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Flux Marching Method for Two-Dimensional Inverse Heat Conduction Problems in the Water Impingement Cooling

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Abstract: Numerous experimental and numerical investigations have been done to uncover the essence of the complex cooling process using circular water jet or planar curtain water jet in steel industry. In this study, the cooling characteristics of water jet, especially the progression of water cooling zones, is described. A new approach named "Flux Marching Method (FMM)" is developed to better suit the features of impingement water cooling. Numerical tests verified that FMM more accurately reflects the actual cooling process in impingement cooling by taking the water movement into account. Special considerations in implementing FMM are discussed.

Key Words: Inverse heat conduction, Water impingement cooling, FEM, Flux marching method

1 Introduction

Water jet cooling is widely used in steel industry productions such as steel strip and plate rolling [1–3]. In order to successfully model and control the cooling process it is necessary to get the accurate heat transfer coefficient or heat flux value and its distribution on the cooled surface. Survey shows that the most of experimental investigations concentrated on heat transfer phenomenon with a single circular or planar jet with stationary or slow moving plates. In the existing research, the temperature at different locations in the cooling zones were commonly measured using thermocouples (TCs) on the opposite side of the cooled surface or inside the test plate (i.e., imbedded TCs) and then surface flux or heat transfer coefficient was determined using various inverse methods [4–7].

Due to the constraints in workpiece geometry and TC installations as well as test cost, a limited number of TCs can be used in tests and the number of measured temperatures is therefore limited. From a mathematical point view, the number of unknowns that can be inversely determined from the measured temperatures should not be more than the number of measurements irrespective of the inverse methods used. This mathematical limitation means that the maximum number of heat fluxes that may be determined is equal to the number of the measured temperatures. A heat transfer finite element (FE) model inevitably required many more heat flux or film coefficient data. The method of assigning or assuming the heat flux distribution on the cooling surface becomes, therefore, a critical and challenging issue for both 2D and 3D inverse heat conduction problems (IHCP).

The aim of this study is to present a new approach referred to as the "flux marching method (FMM)" and discuss its merits by comparison to the common approach referred to as the "flux zoning method (FZM)".

2 FZM and FMM

2.1 Progression of water jet cooling zones

In this section, the progression of water jet cooling zones is discussed for the transient cooling process by using circular jets with stationary plates.

Visual records [8–9] showed that the plate immediately turns grey around the stagnation zone when the water impinges onto the plate surface. Outside the darkened zone the plate is still red hot. Shortly after the start of cooling the grey area around the impingement zone begin to turn black and progresses outwards. This kind of progress to the black color is very rapid. As the black zone is growing within the grey zone the grey zone is also growing outwards but at a considerably lower rate. When the size of the black zone gets close to that of the grey zone the two zones grow approximately at the same speed. Hence, while the plate is being cooled by a circular jet there is a black zone centered at stagnation point followed by a relative small grey circular ring around the black zone and finally by a bright red zone, as schematically shown in Fig. 1. It is clear that the black zone becomes larger and larger and finally covers the entire cooled surface.
From heat transfer mechanism point of view, radiation and air convection cooling is dominant in the red zone and film boiling and/or transition boiling takes place in the grey zone while a single fluid convective heat transfer and/or nucleate boiling may be the critical behaviour for the black zone.

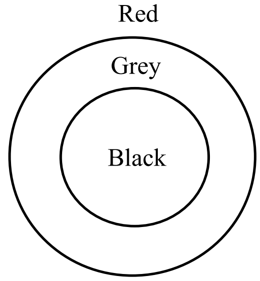

Fig. 1 Progression of cooling zones

Such a progression of cooling zones implies that any material point outside of the impingement zone is first cooled by a water cooling zone with low cooling capacity and its temperature should decrease at a milder rate and the sharp drop of temperature should take place as the front of black zone approaches.

