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Analysis of a mixed convection in a partially porous duct with discrete wall heating

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Abstract: - The purpose of this paper is to analyze numerically the problem of the steady heat transfer by natural convection in a vertical cylinder opened at both ends and filled with a succession of fluid saturated porous elements. The study is carried out using the widely used Darcy-Brinkman flow model with establishment properties at the cylinder exit. The heat transfer coefficients are performed and discussed. In the case of constant wall temperature at the porous elements, the results show two types of flows, with and without fluid recirculation, which depend with the filtration Rayleigh number. This recirculation gives considerable augmentation in heat transfer in the vicinity of the interface porous-fluid.
Key-Words: - Natural convection - Circular duct - Porous elements – Discrete heating.

1 Introduction:

Open channels present a difficult problem to the mathematical formulation. Although many experimental and numerical works were published during the last decades, this problem remains difficult to solve when the formulation is based on the complete form of the conservative equations (the only one which make possible to approach reversal flows) in a geometry limited to the only channel, in particular because the writing of the boundary conditions of entry and of exit remains a largely open question. The heat transfer and the hydrodynamic problem in heated or cooled vertical channel were very largely studied since nearly fifty years. Indeed, the plane channel is representative of several problems like the chimney, the plane solar collector, or the cooling of the electronic components (Incropera, 1988).

One of the first experimental studies of the natural convection in a vertical channel is the paper of Elenbaas (1942), which determined the various modes of flow according to a modified Rayleigh number (Rayleigh number reported to the dimensionless aspect ratio of the channel). The results show that for low Rayleigh numbers, the flow is fully developed, whereas the boundary layer mode characterise the high Rayleigh numbers. Bar-Cohen and Rosenhow (1984) presented, using experimental and numerical results, a whole of valid correlations in various situations, for channels heated symmetrically or asymmetrically, with heat flux or constant wall temperatures imposed. In the symmetrical case, these correlations were recently improved by Olsson (2004). The first numerical studies, based on a parabolic formulation of the equations (axial diffusion neglected), confirmed these results (Sparrow and Azevedo, 1985).

Although many articles present experimental and numerical simulations for fluids in ducts, few results concerning the presence of porous media are available. One can quote the work of Hadim (1994) who considered a plane channel provided with a succession of porous substrates heated by a constant heat flux at bottom and adiabatic on the higher wall.

The aim of the present work is to treat numerically the problem of the steady natural convection, which occurs in a vertical cylinder, opened at both ends and filled with a succession of fluid saturated porous elements. The lateral wall is maintained at a constant temperature at the level of the porous elements and adiabatic condition is taken at the other parts of the walls. The study is carried out using the Darcy-Brinkman flow model and the set of equations is resolved by the finite volumes method.

2 Mathematical formulation:

A schematic of the physical model is shown in Figs. 1. It is assumed that the flow in the cylinder is two-dimensional. The porous medium is considered to be homogeneous, isotropic and saturated with a pure single-phase fluid, which is in thermal equilibrium with the solid matrix. The thermophysical properties of the solid matrix and the fluid are assumed to be constant, except in the body force term of the momentum equations invoking the Boussinesq's approximation. Although for certain porous medium (eg. spherical pearls) porosity can vary because of the Channeling effect close to the walls (Vafai, (1984)), for our part, we consider a confined porous medium, therefore permeability and porosity are constant (Hunt and Tien, (1988)). So, in the energy equation, viscous dissipation is neglected whereas axial conduction is taken into account which is considered can be important for small Reynolds numbers (Hadim and Govindarajan, (1988).

The current study assumes the validity of the Darcy-Brinkman flow model. In these conditions, dimensionless conservative equations are (Lauriat et Prasad, (1989)):

Continuity Equation:

Momentum Equations:

In the longitudinal direction x :

In the radial direction r :

$$\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial r}\right)=-\frac{\partial P}{\partial r}+GR1\left(\frac{\partial^2 v}{\partial x^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right)-\frac{v}{r^2}\right)$$
$$+GR2\cdot v$$

Energy Equation:

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}\right)=GR4\cdot\left(\frac{\partial^2 T}{\partial x^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right)$$

