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Reduced order modelling of compound fluid transmission line systems

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Abstract: - This paper presents a novel approach towards the input-output oriented modelling of hydraulic transmission lines. If the Reynolds number is low enough to justify the assumption of laminar flow and if convective terms are negligible, the governing equations are linear and a very compact description of the input-output behaviour of a transmission line exists in the frequency domain. For a coupled simulation of networks of transmission lines interacting with other, possibly nonlinear components such as valves, there are numerous approaches for the approximation of the transcendental transfer functions arising from the transmission line dynamics by finite dimensional models in the time domain. The proposed method for the derivation of reduced order models offers a trade-off between the degree of accuracy and the system order. Important properties like the passivity of the transmission line model and the exact fulfillment of the Joukowsky relation are guaranteed.

Key-Words: - fluid transmission line, laminar pipe flow, passive model

1 Outline of the problem

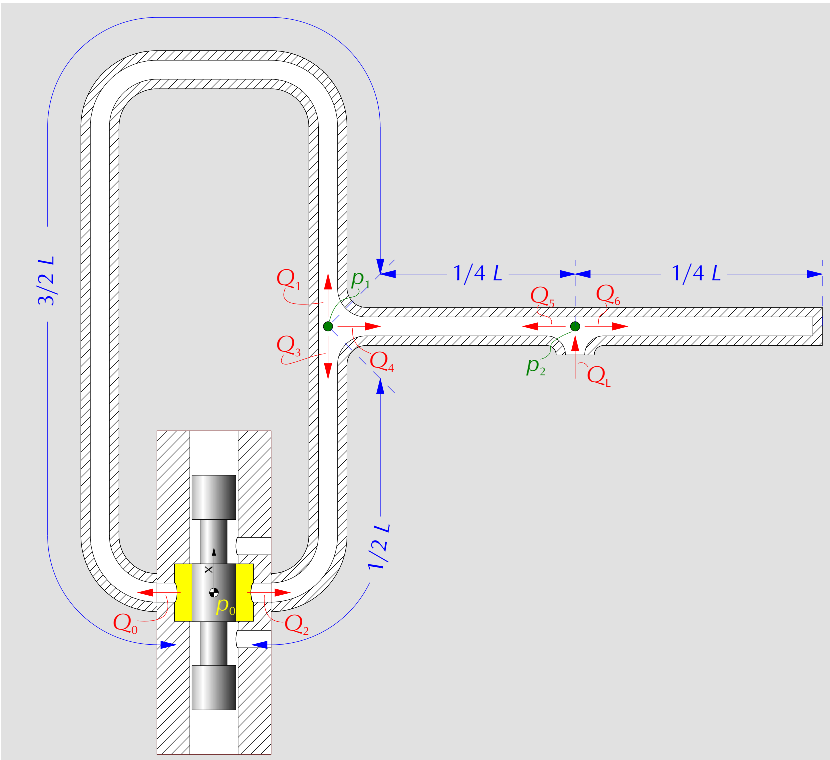

Figure 1: Geometry of the hydraulic wave converter.

This paper considers pipe flow in a system of transmission lines with a constant and circular cross section filled with a weakly compressible Newtonian fluid. The pipe wall is assumed to be rigid and the flow is assumed to be laminar. The system represents the pipework of a so called hydraulic wave converter

[SLGS99] which is as device for energy efficient actuation in fluid power systems. A sketch is given in Figure 1. The radius of curvature of the pipes is assumed to be large enough to justify the validity of the straight pipe input-output model [DO64]

$$\left[\begin{array}{c}\hat{Q}_0\\ \hat{p}_1\end{array}\right]=\left[\begin{array}{cc}\frac{\tanh\left(s\tilde{Z}\right)}{\tilde{Z}} & -\frac{1}{\cosh\left(s\tilde{Z}\right)}\\ \frac{1}{\cosh\left(s\tilde{Z}\right)} & \tilde{Z}\tanh\left(s\tilde{Z}\right)\end{array}\right]\left[\begin{array}{c}\hat{p}_0\\ \hat{Q}_1\end{array}\right] \tag{1}$$

relating the flow rates Q to the pressures p at both ends (index 0 or 1) of a pipe of length L. The Laplace variable is denoted by s and he line impedance Z and propagation operator γ are defined as