2.2 FZM and its features

To apply the developed inverse analysis algorithm to determine the heat flux during water jet cooling process, special procedures for specifying the heat flux distribution on target surface should be set up. As mentioned above, this is not an easy task for both 2D and 3D IHCP.
The commonly used approach is to divide the target surface into several subregions and each subregion may correspond to one temperature measurement. It is possible that the number of subregions would be less than that of measurements when several thermocouples are used [4].
The FE mesh model for FZM used in this analysis is schematically shown in Fig. 2. In this figure, two adjacent subregions are ploted. There are many nodes in each subregion and the measurement point is located in the middle. The numbers of nodes in each individual subregion may have little effect on both the calculation procedure and accuracy and is generally limited by finite element considerations such as the element aspect ratio.
The same value of heat flux is assumed for all nodes in each subregion. However, the heat fluxes for the two subregions are generally not equal, i.e., $q_i \neq q_{i+1}$. That means that two heat fluxes are applied on this part of target surface. The heat flux vector composed of all of heat fluxes on each subregion can be determined using the inverse algorithm in Section 2.This approach is fairly simple and easy to be adopted. It may also be a good practice for the pool cooling process or for cooling surface that is covered simutanously by the water, for example, the impingement zone under the jet water cooling. In fact, the heat flux determined using FZM is the average one for each subregion, that is

$$q_i(t) = \frac{1}{(b-a)} \int_a^b q_i(x,t)dx \qquad (1)$$

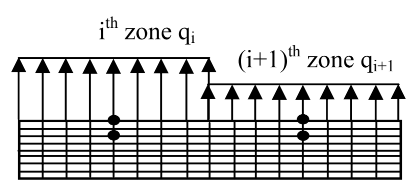

Fig. 2 FZM FE model

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When the heat flux is nearly constant or does not change dramatically in the subregion the obtained heat flux would be a good approximation. Numerical tests had been performed to verify the stability of the inverse analysis algorithm and to investigate various parameters affecting the procedure such as the effects of number of total (future) time steps on the elimination of damping and lagging behavior and the appropriate value of regularization parameters. Detailed results of these numerical tests are reported in reference [7].
However, this approach have two pertinent drawbacks. The first drawback in FZM is that the inversely calculated heat flux depends on the size of subregion even for the symmetrical specification of heat fluxes. The second drawback is that the inversely calculated heat flux depends on whether or not the specification of heat fluxes is symmetrical to the measurements points. For the cases shown in Fig.3, the heat fluxes will be different.

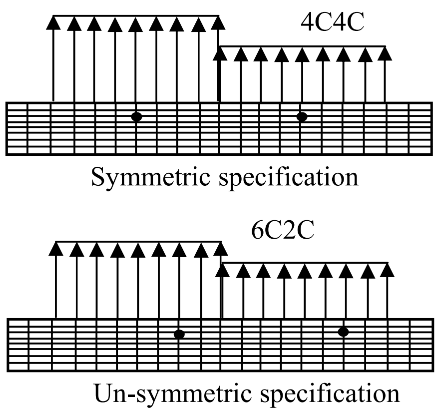

Fig.3 Dependency in FZM

Thus, the FZM may not be an appropraite method for the cooled surface outside of impingement because it does not correctly capture the cooling pattern when the cooling water is progressively covering the surface and sharp drop of temperature takes place at a short time interval, as shown in Table 1.

2.3 FMM and its features

The Flux Marching Method (FMM) is developed based on the above finding. The key points of FMM are highlighted as follow:

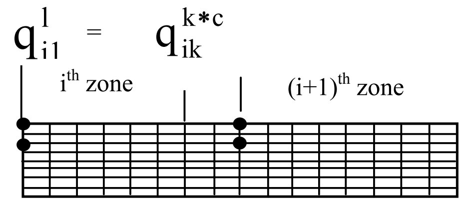

Fig. 4 FMM FE model

The key points of FMM are highlighted as follows:

