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Large eddy simulation of re-suspension of solid particles from an erodible bed

F. GALLERANO, L. MELILLA, G. CANNATA
Dipartimento di Idraulica Trasporti e Strade
Università di Roma "La Sapienza"
Via Eudossiana 18, Roma
ITALY

Abstract: - In the present paper, a LES model, developed for the simulation of turbulent channel flow (in which the generalized SGS stress tensor is related to the SGS turbulent kinetic energy and SGS viscous dissipation), will be extended to include the simulation of re-suspension of solid particles from an erodible bed. The solid particle concentration fields are simulated by numerical integration of the spatially filtered equation of concentration. In this equation the first order tensor (produced by the second order generalized central moment relative to the correlation between velocity and concentration) is related to the gradient of the resolved concentration by means a second order tensor: the coefficient that is present in this closure relation is calculated by a dynamic procedure.

Key-Words: - LES, solid particle, re-suspension

1 Introduction

Recently, the large eddy simulation technique (LES) has been shown to be a promising approach for computational of turbulent transport and dispersion of a passive scalar [1].
The underlying reason for this success is due to the fact that in LES the large scale turbulent mixing is carried out by the resolved field and is not relinquished to a model.
To predict the subgrid-scale variance of a conserved scalar, Cook and Riley [2] proposed a scale similarity model. The model has since been used in a LES of nonpremixed combustion in homogeneous isotropic turbulence [3] and *a priori* tested in a turbulent mixing layer [4].
Pierce and Moin emphasized that the major drawback of this approach is that it requires input from the user in the form of a model coefficient. Furthermore, there is no reason to expect that a "universal value" for the model coefficient exists, except within a well developed inertial subrange. In general, the coefficient could vary with flow type, characteristics of the grid and test filters, Reynolds numbers, etc. [1].
Pierce and Moin overcame the mentioned drawbacks: they applied a dynamic procedure to obtain the model coefficient that appears in the closure relation for the subgrid variance of a conservative scalar.
In the present work, a LES model [5] developed for the simulation of turbulent channel flow (in which the generalized SGS stress tensor is related to the SGS turbulent kinetic energy and SGS viscous dissipation) will be extended to include the simulation of re-suspension of solid particle from an erodible bed in a channel flow.
The solid particle concentration field is simulated by numerical integration of the spatially filtered equation of concentration.
In this equation the first order tensor (produced by the second order generalized central moment relative to velocity and concentration) is related to the gradient of the resolved concentration by means a second order tensor that takes into account the local non uniform characteristics of the spatial filters.
The coefficient, that is present in this closure relation, is calculated by a dynamic procedure.

2 The mathematical model

The unsteady three-dimensional turbulent velocity field is simulated by numerical integration of the spatially filtered momentum equation.
The application of the spatial filter operator (indicated by the overbar $\overline{\quad}$) to the Navier Stokes equations takes to the filtered equations:

$$\frac{\partial \overline{u}_k}{\partial x_k} = 0, \tag{1}$$

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial \overline{u}_i \overline{u}_j}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} + \nu \frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} - \delta_{i3} g \ , \tag{2}$$

In the relation (2) there is the generalized SGS turbulent stress tensor τ_{ij}, that is given by:

$$\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j \tag{3}$$

The SGS turbulent stress tensor can be split into three tensors:

$$\tau_{ij} = L_{ij}^m + C_{ij}^m + R_{ij}^m$$

called, respectively, modified Leonard tensor, modified cross tensor and modified Reynolds tensor and defined as follows [6]:

$$L_{ij}^m = \tau(\overline{u}_i, \overline{u}_j) = \overline{\overline{u}_i \overline{u}_j} - \overline{\overline{u}}_i \overline{\overline{u}}_j \tag{4}$$

$$C_{ij}^m = \tau(\overline{u}_i, u_j') + \tau(u_i', \overline{u}_j) = \overline{\overline{u}_i u_j'} - \overline{\overline{u}}_i \overline{u_j'} + \overline{u_i' \overline{u}_j} - \overline{u_i'}\, \overline{\overline{u}}_j \tag{5}$$

$$R_{ij}^{m} = \tau(u_i^{'}, u_j^{'}) = \overline{u_i^{'} u_j^{'}} - \overline{u_i^{'}}\ \overline{u_j^{'}} \qquad (6)$$