$$Z=Z_0\sqrt{-\frac{J_0\left(\sqrt{-\frac{s}{\nu}}R\right)}{J_2\left(\sqrt{-\frac{s}{\nu}}R\right)}},\gamma=\frac{sL}{c_0}\sqrt{-\frac{J_0\left(\sqrt{-\frac{s}{\nu}}R\right)}{J_2\left(\sqrt{-\frac{s}{\nu}}R\right)}}$$

where the speed of wave propagation c_0 and the characteristic impedance Z_0 are

$$Z_0=\frac{\sqrt{E\rho}}{R^2\pi},\quad c_0=\sqrt{\frac{E}{\rho}}.$$

The internal radius of the pipe is denoted by R and the fluid is characterised by the mass density ρ, the kinematic viscosity ν, and the bulk modulus of compressiblity E. With a scaling of pressure p, flow rate Q, and time t in the form

$$\psi=\frac{p}{p_S},\qquad q=\frac{Z_0}{p_S}Q,\qquad \tau=\frac{c_0}{L}t,\qquad \tilde{s}=\frac{L}{c_0}s$$

where a characteristic pressure p_S is chosen either as the maximum operating pressure of the transmission line or as a typical magnitude of transient pressure excitations acting at the boundary, eq. (1) takes the dimensionless form

$$\left[\begin{array}{c}\hat{q}_0\\ \hat{\psi}_1\end{array}\right]=\left[\begin{array}{cc}\frac{\tanh\left(\tilde{s}\tilde{Z}\right)}{\tilde{Z}} & -\frac{1}{\cosh\left(\tilde{s}\tilde{Z}\right)}\\ \frac{1}{\cosh\left(\tilde{s}\tilde{Z}\right)} & \tilde{Z}\tanh\left(\tilde{s}\tilde{Z}\right)\end{array}\right]\left[\begin{array}{c}\hat{\psi}_0\\ \hat{q}_1\end{array}\right] \tag{2}$$

with the scaled hydraulic impedance

$$\tilde{Z}(s)=\sqrt{\frac{J_0\left(\sqrt{-\frac{\tilde{s}}{D_n}}\right)}{J_2\left(\sqrt{-\frac{\tilde{s}}{D_n}}\right)}}$$

and the dimensionless dissipation number [GL72] $D_n=\frac{\nu L}{c_0 R^2}$. By applying the model (2) to every elementary pipe section in the example system given in Fig. 1 and neglecting concentrated resistances [Man05] at the pipe joints the equation system

$$\hat{q}_0 = \tanh\left(\frac{3}{2}\tilde{s}\tilde{Z}\right)\tilde{Z}^{-1}\hat{\psi}_0-\operatorname{sech}\left(\frac{3}{2}\tilde{s}\tilde{Z}\right)\hat{q}_1 \tag{3a}$$
$$\hat{\psi}_1 = \operatorname{sech}\left(\frac{3}{2}\tilde{s}\tilde{Z}\right)\hat{\psi}_0+\tanh\left(\frac{3}{2}\tilde{s}\tilde{Z}\right)\tilde{Z}\,\hat{q}_1 \tag{3b}$$
$$\hat{q}_2 = \tanh\left(\frac{1}{2}\tilde{s}\tilde{Z}\right)\tilde{Z}^{-1}\hat{\psi}_0-\operatorname{sech}\left(\frac{1}{2}\tilde{s}\tilde{Z}\right)\hat{q}_3 \tag{3c}$$
$$\hat{\psi}_1 = \operatorname{sech}\left(\frac{1}{2}\tilde{s}\tilde{Z}\right)\hat{\psi}_0+\tanh\left(\frac{1}{2}\tilde{s}\tilde{Z}\right)\tilde{Z}\,\hat{q}_3 \tag{3d}$$
$$\hat{q}_4 = \tanh\left(\frac{1}{4}\tilde{s}\tilde{Z}\right)\tilde{Z}^{-1}\hat{\psi}_1-\operatorname{sech}\left(\frac{1}{4}\tilde{s}\tilde{Z}\right)\hat{q}_5 \tag{3e}$$
$$\hat{\psi}_2 = \operatorname{sech}\left(\frac{1}{4}\tilde{s}\tilde{Z}\right)\hat{\psi}_1+\tanh\left(\frac{1}{4}\tilde{s}\tilde{Z}\right)\tilde{Z}\,\hat{q}_5 \tag{3f}$$
$$\hat{q}_6 = \tanh\left(\frac{1}{4}\tilde{s}\tilde{Z}\right)\tilde{Z}^{-1}\hat{\psi}_2 \tag{3g}$$
$$0 = \hat{q}_1+\hat{q}_3+\hat{q}_4 \tag{3h}$$