1. As in the FZM approach, the target surface is divided into several subregions and each subregion corresponds to one temperature measurement, as schematically shown in Fig. 4.
2. There would be generally many nodes in each subregion and the measurement point corresponds to the first node in each subregions. The number of nodes in each individual subregion generally is not equal and may relate to the progressing speed of water cooling zones.
3. The heat flux at the first node is inversely calculated from the internal measured temperature. Because the number of subregions is equal to the number of measurements and only one heat flux in each subregion is considered as unknown, the unknowns are equal to the number of measurements and the inverse problem is solvable.
4. The following schemes may be applied:
   a) The heat flux on a given node at the current time step is generally assumed to be equal to the heat flux value on the neighboring upstream node at the previous time step. This is why we term the approach a flux marching one. The essence of FMM is like wave propagation. The procedure is based on the fact that the surface temperature in each subregion does not change much before the instant at which water covers the surface; and on the assumption that the small change of temperature field in each subregion wouldn't affect the heat transfer along the subregion surface, i.e., the heat transfer at downstream

point that would be cooled at a subsequent time step would be the same as that of an upstream neighbouring point at a prior time step. The heat flux at the first node may be looked at as the impetus of a wave and the flood of heat flux would sweep the points in each individual subregion.

The progressing speed of water cooling zones and the number of nodes in each subregion may be different Thus, a linear interpolation procedure merits attention and is discussed in the following:

i) Use the following equation to determine the time ts_{ij} which is needed for water to move from the first node to the j^{th} node in the i^{th} subregion

$$ts_{ij} = \frac{d_{ij}}{v_i} \tag{2}$$

where d_{ij} is the distance between the first node and the j^{th} node in the i^{th} subregion and v_i is the progressing speed of water cooling zones in the i^{th} subregion.

ii) To find the time difference use

$$t_{ij} = t^k - ts_{ij} \tag{3}$$

where t^k is the total calculation time to the current k^{th} time step and t_{ij} is the time difference to be used for interpolation.

iii) To find the heat flux at the j^{th} node in the i^{th} subregion at the current k^{th} time step, use the following equations:

$$\begin{aligned} &\text{when} \quad t_{ij} \le 0 \\ &q_j^k = q_1^1 \\ &\text{when} \quad t^m \le t_{ij} \le t^{m+1} \\ &q_j^k = q_1^m + \frac{t_{ij} - t^m}{\Delta t^m} * q^{m+1} \\ &\text{where} \quad m < k \end{aligned} \tag{4}$$

This interpolation approach embodies the previous approaches and may be used for all cases.

3 Numerical Tests

In the following sections, the test cases for the assessment of the FZM and FMM approaches are discussed.

3.1 Test procedure and setup

The inverse analysis procedures FZM and FMM are tested with verification cases that are designed to simulate real life situations of water jet cooling process. The focus is placed on the capacity and accuracy of FZM and FMM to recover the input of heat fluxes from the virtual temperature measurements.

The first stage of the verification involves specifying input heat fluxes and solving a direct heat transfer analysis problem to obtain the corresponding temperature field. The second stage involves an inverse analysis. The internal temperatures at target points calculated from the first stage are used as virtually-measured ones to calculate heat fluxes and surface temperatures. Comparisons of the inversely calculated heat fluxes to the inputs as well as the inversely calculated surface temperatures to the directly calculated ones are carried out to investigate the capacity and accuracy of both FZM and FMM.

In the inverse estimation stage, only the data at the current time step is used and no future time steps are assigned; the regularization parameter and the convergence criteria are also fixed for all cases; and except for special specification, no random error is imposed on to the calculated temperature and therefore the vitual measured temperature is without measurement error.