In this paper the closure relation [5] for the generalized SGS turbulent stress tensor τ_{ij} is given by:

$$\tau_{ij} = \left(\frac{2E}{L_{kk}^{m}} \right) L_{ij}^{m} \qquad (7)$$

where E is the generalized SGS turbulent kinetic energy which is calculated by numerically integrating its balance equation. The viscous dissipation ε of the generalized SGS turbulent kinetic energy is calculated by numerical integrating the ε balance equation; the closure coefficients that appears in the closure relation for the unknown tensors in the E balance equation and in the ε balance equation are dynamically calculated by means of the Germano identities. The filtered equation of the concentration of suspended solids is given by:

$$\frac{\partial \overline{C}}{\partial t} + \frac{\partial \overline{u}_i \overline{C}}{\partial x_i} + \frac{\partial}{\partial x_i}\left(\overline{u_i C} - \overline{u}_i \overline{C} \right) = 0 \qquad (8)$$

where $\overline{C}$ represents the spatially filtered concentration field.

The last term of Equation (8) is a first-order tensor given by the generalized central moment relative to the velocity vector and the concentration and is defined as follows:

$$a_i = \tau(u_i, C) = \left(\overline{u_i C} - \overline{u}_i \overline{C} \right) \qquad (9)$$

The above tensor is split in terms of generalized central moments:

$$a_i = L_i^{C} + C_i^{C} + R_i^{C} \qquad (10)$$

where:

$$L_i^{C} = \tau\left(\overline{u}_i, \overline{C}\right) = \overline{\overline{u}_i \overline{C}} - \overline{\overline{u}}_i \overline{\overline{C}} \qquad (11)$$

$$C_i^{C} = \tau\left(\overline{u}_i, C^{'}\right) + \tau\left(u_i^{'}, \overline{C}\right) = \left(\overline{\overline{u}_i C^{'}} - \overline{\overline{u}}_i \overline{C^{'}} + \overline{u_i^{'} \overline{C}} - \overline{u^{'}}\ \overline{\overline{C}} \right) \qquad (12)$$

$$R_i^{C} = \tau\left(u_i^{'}, C^{'}\right) = \overline{u_i^{'} C^{'}} - \overline{u_i^{'}}\ \overline{C^{'}} \qquad (13)$$

The sum of the second and third term on the right hand side of Equation (10) represents an unknown first-order tensor. In this paper a new closure relation for this unknown tensor is proposed.

The closure relation between the sum of the unknown unresolved tensors $C_i^{C} + R_i^{C}$ and the filtered concentration is given by:

$$C_i^{c} + R_i^{c} = \nu_{ij}^{C} \frac{\partial C}{\partial x_i} \qquad (14)$$

where ν_{ij}^{C} represents a second order tensor that is proportional to the turbulence subgrid velocity scale and the turbulence subgrid length scale. This tensor is defined by:

$$\nu_{ij}^{C} = c_c \sqrt{E} d_{ij} \qquad (15)$$

In order to take into account the local non uniform characteristics of the spatial filters, the second-order tensor d_{mn} is defined as:

$$d_{mn} = \Delta_m \Delta_n / \left(\Delta_1 \Delta_2 \Delta_3 \right)^{\frac{1}{3}} \qquad (16)$$

in which Δ_i is the vector of which the components are the filter dimensions in the three coordinate directions.