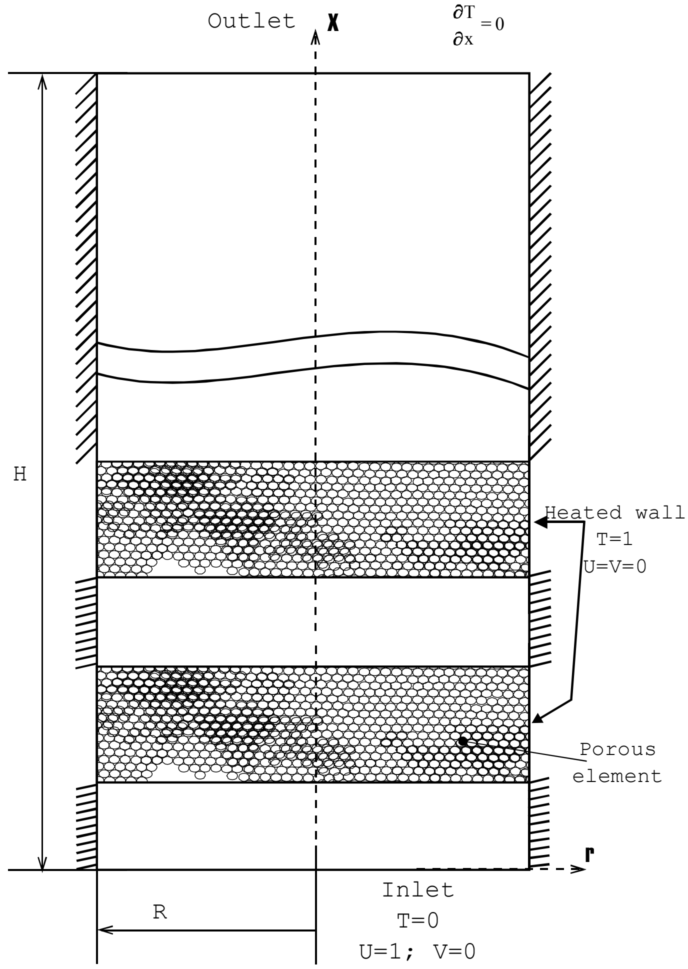

Fig1. Geometrical Configuration

Where the dimensionless groups in the equations are recapitulated as follows:

$$\begin{cases} GR1=\frac{R_v\,\varepsilon^2}{Re};\ \text{porous} \\ GR1=\frac{1}{Re};\ \text{fluid}\end{cases};\quad \begin{cases} GR2=-\frac{\varepsilon^2}{Da\cdot Re};\ \text{porous} \\ GR2=0;\ \text{fluid}\end{cases};\quad \begin{cases} GR3=\frac{\varepsilon^2 Ra}{Pr\,Re};\ \text{porous} \\ GR3=\frac{Ra}{Pr\,Re};\ \text{fluid}\end{cases};\quad \begin{cases} GR4=\frac{R_k}{Pr\cdot Re};\ \text{porous} \\ GR4=\frac{1}{Pr\cdot Re};\ \text{fluid}\end{cases}$$

3 Numerical Resolution:

The governing equations with the associated boundary conditions are solved by the finite volumes method, initiated by Patankar (1980), which is based on the solution of difference equations obtained by integrating the differential equations of conservation over control volumes enclosing the nodal points. The numerical program was validated for the classical forced convection compared to the analytic solution proposed by Ameziani and Bouhadef (2001)

4 Results:

We have presented the results of the numerical simulations in terms of curves showing evolution of space averaged Nusselt numbers and axial velocity at the centre evolution. Stream function map are also presented. In this paper, all the calculations have been performed for the air Pr=Sc=071, Aspect ratio A=10, Rk=1.