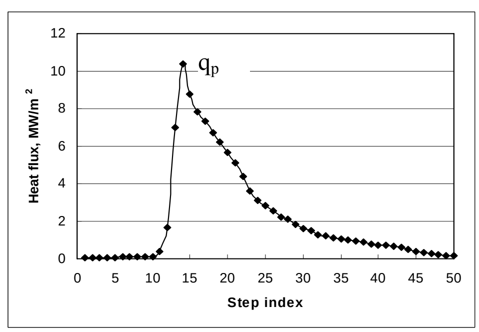

Fig. 5 Input heat flux profiles

Fig. 5 shows the crucial part of a typical profile of heat fluxes in water impingement cooling that is used as an input for the direct analyses while the direct calculation results are used in the verification of the inverse analysis approaches. The nodes in the impingement zone experiences the given heat flux change simultaneously while the nodes outside the impingement zone will experience this setup of heat fluxes with a time shift, which simulates the progression of water cooling zones.

Seven cases are studied. FZM results of the first seven cases are shown in Table 1. In these cases, four peak values q_p and four moving speeds v are adopted. Case 2 will be discussed in detail and the other three cases are presented in Section 4.

Table 1 Test scheme and FZM results

Case	q_p, MW/m^2	V, mm/s	Inversely calculated q_p, MW/m^2			
			TC2		TC3	
			Value	Ratio	Value	Ratio
1	10.4	1.25	6.86	0.660	4.68	0.450
2	10.4	2	6.88	0.662	5.58	0.537
3	10.4	5	7.24	0.696	7.28	0.700
4	10.4	10	8.55	0.822	8.82	0.848
5	7.8	2	5.16	0.662	4.25	0.545
6	5.2	2	3.44	0.662	2.78	0.535
7	2.6	2	1.72	0.662	1.38	0.531

3.2 FE model

The finite element model used for both direct and inverse analyses in these numerical tests is a 2D axisymmetrical one shown in Fig. 6 (Note that the scale is not the same in the *x*- and *y*-directions).

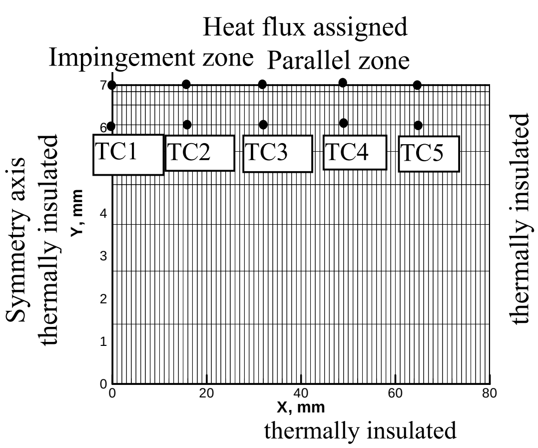

Fig. 6 FE-model for numerical tests

The domain size is 7 mm in thickness and 80 mm in the radial direction. There are 80 elements in the radial direction and 9 elements in the thickness direction. The elements are uniform in the radial direction to ensure an unbiased simulation of the water movement whereas they are variable in thickness direction with relatively denser mesh close to the top surface.

As illustrated in Fig. 6, the domain is evenly divided into five subregions and each subregion has 16 elements. The first subregion is assumed as impingement zone and the other four subregions are parallel zone. Five sampling locations are set up and the first one is at the stagnation and the other four sampling locations lie at the intersection between the subregions. At each location, the temperatures at two points, one on top surface and one at 1 mm beneath the surface point, are sampled in the direct calculation. The temperatures at the internal points are used in the inverse analysis to inversely estimate the heat flux and the temperature of the points on the top surface.

There are totally 81 nodes on the top surface and they are listed here as 1 to 81 and may be in different calculation zone as stated in Table 2.

Table 2 Node in calculation zone

Zone	1	2	3	4	5
Sampling	1	17	33	49	65
Direct	1 ~16	17~32	33~48	49~64	65~81
FZM	1 ~8	9 ~24	25~40	41~56	57~81
FMM	1 ~16	17~32	33~48	49~64	65~81

The sampling nodes are in the middle of each zone for FZM (this is an axisymmetrical problem and node 1 is in the middle of the first zone) while the other four sampling points are the first node for direct calculation and FMM.