By introducing Eq. (15) in (14), the following closure relation is obtained:

$$C_i^{C} + R_i^{C} = c_c \sqrt{E} d_{in} \frac{\partial \overline{C}}{\partial x_n} \qquad (17)$$

Let $\widehat{(\cdot)}$ be the symbol that indicates the filter operation at the test level and let $T\left(u_i, C\right)$ be the generalized central moment at the test level related to the velocity vector and the concentration, the following first-order tensor is defined

$$A_i = T\left(u_i, C\right) = \widehat{\overline{u_i C}} - \widehat{\overline{u}}_i \widehat{\overline{C}} \qquad (18)$$

which in terms of generalized central moments can be split as:

$$A_i = L_i^{C^T} + C_i^{C^T} + R_i^{C^T} \qquad (19)$$

where:

$$L_i^{C^T} = T\left(\overline{u}_i, \overline{C}\right) = \widehat{\overline{u}_i \overline{C}} - \widehat{\overline{u}}_i \widehat{\overline{C}} \qquad (20)$$

$$C_i^{C^T} = T\left(\overline{u}_i, C^{'}\right) + T\left(u_i^{'}, \overline{C}\right) = \left(\widehat{\overline{u}_i C^{'}} - \widehat{\overline{u}}_i \widehat{C^{'}} + \widehat{u_i^{'} \overline{C}} - \widehat{u^{'}}\ \widehat{\overline{C}} \right) \qquad (20)$$

$$R_i^{C^T} = T\left(u_i^{'}, C^{'}\right) = \widehat{u_i^{'} C^{'}} - \widehat{u_i^{'}}\ \widehat{C^{'}} \qquad (21)$$

The sum of the last two terms on the right hand side of Equation (19) is an unknown quantity that is modeled with an expression analogous to Equation (17):

$$C_i^{C^T} + R_i^{C^T} = c_c \sqrt{E^T} d_{in}^{T} \frac{\partial \widehat{\overline{C}}}{\partial x_n} \qquad (22)$$

The calculation of the coefficient c_c is carried out by using the following identity

$$A_i - \widehat{a}_i = \widehat{u_i C} - \widehat{u}_i \widehat{C} \qquad (23)$$

Equation (23), with the use of the closure relations expressed by Equations (17) and (22), becomes

$$c_c \sqrt{E^T} d_{in}^{T} \frac{\partial \widehat{\overline{C}}}{\partial x_n} - c_c \widehat{\sqrt{E} d_{in} \frac{\partial \overline{C}}{\partial x_n}} = \widehat{u_i C} - \widehat{\overline{u}}_i \widehat{\overline{C}} - \widehat{\overline{u}_i \overline{C}} + \widehat{\overline{u}}_i \widehat{\overline{C}} + \widehat{u_i \overline{C}} - \widehat{u_i \overline{C}} \qquad (24)$$

where E^T indicates the SGS kinetic energy relative to the test-filter and d_{in}^{T} indicates the second-order tensor associated with the turbulence length scales relative to the test-filter.

Equation (24) allows the dynamic calculation of the coefficient c_c.

The numerical integration of Equation (8) for the simulation of the filtered concentration field of suspended solids may be carried out once the boundary conditions have been defined.

The plane, in proximity to the bottom, which defines the boundary for the concentration field is placed immediately above the viscous sublayer, inside the buffer layer: the

"Reference Concentration", C_r, is imposed on this plane as a boundary condition.

This reference concentration is calculated as a function of the resolved tangential stress at the bottom and the critical stress representing the threshold beyond which the movement of solid particles from the bottom is produced.