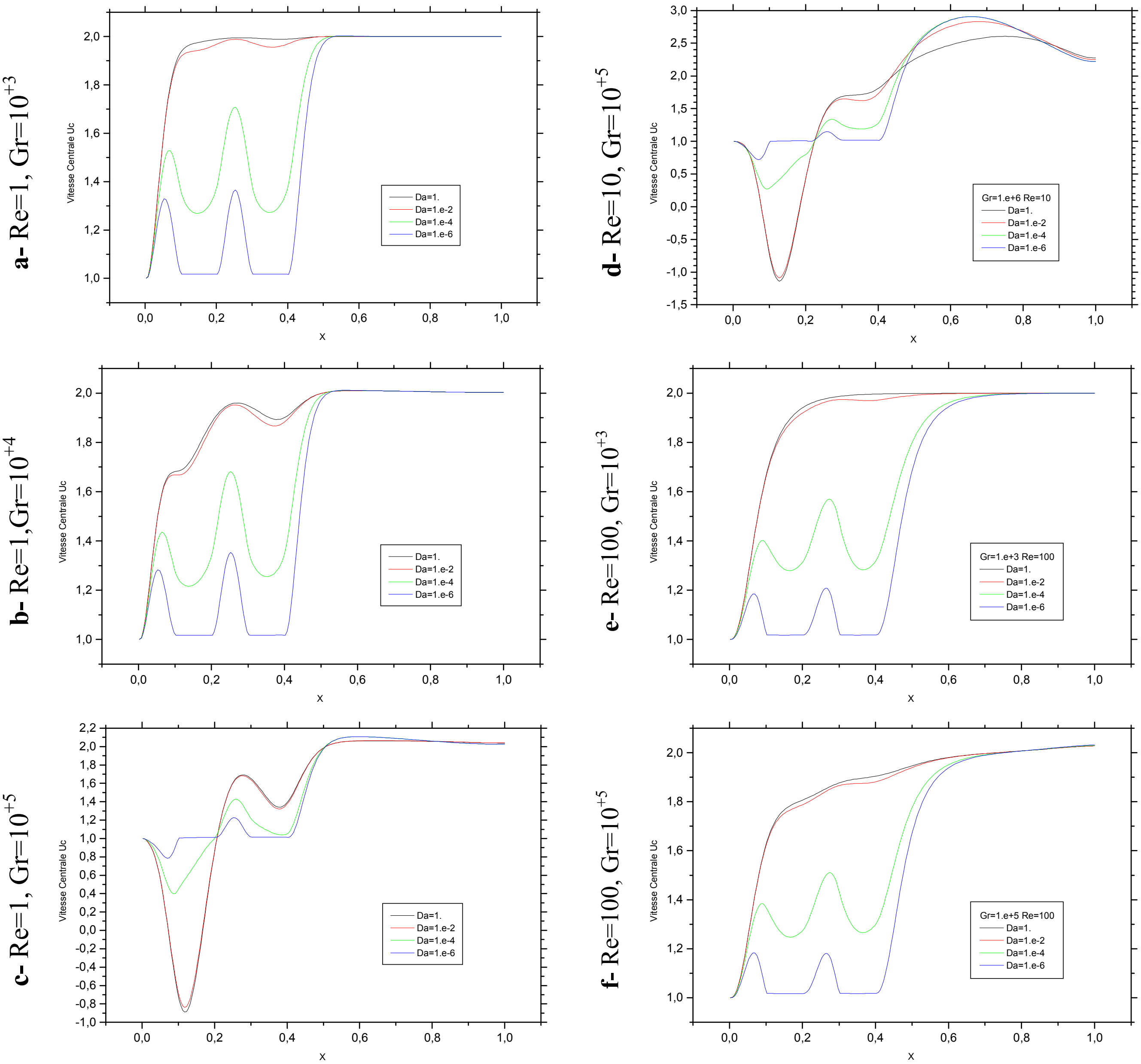

Figure 2 Evolution of the axial velocity along the duct.

Dynamical results:

Figures (2 a-f) illustrate the evolution of the axial velocity (Uc) along the cylinder for several values of the Darcy, Reynolds and Grashoff numbers.

For Re=1 and low values of the Grashoff number ($10^3 - 10^4$), figures (2-a) and (2-b) show a boundary layers development of the central velocity evolution, which develops starting from the flat profile at the entry ($Uc = 1$) and progressively tending towards an established profile of axial maximal value equal to twice the value of entry for $Da = 1$, representing the limit of forced convection in the fluid case.

The effect of the porous matrix appears clearly on these plots, where one can mention the reduction of the axial velocity due to braking in the porous elements to reach a total braking for $Da = 10^{-6}$, resulting in axial velocity values, in these matrices, of $Uc = 1$ (Darcian regime).