The material is assumed have a density of 7800 kg/m^3, a specific heat of 480 J/kg°C and a thermal conductivity of 20W/m°C.

3.3 Specification of boundary conditions

The main objective of the current work is to investiagte the suitability and accuracy of FMM for the pararrel water jet cooling. The assumed boundary conditions as

shown in Fig. 6 are such that all sides except top surface are thermally insulated. This may not be practically accurate, but it should not influence the desired conclusions.
The applied heat flux boundary conditions are calculated and applied on the nodes on the top surface of the plate. The applied scheme insures space and time marching that simulates the actual cooling conditions and is briefly explained in the following.
At the first stage (here 30 steps, 3s), air-cooling is assumed all-over the top surface and uniform heat flux ($\mathbf{q}_1$) is applied. At the second stage the water cooling starts and a heat flux value ($\mathbf{q}_2$) is read according to the assumed curve input (see Fig. 5) and is applied to all the nodes in the impingement zone as well as the first node at the second subregion (later these nodes are called as impingement nodes). All other elements would still be subjected to air-cooling, i.e., heat flux value of ($\mathbf{q}_1$). At the next time step, another heat flux value ($\mathbf{q}_3$) is read in and is applied to the impingement nodes. The heat flux history on the impingement nodes is shifted to the other nodes in the parallel zone according to the assigned speed and element length. This space-time-marching approach implies that the cooling condition for all points is almost identically except for a time shift and small temperature drop due to the longer air-cooling period.

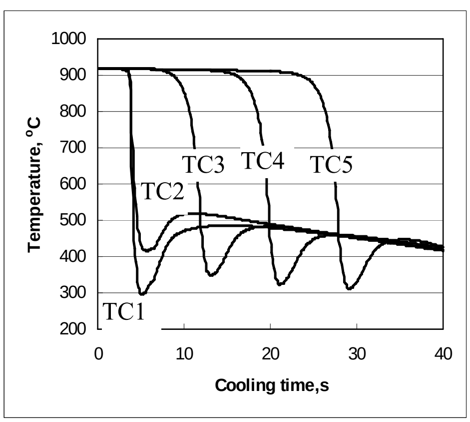

Fig. 7 Internal temperatures directly calculated in case 2

The calculated internal temperatures from direct simulation are shown in Fig. 7. The changes of internal temperatures are fairly similar to those obtained from the experimental measurements in the stationary plates except for the large temperature recovery, which occured in the moving plate cases. It may be stated that the temperature profiles assemble the features for both stationary and moving plates.
This calculated internal temperature is used as virtually measured one in inverse calculation.

4 Results and Discussions

We first note that if the marching speed of heat flux matches the progressing speed of water cooling zones, the input heat flux may be exactly recovered (both peak value and profile) . All calcualtions relating to the input flux, incliding suface temperature predictions, would be produced with very good accuracy using FMM. For the case considered, the FMM surface temperature predictions only differ by about 10 °C with the same accuracy of ±1°C for the input temperatures. This prediction accuracy is quite acceptable and will not affect the relationship between surface temperature and heat flux.
A crucial point in using FMM is to determine and to closely match the progressing speed of water cooling zones. Speed mismatch may cause problem in recovering the requird heat fluxes. The influences of speed mismatch are numerically investigated.
Fig. 8 shows the calculated heat fluxes at TC2 through TC5 with an increase 5% of marching speed for the second zone only. This figure indicates that the heat flux at TC2 has little change while all the heat fluxes at TC3 and TC5 are severely affected. At TC3, the heat flux first takes a negative value, indicating a heating process before it starts to increase. Because the temperature at TC3 is reduced by water earlier than it should be, an amount of heat should be input to keep the temperature and, therefore, a negative heat flux is decipted. This heating process would create a wrong temperature field

around the sampling point and the peak value of heat flux would be mis-predicted.
It is expected that the temperature would be affected by the precedent heat fluxes. When the inaccurate value of the predicted heat flux at TC3 marches to TC4 it will deform the temperature field around TC4. Thus the heat flux at TC4 will be different from the input. In this location the heat flux shows a small positive increase to blance the effect from the negative one due to the marching scheme of heat flux. The same phenominon repeats to some extent at locations TC4 and TC5.
If a mismatch of 5% is applied to all locations TC3 to TC5, the distortions of the predicted heat fluxes would be intensified.