In particular, in the proposed model the value of the local and instantaneous reference concentration, C_r, is related to the resolved velocity field at the bottom by means of the formula proposed by Van Rijn [7]. The aforesaid Van Rijn formula reads:

$$C_r = 0{,}015\left(\frac{D_p}{a}\right)\left(\frac{T^{1,5}}{D_*^{0,3}}\right) \qquad (25)$$

in which

a: distance from the bottom at which C_r is calculated

D_p: diameter of the solid particles

$$D_* = D_p\left[\frac{(\delta-1)g}{\nu}\right]^{1/3} \quad \begin{cases} \delta: \text{relative density of the particle} \\ \nu: \text{kinetic viscosity} \end{cases} \qquad (26)$$

$$T = \frac{\bar{u}_*^2 - u_{*crit}^2}{u_{*crit}^2} \qquad (27)$$

and the quantity $\bar{u}_*^2$ which appears in Equation (27) is linked to the tangential stress at the bottom, τ_f, produced at each time step by the tangential resolved velocity component at the bottom, $\bar{u}$, by means of the following relation:

$$\bar{u}_*^2 = \frac{\tau_f}{\rho} = \nu\frac{\partial \bar{u}}{\partial y} \qquad (28)$$

3 Results and discussion

The symbol $\langle . \rangle$ represents the Reynolds average and the friction velocity based Reynolds number is given by:

$Re_\tau = \langle u_* \rangle L/\nu$

The proposed model is used to simulate the phenomenon of the re-suspension (from the bottom) of solid particles in a channel flow.

The values of the diameter and the relative density of the solid particles are, respectively, equal to 200 μm and 1.40.

The "large eddy simulation" of the velocity and concentration fields, in a channel flow, is performed at Reynolds number $Re_\tau = 395$.

In Figures 1-5 a sequence of longitudinal sections of instantaneous solid particles concentration fields (relative to the transitory interval during which the phenomenon of the re-suspension begins and develops from the bottom and the transport of solid particles develops inside the channel) is shown. The mean flow is directed towards the growing x axis. Periodic boundary conditions on the velocity and concentration fields in the streamwise direction allow the numerical simulation of the concentration field which occurs in a channel of infinite length having an erodible particle bed.

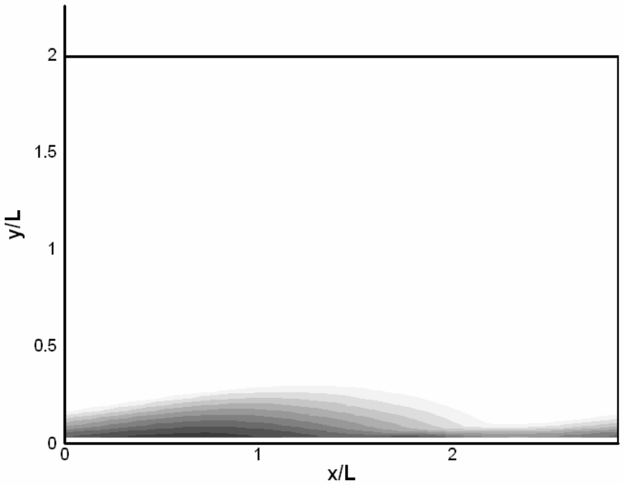

Figure 1. Longitudinal section of the filtered concentration field. Channel flow, $Re_\tau = 395$.

Figure 1 shows the filtered concentration field (at an instant of the numerical simulation): from the figure it is possible to deduce how some vortices (characterized by a high content of the turbulent kinetic energy) produce instant values of friction velocity that is able to start the re-suspension of the solid particles.

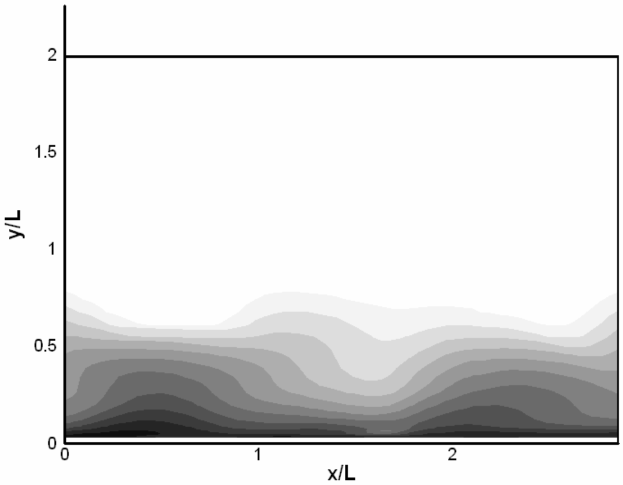

Figure 2. Longitudinal section of the filtered concentration field. Channel flow, $Re_\tau = 395$.