describes the dynamics of the compound pipeline system in the frequency domain. The system is excited by the control valve delivering a flow Q_V into the volume V_V. The evolution of the pressure p_0 this volume is described by

$$C\,\tilde{s}\,\hat{\psi}_0=\hat{q}_V-\hat{q}_1-\hat{q}_2. \tag{4}$$

At the other end of the converter, the pressure p_2 is assumed to be prescribed by the load. The load flow rate (defined as the volumetric flow rate entering the converter from the load) is given by

$$\hat{q}_L=\hat{q}_5+\hat{q}_6. \tag{5}$$

From the set of equations (3, 4, 5) an input output description

$$\left[\begin{array}{c}\hat{\psi}_0\\ \hat{q}_L\end{array}\right]=\mathbf{G}\left(\tilde{s}\right)\left[\begin{array}{c}\hat{q}_V\\ \hat{\psi}_2\end{array}\right]$$

can be computed.

It is important to note that the static gain of the the transfer matrix $\mathbf{G}(s)$ represents the pressure drop due to stationary Hagen-Poisseuille flow

$$\lim_{\tilde{s}\to 0}\mathbf{G}\left(\tilde{s}\right)=\left[\begin{array}{cc}5D_n & -1\\ 1 & 0\end{array}\right] \tag{6}$$

and the direct feedthrough term

$$\lim_{\tilde{s}\to\infty} \mathbf{G}(\tilde{s}) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{7}$$

exactly fulfils the Joukowsky relation for the scaled model at the load end of the converter. The goal is now to find a state-space approximation

$$\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\begin{bmatrix} q_V \\ \psi_2 \end{bmatrix} \tag{8a}$$

$$\begin{bmatrix} \psi_0 \\ q_L \end{bmatrix} = \mathbf{C}\,\mathbf{x} + \mathbf{D}\begin{bmatrix} q_V \\ \psi_2 \end{bmatrix} \tag{8b}$$

for the input-output behaviour described by eq. (1). The system (8) should exactly fulfill the limit properties (6) and (7), i.e.

$$\mathbf{C}(\mathbf{I}-\mathbf{A})^{-1}\mathbf{B} = \begin{bmatrix} 5D_n & -1 \\ 1 & 0 \end{bmatrix} \tag{9}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{10}$$

Furthermore, it has to reflect an important property (passivity) of the real system by meeting the criteria for positive realness [Kha96] and the approximation error - measured in an appropriate norm - between (8) and (1) should be as low as possible for a given system order. The passivity of the model is so important because a coupled simulation containing a non-passive transmission line model can become unstable when only passive components like pipelines, hoses, and concentrated hydraulic volumes are plugged together in a simulation system [Man04].

2 Formulation as a nonlinear programming problem

The problem described in the last section can be treated by a number of different approaches. Either a very accurate and passive model is found and reduced by a passivity preserving order reduction method, or passivity is enforced as a constraint in the reduction method. The first approach requires the availability of an accurate and passive model.

The proposed method has already been published in a discrete-time variant in [Man]. For discrete-time models, a method of characterisitcs with an accurate friction model like the approach by Suzuki et. al. [Zie68, STS91] could be used for the generation of this model. However, the order of this model is very high [Man01] which results in a large computational effort for the passivity preserving order reduction. A similar situation can be expected with continuous-time models derived for instance by the Galerkin method. Therefore, the second approach – i. e. the enforcement of passivity by constraints in a nonlinear programming problem – is used in the following.

The matrices $\mathbf{A} \in \mathbb{R}_n^n$, $\mathbf{B} \in \mathbb{R}_n^2$, $\mathbf{C} \in \mathbb{R}_2^n$, and $\mathbf{D} \in \mathbb{R}_2^2$ have to be found for a given system order n. The frequency response function for the system (8) is defined by

$$\mathbf{G}_{approx}(\tilde{s}) = \mathbf{C}(\tilde{s}\mathbf{I}-\mathbf{A})^{-1}\mathbf{B} + \mathbf{D}\,. \tag{11}$$

