Mathematical Modelling of Living Room with Different Type of Heating

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Abstract: The paper deals with the distributions of temperature and averaged turbulent airflows in living rooms in a 3D approximation using Ansys/CFX software. The heat balance of a room and its dependence on various external factors are also considered. As physical parameters of thermal comfort conditions, the airflow velocities and indoor temperatures with its gradients are analysed. The distributions are calculated depending on the heating type (convector or floor heating) and the placement of the heater. The influence of these factors on the air circulation and temperature field, as well as the related heat flows through building structures are analysed. It is shown that it is possible to reduce heating power, maintaining the conditions of thermal comfort in the room at the same time. Also, an optimal location of the heater and the best type of room's heating are discussed from different viewing angles.

Key-Words: Mathematical Modelling, Living Room, Thermal Comfort, Heater, Floor Heating, Heating Power.

1 Introduction

Personal feeling of comfort is generally impressed by many objective and subjective factors [1]. Physical parameters like velocity of airflows, absolute temperature and amplitude of the vertical temperature gradient in the room are very important to provide an optimal thermal comfort conditions, thus it is necessary to analyse these factors in different models of a living room

An optimal arrangement of the heater and appropriate installation of controllable venting system allows maintenance of thermal comfort in the living room with reduced heat consumption. A physical model of heat balance for a living room with various physical conditions and different geometries is used, which allows analysing the distributions of the airflows and temperature. The mathematical modelling enables to choose an optimal type of heating and a placement of the convector, in order to decrease heat losses and to improve conditions of thermal comfort.

2 Problem Formulation

The room with different boundary conditions (convection, surface temperature, air openings) is modelled helping to understand the features of heat transfer process in the room as well as distribution of various characteristic quantities and their dependence on the different conditions. A placement of the heating element (convector) is varied and a 3D model with the floor heating is developed and their influence on the distributions of temperature and velocity fields is analysed, characterising the conditions of the thermal comfort. For developing mathematical models and numerical calculations, software Ansys/CFX is used.

The calculations have been performed for the room shown in Fig.1, filled with an air. The window and the wall to the exterior air are modelled using different materials with heat transmittance U for the window 2.5 W/(m²·K) and for the wall – 0.35 W/(m²·K). Such values are chosen similar to the room with good insulated outer wall and an ordinary double-glazed window. Between window and wall a small cranny is created to model real gaps in old window-frames, but, in the opposite wall, there is a ventilation opening.

On outer rooms' boundaries, convection boundary conditions are set with according surface heat transfer coefficients. There is conditionally assumed that the surrounding rooms (upstairs, downstairs and side rooms) have the comfortable temperature of 20 °C, but the end wall is contiguous with a corridor or a staircase where the temperature is lower (15 °C). The outdoor temperature is chosen corresponding to the winter conditions (-10 °C). On the surfaces of crannies in window-frame and ventilation system's opening boundary conditions with constant pressure and accordant temperature of -10 °C and 15 °C are defined. Pressure difference ΔP between opposite walls is set to 0 Pa.



Fig. 1. Layout of a modelled room

Four developed models with different type of heating and location of the convector are as follows:

- 1 convector placed near exterior wall;
- 2 convector placed near wall to the corridor;
- 3 convector placed near side wall;
- 4 floor heating (without convector).

Surface temperature of the heater is set to constant 50 °C for variants with convector heating and to 25 °C for model with floor heating. For all surfaces, except openings, non-slip boundary conditions are used.

In this problem formulation, the airflow in the room depends both on the convection created by the temperature difference and on the air exchange between openings in the structures. To describe the quasi-stationary behaviour of temperature and averaged turbulent flows, traditional differential equations are employed [2]:

- Reynolds averaged momentum equation;
- continuity equation;

• equations for specific turbulence energy k and dissipation rate of this energy ε ;

• energy conservation equation.

The turbulent viscosity is calculated using the k- ϵ turbulence model.

The discretisation was performed with tetrahedral elements of varying size; boundary layers are discretised with smaller prismatic elements. The characteristic size of finite elements is from 10 cm in the middle of the room to 0.3 mm in the vicinity of the heating element and for the openings in the walls. Therefore, the total number of elements reaches $5 \cdot 10^6$ depending on geometry. The boundary conditions of the third type (convection from walls) and the low viscosity of air essentially worse the convergence of an iteration process. The time required for calculations with a 3 GHz computer is about 5 days. The calculated difference between the heat amount from the heater and the losses from the outer surfaces and openings decreases below 5 %.