In Figures 2 it is shown how the concentration field values change over time: these variations are the result of the streamwise movement of the aforesaid vortices: the increases of the instantaneous friction velocity (associated to the vortices) and the streamwise movement of the vortices, with high value of kinetic energy, produce an increase of the value of the reference concentration and a streamwise movement of the bottom regions characterized by the re-suspension of the solid particles.

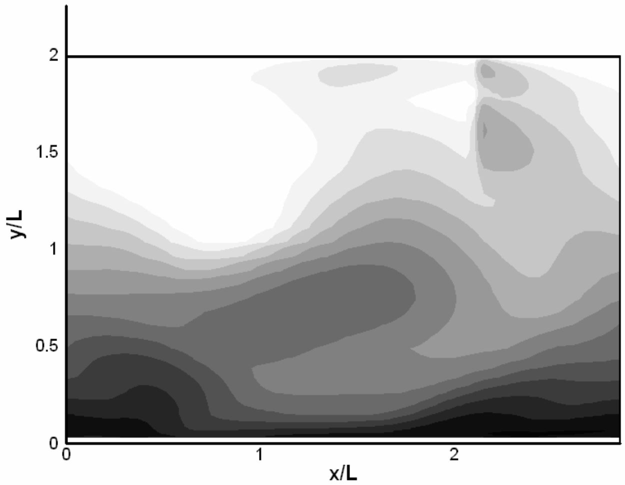

Figure 3. Longitudinal section of the filtered concentration field. Channel flow, $Re_\tau = 395$.

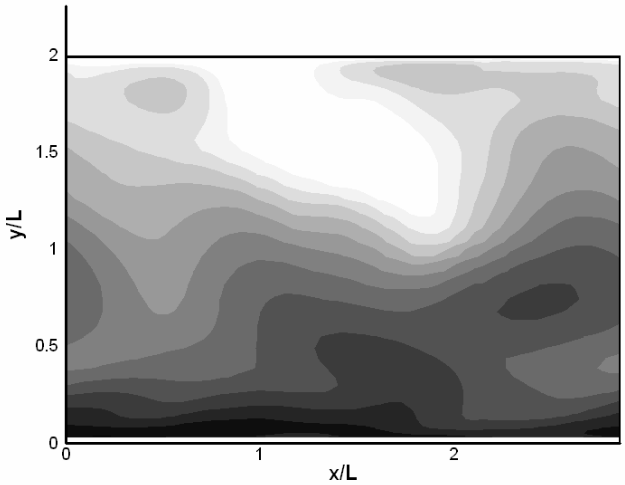

Figure 4. Longitudinal section of the filtered concentration field. Channel flow, $Re_\tau = 395$.

In figure 3 it is shown the filtered concentrations field at a successive step of the simulation: from the figure its possible to see that the solid particle (that have been suspended from the bottom) come to the central region of the channel and are transported downstream by the high velocity of the current.

Figure 6 shows a spanwise section of the filtered concentration field (at an instant of the numerical simulation).

After a high number of iterations a statistically steady condition for both the solid and the liquid phase is verified.

By applying a Reynolds average to the spatially filtered concentration values (for a time higher than the integral time scale of turbulence) a statistically steady vertical concentration profile of the solids in suspension is obtained:

$$\left\langle \overline{C}(y) \right\rangle = \int_0^{T_0} \overline{C}(y,t)dt \tag{29}$$

where y is the vertical coordinate and T_0 the time over which the average is calculated.