The transfer function matrix $\mathbf{G}(s)$ of eq. (1) is to be approximated by (8). According to Parseval's theorem, a minimisation of the sum of squared errors between the frequency response function $\mathbf{G}_{approx}$ and $\mathbf{G}(\tilde{s})$ will result in a minmisation of the ITSE error criterion applied to the impulse response. The quadratic cost function is written as

$$\int_0^\infty tr\left(\mathbf{G}_{approx}(j\tilde{\omega}) - \mathbf{G}(j\tilde{\omega})\right) \cdot \tag{12}$$

$$\left(\mathbf{G}_{approx}(j\tilde{\omega}) - \mathbf{G}(j\tilde{\omega})\right)^T d\omega \tag{13}$$

While the cost function (12) measures the quadratic error of the impulse response, it is also possible to minimize the quadratic error for a step input signal by using the frequency weighted error

$$\frac{1}{j\tilde{\omega}}\left(\mathbf{G}_{approx}(j\tilde{\omega}) - \mathbf{G}(j\tilde{\omega})\right) \tag{14}$$

in lieu of the simple difference $G_{approx} - G$. An alternative to the $\mathcal{H}_2$ approach proposed in this paper is the use of the $\mathcal{H}_\infty$ norm of the error $\mathbf{G}_{approx} - \mathbf{G}$ in order to minimize the worst case error instead of the error for a specific test signal. However, most experiments published in the fluid power literature are conducted by rapidly opening or closing a valve within a transmission line system resulting in step response data. Therefore, most time domain comparisons show the step responses of transmission line models. In order to compare against other author's results, in this paper the step response of the model is optimized, i. e. the cost function (12) is used with the modified error term (14).

While eq. (10) simply gives the solution for the direct feedthrough matrix $\mathbf{D}$, the gain condition (9) has to be built into the nonlinear programming problem. Due to the nature of the pipeline model with mixed boundary conditions, all poles of the transfer functions in $\mathbf{G}(\tilde{s})$

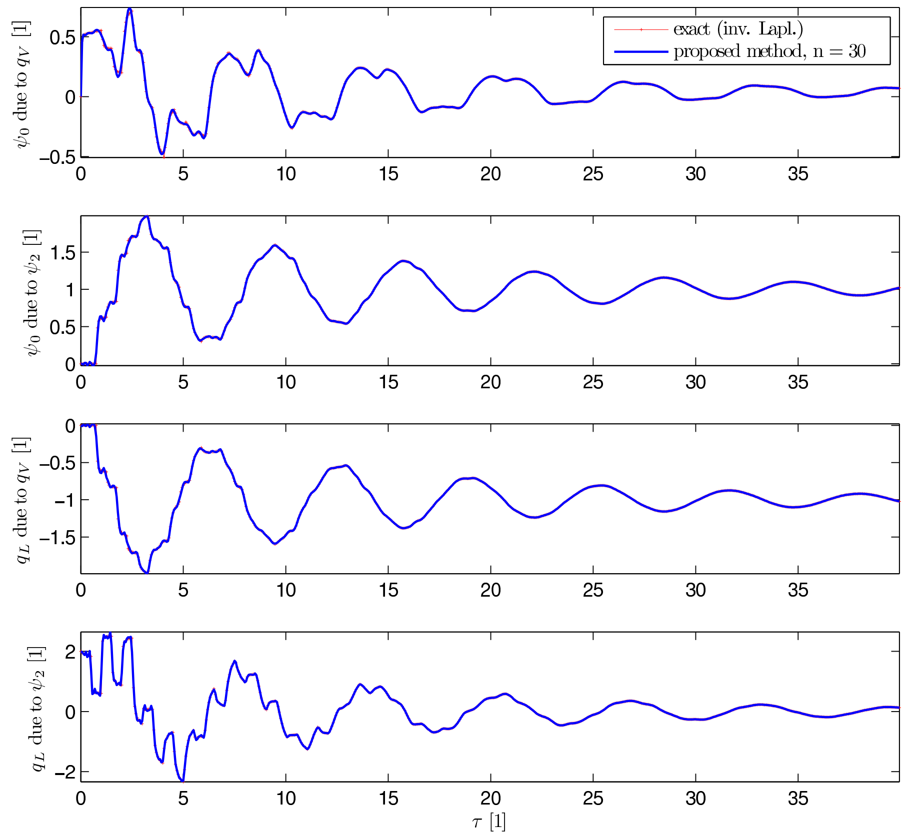

Figure 2: Comparison of step responses.