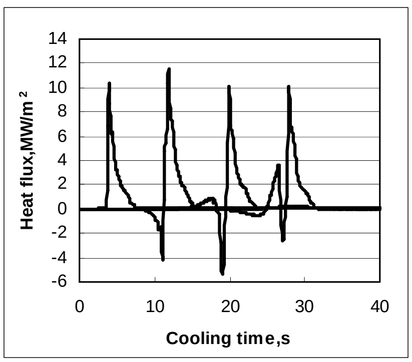

Fig. 8 Effect of higher speed mismatch using FMM

The influencing intensities of mismatch level on the calculated heat flux accuracy are depicted in Fig. 9 to Fig. 11 and Table 3. For all cases the peak value of input heat flux is 10.4MW/m^2 in direct calculation and the marching speed at the second subregion is artificially shifted with a deviation. The results at location TC3 are presented.
Fig. 9 shows the results with a higher speed mismatch. With the increasing of speed mismatch from +5% to +20% the unreal negative heat flux increases from -4.2 MW/m^2 to -8.6 MW/m^2, the later is close to the input peak value. Because of the negative portion the positive heat flux becomes higher than the input value with a 23% increase. It is to note that the positive portions of heat fluxes are quite close to the input heat flux if the negative portions are neglected.

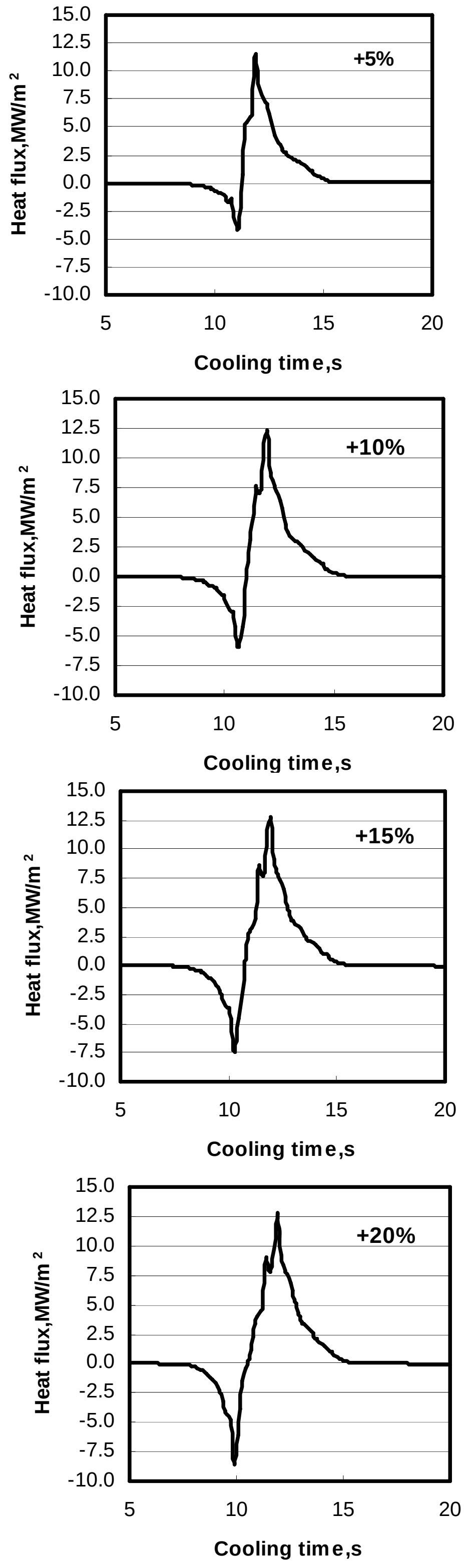

Fig. 9 Calculated heat fluxes with higher speed mismatch

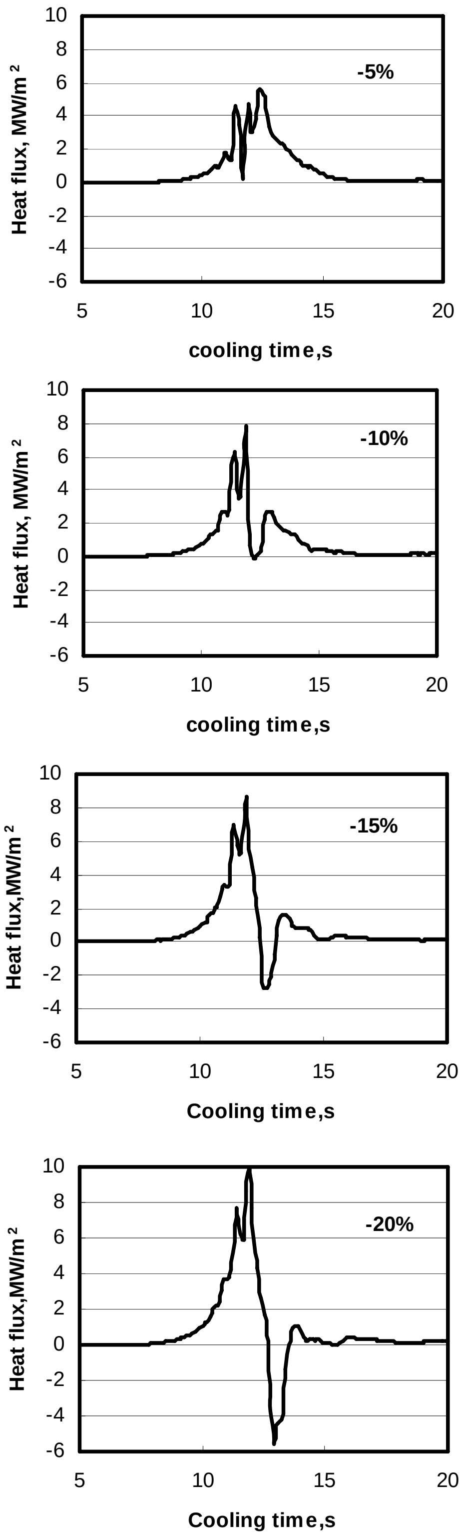

Fig. 10 Calculated heat fluxes with lower speed mismatch

The deformations of heat flux at TC3 are obtained if the marching speed is less than the progressing speed. Fig. 10 shows the deformations of heat flux at TC3 with various values of imposed mismatch at the second subregion. When the speed at the second subregion is slightly smaller than the input value, the marching heat flux from TC2 increases at a later time than the setting and benefits the cooling down of temperature at TC3. Thus less heat should be extracted away by the heat flux at TC3 itself. Therefore, the heat flux at TC3 is extremely deviated from the input one. As the slower speed mismatch increases, however, the deviation of the calculated heat flux to the input one becomes less and negative heat flux appears after the positive one. When the marching speed is 20% less mismatched the positive portion of heat flux turns closer to the input one.

The accuracy of calculate heat flux for above cases are presented in Table 3 and Fig. 11 Data shows that the calculated heat flux peak value at TC3 is about 11% to 23% higher than the input one with a higher speed mismatch. Compared to a 46.3% deviation with FZM, this accuracy is acceptable.

It is to note that the heat flux at TC2 with FMM is only slightly affected by the speed mismatch while that with FZM is also greatly influenced.