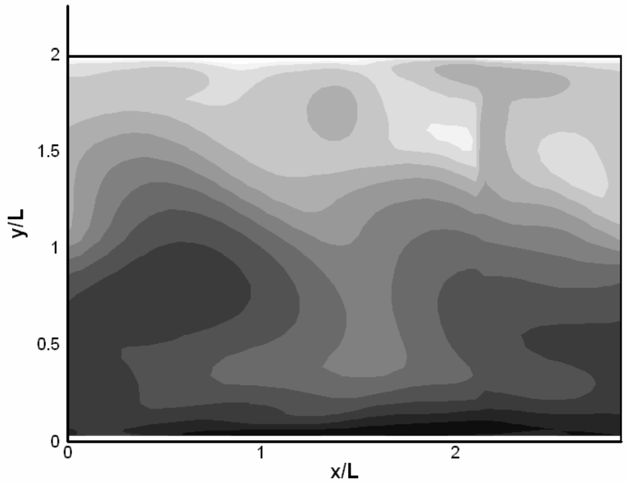

Figure 5. Longitudinal section of the filtered concentration field. Channel flow, $Re_\tau = 395$.

In figure 5 it is shown the fully developed resolved concentration field.

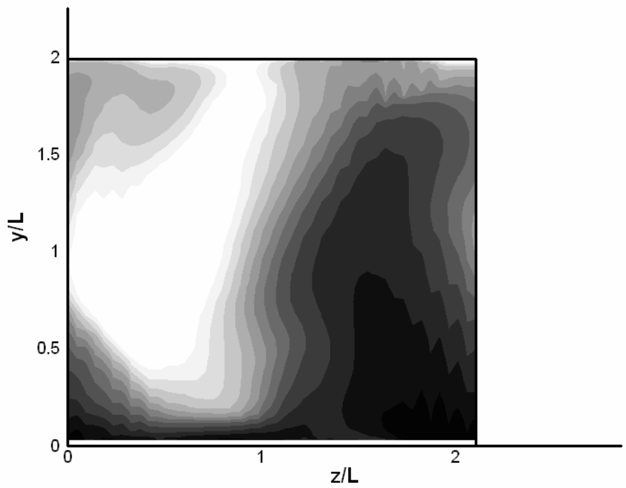

Figure 6. Spanwise section of the filtered concentration field. Channel flow, $Re_\tau = 395$.

The vertical concentration profile, obtained by averaging (over time) the spatially filtered concentration values (calculated by means of the proposed numerical model), are compared with the theoretical profile of the Reynolds averaged concentration values in the same average hydrodynamic conditions and for the same particle characteristics.

The theoretical profile of the Reynolds averaged concentration values is calculated by adopting the following Van Rijn's formula:

$$\frac{\langle C \rangle}{\langle C_r \rangle} = \left[\frac{a(H-y)}{y(H-a)} \right]^{Z'} \tag{30}$$

where

$$Z' = Z + \tau \tag{31}$$

$$\tau = 2.5 \left[\frac{\omega_0}{\langle u_* \rangle} \right]^{0.8} \left[\frac{\langle C_r \rangle}{0.65} \right]^{0.4} \tag{32}$$

$$Z = \frac{\omega_0}{K_\beta k u_*} \quad (33)$$

$$K_\beta = 1 + 2\left[\frac{\omega_0}{\langle u_* \rangle}\right]^2 \quad (34)$$