When the Grashoff number value increases, the effects of forced convection tend to vanish, to leave place to reversal fluid motion (natural convection prevalent) which is very accentuated with the increase of permeability. These flows downward have smaller values for lower permeabilities, fact precisely due to the braking when crossing the porous matrix.

When the Reynolds number is weak ($Re = 1$, fig. 2-a-c), one notices that the effects of the permeability remain visible for values of the Darcy numbers lower than the unity value (Da≤1), for low values of the Grashoff number ($Gr \leq 10^4$). When the Reynolds number becomes more important (Re=10), the effects of forced convection remain prevalent up to values of Gr of about 10^4.

For larger Reynolds numbers (Re=100), it clearly appears a prevalence of the forced convection until Grashoff numbers of about 10^5. The combined effects of mixed convection appear then, characterized by recirculation flow, in particular with high permeability (Da=1), and an acceleration of the fluid towards the exit of the channel (2-f figures)).

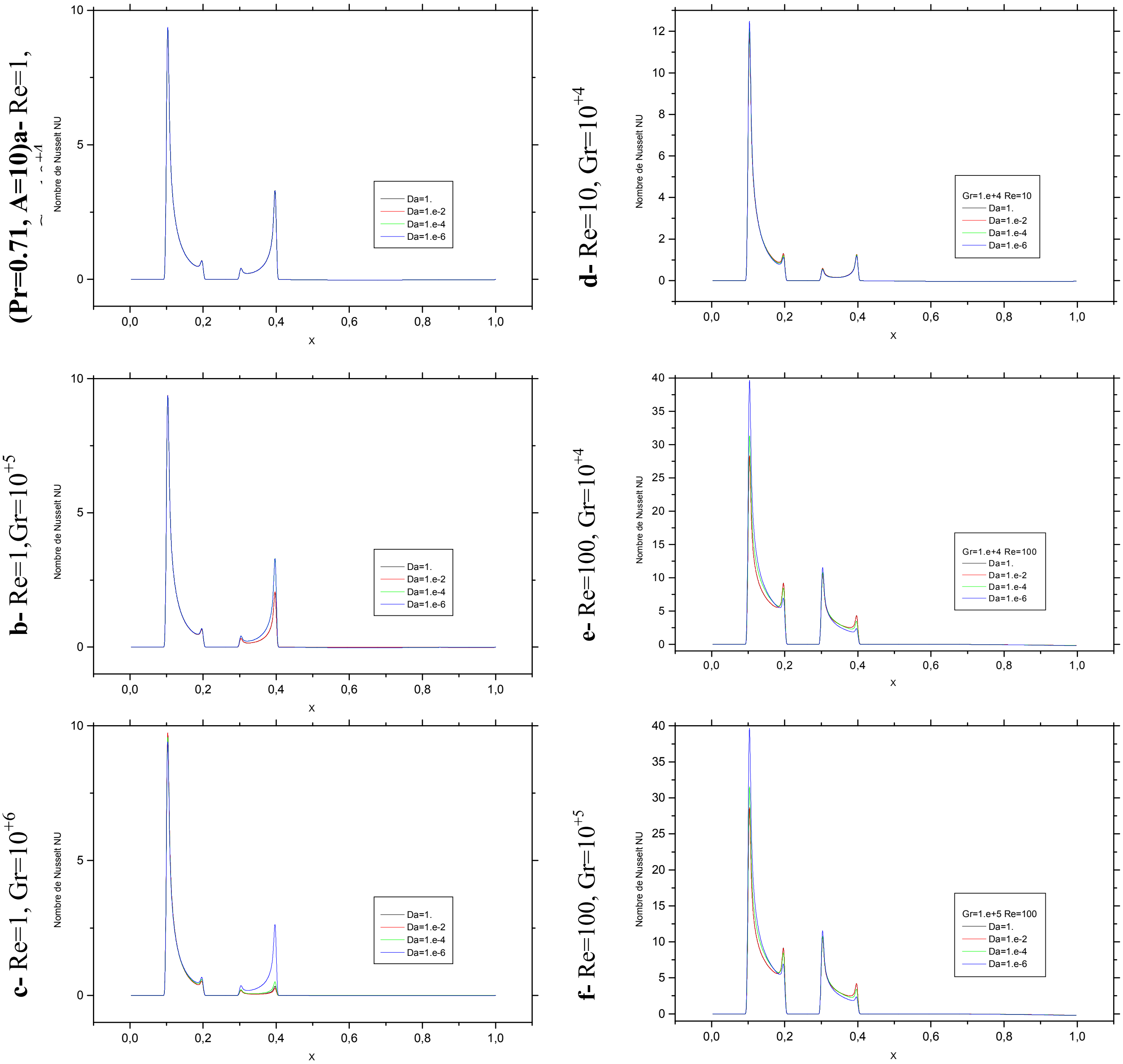

Figure 3 Evolution of the local Nusselt numbers along the duct. (Pr=0.71, A=10)

Thermal results:

On the figures (3-a-f) are represented the evolutions of the local Nusselt numbers according to the axial coordinate for various values of Grashoff and Darcy numbers.

It is noticed that for Re=1, the evolution of the transfer coefficient reveals an important value when one approaches the first porous element, this is due to the important variations in temperature between the wall and the entering fluid (first heat source). The transfer is then decreasing when it crosses this porous obstacle. Thereafter the other end of the source presents another peak of increase in transfer which corresponds to a temperature gradient due to the important variations in temperature between the fluid and the wall in the vicinity of the interface porous-fluid. Indeed, alternation between the porous zones corresponding to parts of hot walls and the adiabatic colder zones, causes a downward return of the fluid particles in the vicinity of the interface porous-fluid.

On the figures (3-d-f) the evolutions of the local Nusselt number along the duct are represented, for two other values of the Reynolds number (10 and 100). It is noticed that, for all the other fixed parameters, the exchanges are more important as the Reynolds number is high, which is physically in conformity insofar as the Re increases augments the inertial effects and the fluid flow and thus increases the quantities of energy extracted by convection. Nevertheless, when