Table 3 Calculated heat flux accuracy at TC3 with speed mismatch

Speed	5%	10%	15%	20%
Faster	11.6	12.4	12.7	12.8
	111.5%	119.2%	122.1%	123.1%
Slower	5.59	7.75	8.69	10.0
	53.8%	74.5%	83.6%	96.2%

It is clear from these calculations and analyses that the deformed profiles of heat flux at TC3 are different for an overestimation or an under-estimation of speed. This result is extremely informative and is useful for figuring out whether the speed is overestimated or under-estimated. The guideline for setting up the marching speed of heat flux may be in this way that the marching speed should be set up at a slightly a higher value rather than lower value.

5 Summary and Prospects

In order to uncover the essence of the complex water jet cooling widely used in industry, more experimental and numerical investigations are still required. The new approach "Flux Marching Method (FMM)" may allow obtaining a more accurate heat flux inversely calculated from the measured temperature and realizing the above objective.

"Flux Zoning Method (FZM)" is the common approach used to far for assigning the heat fluxes on the cooling surface in inverse calculation. It is simple and easy to use and can be used in the case whereas the target surface is simultaneously cooled by a same medium.

A main feature of FMM in comparison to FZM is the essence of wave propagation. In this method, the cooling surface is still divided into several subregions and each subregion is also corresponding to one temperature measurement. The heat flux at the first node for each subregion is inversely calculated from the internal measured temperature and functions as the impetus of wave and the flood of heat flux would sweep the points in each individual subregion. Thus FMM takes the water movement into account, and therefore may be more accurate to reflect the actual cooling process in impingement cooling.

Using FMM needs more attentions when determining the marching speed of heat flux and smoothing the jumping of heat flux. Both the overstepping of heat flux may cause the disturbance in heat flux profile with a virtual negative heat flux before the real sharp increase of heat flux. Owe-stepping may damp the real sharp increase of heat flux and cause a virtual negative heat flux after the real sharp increase of heat flux. Slight adjustment of marching speed of heat flux may eliminate or greatly reduce the effect from the speed mismatch.

In this study, although it is implemented in a FE program FMM is a general approach to assign heat fluxes along the cooling surfaces or subregions from the design variables, and can be used in other methods mentioned in the introduction section. Moreover, it may be also possible first to assume a heat flux profile with constants and to assign the heat fluxes from FMM, and then to determine these constants by an inverse method.

References

[1] M. Korchynsky, Development of "controlled cooling" practice, *Accelerated Cooling of Steel*, P. D. Southwick (ed.), 1986, pp. 3–14.

[2] D. H. Wolf, F. P. Incropera and R. Viskanta, Jet impingement boiling, *Advances in Heat Transfer*, Vol.23, 1992, pp. 5–132.

[3] R. M. Guo and S. T. Hwang, Investigation of strip cooling behavior in the run-out section of hot strip mills, *Journal of Materials Processing and Manufacturing Science*, Vol.4, 1996, pp. 339–351.

[4] J. V. Beck, B. Blackwell and C. R. S. Clair Jr., *Inverse Heat Conduction: Ill-posed Problem*, Wiley-Interscience Publication, New York, 1985.

[5] H. K. Kim, Evaluation of heat transfer coefficient during heat treatment by inverse analysis, *Journal of Material Processing Technology*, Vol.112, 2001, pp. 157–165.

[6] Z. Liu, D. Fraser and I.V. Samarasekera, Experimental study and calculation of boiling heat transfer on steel plates during runout table operation, *Canadian Metallurgical Quarterly*, Vol.41, 2002, pp. 63–74.

[7] M. S. Gadala and F. Xu, A FE-based sequential algorithm for the determination of heat flux during impingement water cooling, *International Journal of Numerical Methods for Heat & Flow Fluid*, Vol. 16, No. 3, 2006, pp. 356–385.

[8] A. T. Hauksson, Experimental study of boiling heat transfer during water jet impingement on a hot steel plate, *Master Thesis*, the University of British Columbia, 2001.

[9] Q. Meng, Experimental study of transient cooling of a hot steel plate by an impinging circular jet, *Master Thesis*, the University of British Columbia, 2002.