ω_0 = terminal velocity (35)

k = Von Karman constant (36)

and $\langle C_r \rangle$ is the reference concentration calculated by means of Equation (25) in which

$$T = \frac{\langle u_* \rangle^2 - u_{*crit}^2}{u_{*crit}^2} \quad (37)$$

and the parameter a, that appears in the Equation (25) and (30), is assumed equal to the distance from the bottom where the plane on which the Reference Concentration is assigned. In this case a is assumed equal to 12 wall units, where the wall unit

$$y^+ = \frac{y \langle u_* \rangle}{\nu} \quad (38)$$

is calculated using the friction velocity obtained with the Reynolds number of the numerical simulation of the channel flow:

$$\langle u_* \rangle = \frac{Re_\tau \nu}{L} \quad (39)$$

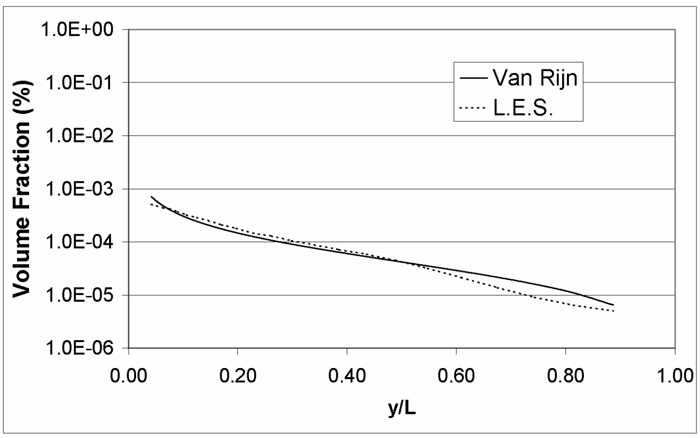

Figure 7. Comparison between Van Rijn's formula values and Reynolds averaged LES values.

In Figure 7 the vertical concentration profile, obtained by averaging (over time) the spatially filtered concentration values (calculated by means of the proposed numerical model), is compared with the vertical concentration profile calculated theoretically by means of Van Rijn's formula.

4 Conclusions

In the present paper, a LES model, developed for the simulation of turbulent channel flow (in which the generalized SGS stress tensor is related to the SGS turbulent kinetic energy and SGS viscous dissipation), is extended to include the simulation of re-suspension of solid particle from an erodible bed. The solid particle concentration field is simulated by numerical integration of the spatially filtered equation of concentration. In this equation the first order tensor (produced by the second order generalized central moment relative to the correlation between velocity and concentration) is related to the gradient of the resolved concentration by means a second order tensor: the coefficient that is present in this closure relation is calculated by a dynamic procedure.

The proposed model is used to simulate the phenomenon of the re-suspension (from the bottom) of solid particles in a channel flow.

The values of the diameter and the relative density of the solid particles are, respectively, equal to 200 μm and 1.40.

The "large eddy simulation" of the velocity and concentration fields in a channel flow is performed at $Re_\tau = 395$.

The vertical concentration profile, obtained by averaging (over time) the spatially filtered concentration values (calculated by means of the proposed numerical model), are in agreement with the theoretical data.

References:

[1] Pierce C.D., Moin P., A dynamic model for sub-grid scale variance and dissipation rate, *Physics of Fluids*, Vol.10 (12), 1998, pp. 3041-3044.

[2] Cook A.W., Riley J.J., A subgrid model for equilibrium chemistry in turbulent flows, *Phys. Fluids*, Vol. 6, 1994, pp. 2868.

[3] Reveillon J., Verisch L., Response of dynamicLES model to heat release induced effects, *Phys. Fluids*, Vol. 6, 1996, pp. 2248.

[4] Jimenez J., Linan A., Rogers M.M., HigueraF.J., A priori testing of subgrid models for chemically reacting nonpremixed turbulent shear flows by assumed PDF methods, *Engineering Applications of Large Eddy Simulations*, ASME, 1993.

[5] Gallerano F., Pasero E. & Cannata G., A dynamic two-equation Sub Grid Scale model, *Continuum Mech. Thermodyn*, Vol. 17, 2005, pp. 101-123.

[6] Germano M., Turbulence: the filtering approach., *J. Fluid Mech.*, Vol. 238, 1992, pp. 323-336.

[7] Van Rijn L.C., Sediment transport, *Delft Hydraulic Laborator*. Vol. 334, 1985.