the Reynolds number is weak, the peak of increase in the transfers at the end, in particular of the second porous matrix, allotted to the fluid recirculation is relatively greater than for the cases with higher values of Reynolds. This is explained by the fact that the movements of recirculation of natural convection, which cool the fluid in this vicinity, are more active with low velocity ascending flow.

For high Reynolds numbers (Fig. 3-e,f), one notices that the peaks of the end of matrices tend to become relatively weak, which can be also explained in this case, with the fact that the upswing of forced convection will override the effects of natural convection. It is clear on these figures that the permeability presents a considerable effect on the heat transfer. It is noticed that these exchanges decrease with the reduction of the Darcy number. For very weak Darcy numbers, the convective transfer vanishes more and more and tends towards diffusive transfers, in particular for Grashoff numbers, Gr≤10+4. This decrease of the transfers is due to the weakening of the flow with the decrease of the permeability; the convective flow is then less and less important and induces a conductive mode.

5 Conclusion :

The problem of the steady heat transfer by natural convection in a vertical cylinder opened at both ends and filled with a succession of fluid saturated porous elements is numerically analysed. The heat transfer coefficient is performed and discussed. In the case of constant wall temperature at the porous elements, the results show two types of flows, with and without fluid recirculation, which depend with the Grashoff number.

For low Grashoff numbers $Gr \leq 10^4$, even for small Reynolds numbers (Re=1), the axial velocity evolution have a behaviour of boundary layer development in forced convection. In fact, it is the effect of the forced convection which overrides the effects of the buoyancy forces which are not enough powerful to start the process of the natural convection. This behaviour is confirmed by the stream function maps (fig. 4-a).

The growth of the Grashoff number increases maximum central velocity, and this increase is more important as the Reynolds number augments.

Let us note that starting from a value of the Grashoff number of about 10^5 (fig. 4-b-e), recirculation zones appear, traduced by a negative central velocity *Uc* and a prevalence of the natural convection effects. This phenomenon is confirmed by the stream function maps on which the swirls of turnoff flow appears clearly. These latter are more accentuated in the areas close to the porous obstacles, when the permeability is important.

