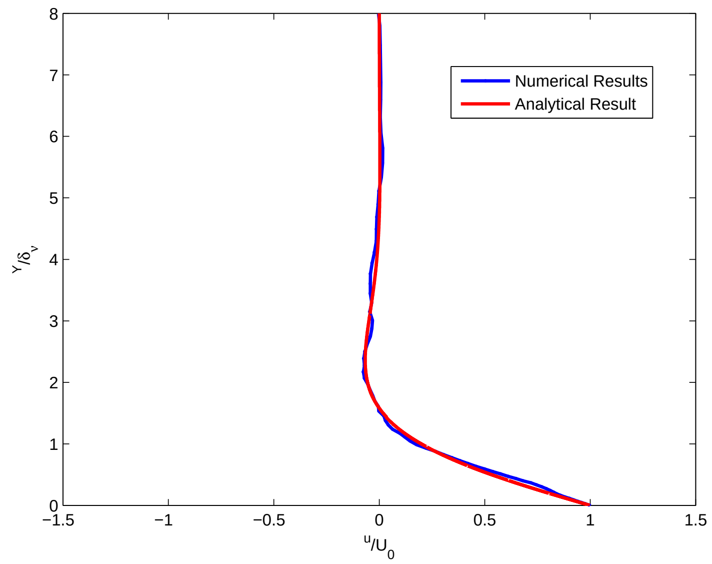

fig. 5 Analytical and Numerical results of velocity profile in $\omega t = \frac{5\pi}{2}$

fig. 6

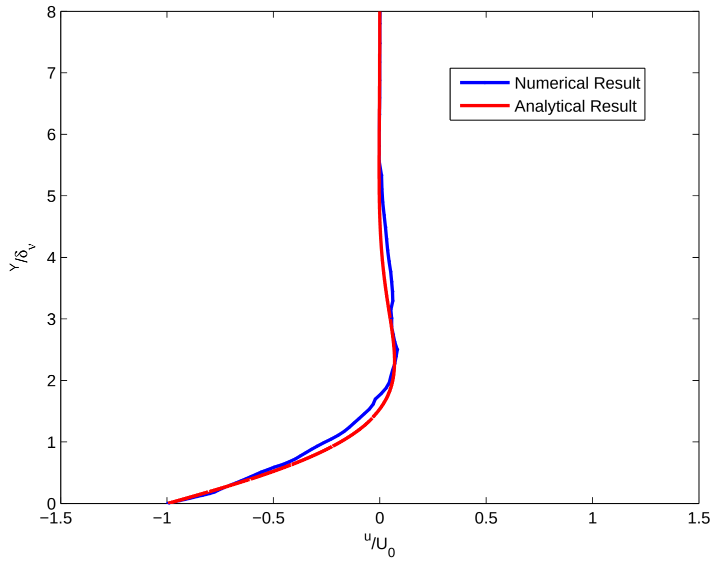

fig. 7 Analytical and Numerical results of velocity profile in $\omega t = \frac{7\pi}{2}$