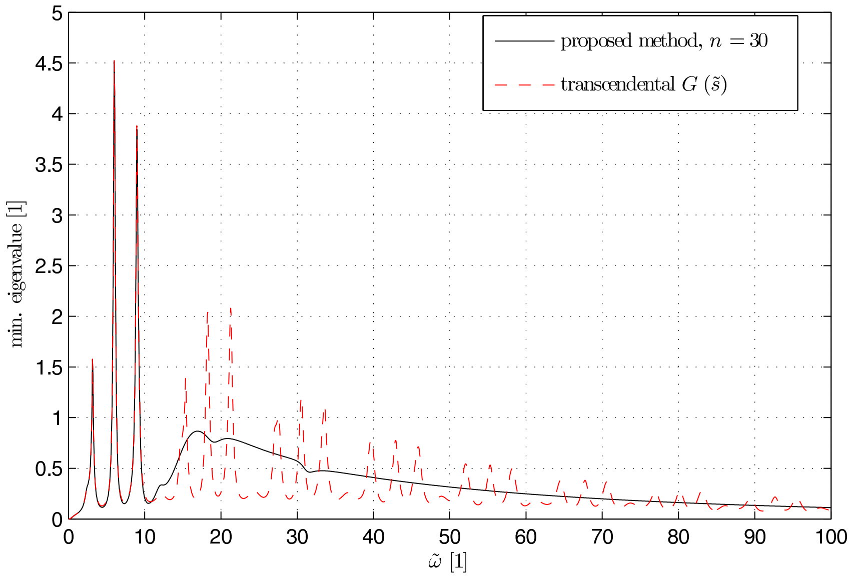

Figure 3: The minimum eigenvalue of $\mathbf{G}_{approx}\left(j\tilde{\omega}\right) + \mathbf{G}^{*}_{approx}\left(j\tilde{\omega}\right)$ over $0 \leq \omega \leq 5\omega_{max}$.

have strictly negative real parts. Therefore, the poles of the transfer function (11) are now restricted to lie in the left open half plane. With this assumption, the model is passive if and only if [Kha96]

$$\mathbf{G}_{approx}\left(j\tilde{\omega}\right) + \mathbf{G}^{*}_{approx}\left(j\tilde{\omega}\right) \geq \mathbf{0} \qquad (15)$$

for all real $\tilde{\omega}$.

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3 Order reduction with passivity constraints

The proposed nonlinear programming approach does not contain any information about the choice of the reduced system order n. One approach would be to use some estimate on the system order and then to solve the optimization problem for fixed n. The method proposed in this paper uses the fact that with increasing system order the approximation error must decrease. An iterative procedure is started with a low approximation order n. The solution of this low order problem is then used for constructing the starting point for the next problem with order $n+1$. This process is repeated until either the approximation error falls below a desired error margin or the order gets too high.

Before the objective function for the optimization problem is coded, a parameterization of the system (8) must be chosen. The general form using all entries of the unknown matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ is a bad choice due to the high number of parameters. While the n by n matrix $\mathbf{A}$ containts n^2 entries, the n eigenvalues of $\mathbf{A}$ sufficiently characterise the influence of $\mathbf{A}$ on the input-output behaviour of the system (8). Among possible candidates for a parameterization of the system (8) are the non-zero matrix entries of the modal or companion form of the system as well as the coefficients of the corresponding transfer function. Numerical experiments conducted during the implementation of the proposed method showed the best results with the modal form. The price to be paid for the benefitial numerical behaviour of the modal form is that the structure of the parameterization changes with the eigenvalue structure of $\mathbf{A}$. The case $n = 4$ for instance offers three possibilites for the structure of the matrix $\mathbf{A}$ in modal form. The first one features four real eigenvalues

$$A = \begin{bmatrix} a_1 & & & \\ & a_2 & & \\ & & a_3 & \\ & & & a_4 \end{bmatrix}$$

the second one has a complex conjugate pair of eigenvalues and two real eigenvalues

$$A = \begin{bmatrix} a_1 & -a_2 & & \\ a_2 & a_1 & & \\ & & a_3 & \\ & & & a_4 \end{bmatrix}$$

and the third one has two pairs of complex conjugate eigenvalues

$$A = \begin{bmatrix} a_1 & -a_2 & & \\ a_2 & a_1 & & \\ & & a_3 & -a_4 \\ & & a_4 & a_3 \end{bmatrix}$$