3 Problem Solution

Figs. 2 and 3 show characteristic velocity fields and temperature contours from 10 to 20 $^{\circ}$ C for considered models. The main results are also summarised in Table 1 – these can be divided into two significant groups for detailed analysis:

• heat balance of the room – heating power needed for temperature maintenance Q (W) and air exchange rate connected with convective heat losses through openings in building envelope;

• thermal comfort conditions – average velocity v (cm/s), mean temperature T (°C), as well as vertical and horizontal temperature differences ΔT (°C).

It is conveniently to analyse each of these result groups separately, in order to choose the best room model from the viewpoint of energy consumption and with better thermal comfort conditions. As one can see from the result summary, it is very difficult to satisfy both of these requirements at the same time.

3.1 Heat Balance Analysis

Since the convective heat transfer from the convector is essentially dependent on the air flow intensity near its surface, it is obvious that, despite its constant temperature, the maximum heat will be taken off when a heavy air motion occurs along it for model 1 up to 20 cm/s near the convector and about 5 cm/s in the middle of the room. However, in case of placing the heater by a wall to the corridor (model 3), the heating power is only 84% of the above mentioned - see Table 1 and Fig. 4. Also floor heating is not an optimal solution from the viewpoint of heat consumption due to its large warm area – the heating power for this type of heating in model 4 is a bit less than with convector heating in model 1. But due to a greater electricity costs, a floor heating expenses will be also greater than for case with central hot water heating.



Fig. 2. Characteristic velocity vector field for models 1 (a), 2 (b), 3 (c) and 4 (d)

Fig. 3. Temperature contours from 10 to 20 °C for models 1 (a), 2 (b), 3 (c) and 4 (d)

Table 1.	Geometrical	properties	and the	calculation	results for	different	develope	d models
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Model	Placement of the heater	Total heat- ing power $Q(W)$	Air ex- change rate <i>n</i> (1/h)	Average velocity v (cm/s)	Average temperature T (°C)	Vertical* tem- perature differ- ence ΔT (°C)	Horizontal* tem- perature differ- ence ΔT (°C)
1	W	164	0.52	5	17.6	0.7	6.4
2	С	138	0.15	2	18.3	2.0	4.6
3	S	145	0.14	4	19.2	1.9	2.8
4	F	154	0.20	3	19.0	0.8	2.5

W – near window, C – near opposite wall (to the corridor), S – near side wall, F – floor heating * in the middle if the room

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Fig. 4. Total heating power and characteristic average temperature in the middle of the room for different models

Another significant factor related to the heat losses is an air exchange rate characterising convection through openings in the room's boundary structures. However, we can not just close the openings, which are necessary for maintaining the content of oxygen inhaled by people.

As the normal value characterising air exchange intensity in the rooms without forced ventilation, the air exchange rate $n\approx0.7$ (1/h) is accepted. Taking into account the air inflows and outflows through slots in the window-frame and through the ventilation opening, it is obvious that in model 1 the air exchange is nearly sufficient (Table 1), however, it means increased convective heat losses. In turn, an air exchange below the normal (models 2-4) would decrease heat energy losses, without making people feel better at the same time – a ventilation system or natural airflows due to the pressure difference between exterior and opposite walls are needed for that a rooms.

Therefore, model 3 will ensure a relatively high average temperature in the room (above 19 °C) with less energy consumption, but the model 1 is the most disadvantageous from the viewpoint of heating energy consumption.

3.2 Thermal Comfort Analysis

One of the aspects important for the thermal comfort is the temperature difference in the vertical direction; it should be as small as possible, but not greater than 2 °C [1]. In models 2 and 3 (with a heater placed near the wall to the corridor and side wall accordingly), air stratification with the vertical temperature difference in the middle of the room about 2 °C is observed, but in models 1 and 4, temperature difference is below 1 °C (Table 1 and Fig. 3). In the first case such temperature gradient is created due to essential air circulation in the whole room (Figs. 2b, c and 3b, c), Fig. 5 shows an example of hot air uprising from the convector.

In the model 1, two considerable airflows exist near exterior opposite walls (Figs. 2a, 3a), hence a temperature changes in the middle of the room are minimum – only 0.7 °C. However, temperature fluctuations near the outer wall are notable: this is caused by active cold and hot air flows, which are partially separated by a windowsill and directed horizontally. When the air warmed by a heater is moving along its surface upward, it meets an obstacle – a windowsill, as a result of which the direction of a hot air stream is changed. But at the opposite wall of the room, there is downward inflow of cooler air through the ventilation opening. Thus, the lower absolute temperature value is observed for this model.



Fig. 5. Temperature isosurface of 20 °C for model 3

At the same time, use of floor heating reduce vortex created by the convector with notable high temperature. Temperature profile here is quite vertical and its amplitude is below 1 °C, it is well coinciding with the results of other investigations of floor heating problems [4], also in horizontal direction oscillations is below 3° C (Table 1).