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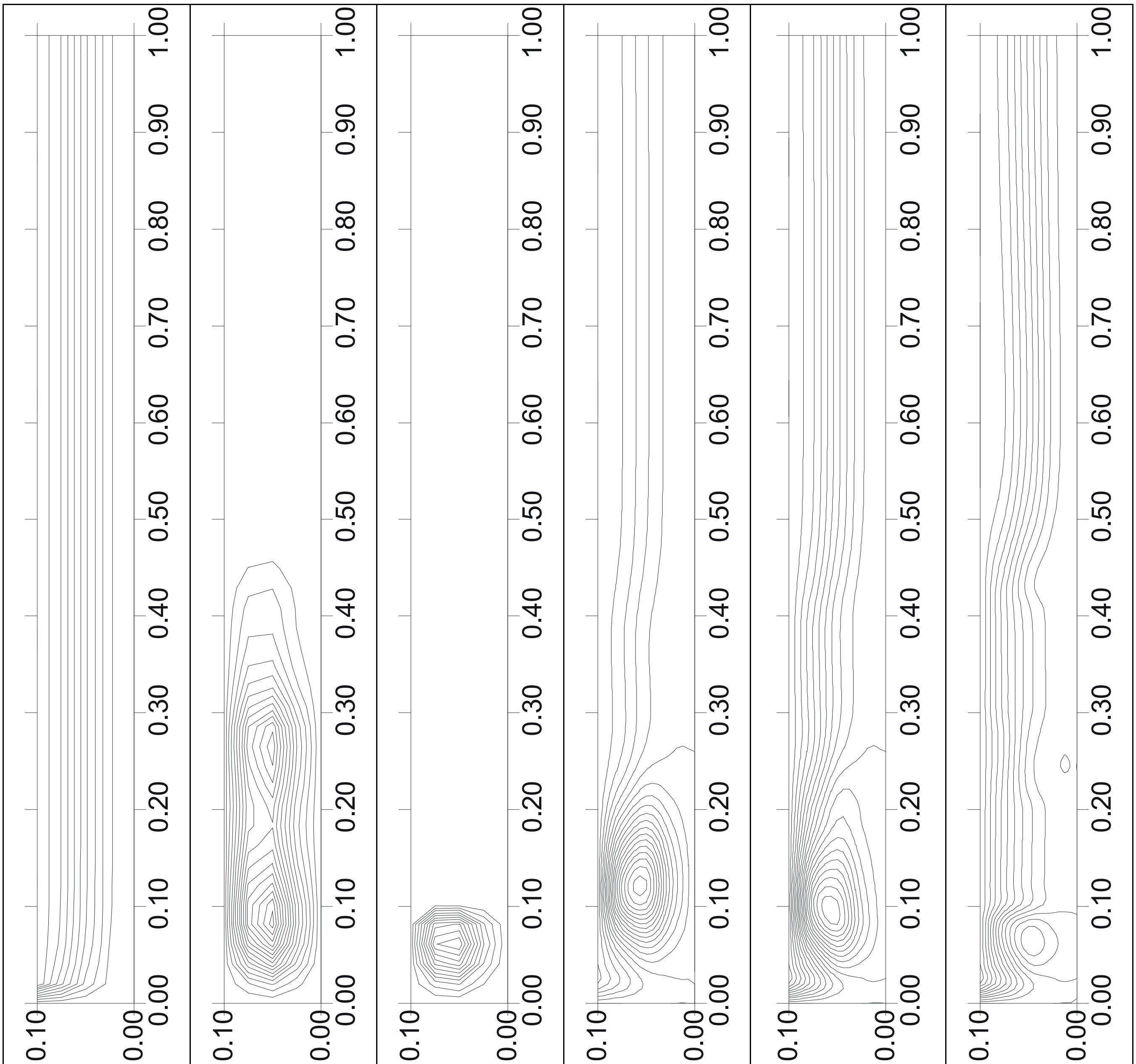

a-Da=1-10^{-6} Re=1-10	**b**-Da=10^{-4} Re=1	**c**-Da=10^{-6} Re=1	**d-** Da=1	**e-** Da=10^{-4}	**f-** Da=10^{-}
Gr=10^{+3}	**Gr=10^{+5}**		**Re=1, Gr=10^{+6}**		

Figure 4 - Stream function maps for different situation. (Pr=0.71, A=10)