The matrices $\mathbf{B}$ and $\mathbf{C}$ are simply parameterized by all their entries. In this canonical form, the matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ contain a number of $5\,n$ parameters to be optimized in the nonlinear programming problem. The frequnecy axis is restricted to the interval $\tilde{\omega}_{min} \leq \tilde{\omega} \leq \tilde{\omega}_{max}$and discretized by a number of N_ω equally spaced points

$$\tilde{\omega}_k = \tilde{\omega}_{min} + \frac{k-1}{N_\omega - 1}\left(\tilde{\omega}_{max} - \tilde{\omega}_{min}\right), \qquad k = 1, 2, \ldots N_\omega \quad (16)$$

The integral in the cost function (12) is replaced by a sum of quadratic errors at the points $\tilde{\omega}_k$ and the passivity criterion (15) is introduced as a constraint at every point $\tilde{\omega}_k$. This simple strategy will fail to work if the passivity criterion is violated between any two frequency points $\tilde{\omega}_k$ and $\tilde{\omega}_{k+1}$ as well as for $0 \leq \tilde{\omega} \leq \tilde{\omega}_{min}$ and $\tilde{\omega} > \tilde{\omega}_{max}$. A continuation method using a relaxed passivity criterion

$$\mathbf{G}_{approx}\left(\delta + j\tilde{\omega}_k\right) + \mathbf{G}^*_{approx}\left(\delta + j\tilde{\omega}_k\right) \geq 0, \qquad k = 1, 2, \ldots N_\omega \quad (17)$$

is used to overcome the first problem, i.e. the passivity constraint violation between the grid points. The nonlinear programming problem is first solved for a value $\delta = \delta_0 > 0$ and the result is used as a start value for a problem with a slightly reduced relaxation parameter δ. While the cost function always uses the discretization of ω defined in (16), the points at which the constraints (17) are evaluated are altered after the initial problem with $\delta = \delta_0$ is solved. For the computation of the new constraint grid, the passivity criterion is evaluated on a very fine, equally spaced grid over the frequency range from $\tilde{\omega}_{min}$to $\tilde{\omega}_{max}$. The updated grid has the same number N_ω of points but is dense where the criterion is violated and coarse where the minimum eigenvalue of $\mathbf{G}_{approx} + \mathbf{G}^*_{approx}$ is positive. A variable step continuation method for the relaxation parameter δ is used to decrease the value of δ gradually down to 0. The constraint grid update is performed in each iteration of the continuation method. Clearly, this approach results in an expensive series of numerical computations. However, the goal of this paper is not to find a fast method for the derivation of a transmission line model, but to find the optimal approximation of the transmission line dynamics for a given order of the reduced system. This will allow for the assessment of existing models with respect to computational efficiency.

The resulting large scale nonlinear programming problems are solved with the sequential quadratic programming code SNOPT [HGE05]. In the implementation of the constraint function (17) an analytical solution for the smaller eigenvalue of the 2 by 2 matrix $\mathbf{G}_{approx} + \mathbf{G}^*_{approx}$ is used. The gradients both for the cost function as well as for the constraint functions are computed in analytical form and made available to the SNOPT solver.

The results shown in this paper have been obtained with $\tilde{\omega}_{min} = 0.1$, $\tilde{\omega}_{max} = 20$ and $N_\omega = 500$. Figure 2 shows the step responses for reduced models of order $n = 13$ compared to "exact" step responses computed from the transcendental transfer function matrix (11) by numerical inverse Laplace transorm [Wee66]. The minimal eigenvalue associated with the criterion (15) is shown over the frequency range $0 \leq \tilde{\omega} \leq 5\,\tilde{\omega}_{max}$ in Fig. 3. The frequency range above ω_{max} does not contain any new information, in particular no violation of the passivity constraint. This has been checked after the optimization.

4 Conclusions

A method for the calculation of a linear, continuos-time, reduced order, passive model for laminar, transient pipe flow has been proposed. For a compound transmission line system representing the pipework of a hydraulic wave converter, a reduced order model of order 13 has been shown to give a good approximation of the step responses calculated by numerical inverse Laplace transform. Thus, it is possible to describe the input-output behaviour of transmission lines featuring the so called 'frequency-dependent' friction by low order models. Particularly important is the preservation of the passivity of the real system in the mathematical modelling approach.

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