Fig. 6 shows visualisation of the temperature distribution in the room from 18 to 20 °C and an isosurface of the 18 °C temperature front for models with side placed convector and for floor heating. One can see there that, for model 3, significant air temperature stratification is established, while for model 4, this stratification is inconsiderable. In both models, cold air inflow through opening in the window-frame near the windowsill is analogous, but

in the first case, a cold airflow from the outside is present also near wall opposite to convector (Fig. 6a). On the other hand, situation is symmetrical for the floor heating (Fig. 6a).

Fig. 7 comparatively shows temperature profiles along vertical line in the middle of the room for all models. As it is evident from this visualisation and Table 1, average temperatures in the room for models 3 and 4 are the same, but floor heating provides better comfort conditions thanks to insignificant fluctuations in the temperature profile.

From the viewpoint of mean temperature and temperature fluctuations, model 4 is the best suited for human living, nonetheless the others models also meet the requirements of the conditions for thermal comfort [1]. In this way, complex analysis of the temperature field and the other parameter – airflows is needed.

According to the specification of the conditions for thermal comfort [1], maximum airflow velocity in the heated room is limited to 10 cm/s, but it must be as small as possible in practice, expect areas near openings are not used for human occupancy. As one can see from the results summarised in Table 1, the intensity of air flow is low (2...5cm/s) actually throughout the room for all models, but the lowest velocities are observed in models 2 and 4.

Complex analysis of the both thermal comfort conditions – temperature amplitude and airflow velocities (Fig. 8) shows that only one of these factors is at minimum in models 1 and 2; in model 3 – none of the parameters is at minimum; however, the best conditions for human living are observed in the model 4 with floor heating. At the same time, this type of heating uses electric power and therefore it is related with greater expenses.



Fig. 6. Temperature contours from 18 to 20 °C and temperature isosurface of 18 °C and for model 3 (a) and model 4 (b)



Fig.7. Temperature profiles in the middle of the room for different models



Fig. 8. Characteristic airflow velocity and vertical temperature difference in the middle of the room for different models

3.3 Risk of Condensation

A dewpoint in living rooms can be reached near cold surfaces; particularly high risk of condensate appearance exists for the outside building structures having a high permeability of heat. In this aspect, the most critical construction for simulated models window with the heat transmittance is $U=2.5 \text{ W/(m^2 \cdot K)}$. Such a risk is increasing with difference between the characteristic temperature of the room and the temperature of the window surface: in model 1, the characteristic room temperature exceeds 17,6 °C, while at the upper edge of the window the temperature falls down to 10 °C (Fig. 3a). As a result, the condensation on the window surface is highly probable; if the relative air humidity in the room is 65 %, the condensation will begin at a temperature below 11 °C. It should however be noted that through the slots in window joints, there is inflow of cold air with lower absolute moisture content.

Somewhat unexpected has been the result that the risk of condensation on the window surface is practically absent in the case when the heater is placed by the wall opposite to the window (model 2). This is determined by the warm air flow along ceilings in the direction to the outer wall and relatively immobile warm air masses in the upper part of this wall above the window (Fig. 3b). In models 3 and 4, the risk of condensation is negligible due to the high mean temperature and slow air motion. Since the heat insulation of the outer wall is chosen relatively good with heat transmittance $U=0.35 \text{ W/(m^2 \cdot K)}$ for all models, the probability of condensation is low nearby it. However, the risk of condensation could increase in the bottom part of the outer wall if its heat permeability is increasing.

4 Conclusion

3D numerical calculations of temperature and airflow distribution in a living room with opening for air exchange show the essential influence of heater dislocation and its type on thermal comfort conditions in the room as well as on heat transfer from the heater with constant surface temperature. Obtained temperature distributions help to forecast critical places near boundary constructions where there is a high risk of condensation.

To summarise among all the considered models, the least advantageous from a viewpoint of thermal comfort conditions are the models with convector placed near the wall to the corridor and the side wall (due to great vertical temperature difference), but the most advantageous – the model with floor heating. In the last one, temperature vertical difference is below 1 °C and air velocities are only 3 cm/s, therefore the requirements of thermal comfort is satisfied best of all. However, notable heat losses are observed here and since electricity is usually used for the floor heating, expenses for this type of heating are serious. Therefore, one can choose the most significant target: to minimise heating power or to improve thermal comfort in the room.

Detailed analysis of modelling results for living rooms with different boundary constructions' heat transfer coefficients, varying heater surface temperature, geometry configurations and corresponding comparison of heat balances as well as influence on thermal comfort conditions are analysed in publications [5, 6].

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