Proceedings of the 4th WSEAS International Conference on Fluid Mechanics and Aerodynamics, Elounda, Greece, August 21-23, 2006 (pp337-342)

Conservation Law of Centrifugal Force and Mechanism of Energy Transfer Caused in Turbomachinery

TAKAHARU TANAKA
Department of Mechanical Engineering
Kobe University
Rokko, Nada, Kobe
JAPAN

Abstract: - Conservation low of Centrifugal force indicates that the fluid particle, which is forced to rotate in the rotating flow passage, causes two different kind rotational motions. One is the appearent rotational motion around the axis of rotation caused together with impeller blades. And the other is the substantial rotational vortex motion around itself. Head rise is caused by the infinitely small but infinitely large number of fluid particles substantial rotational vortex motion around themselves caused in the rotating flow passage by the impelling action.

Key-Words: - Theoretical pump head, Energy change, Conservation law on Centrifugal force.

1 Introduction

Theoretical head is one of the most fundamental and important subject to be established in the study of turbomachinery. Stepanoff, A. J. [1] considered that mechanical energy is supplied to fluid particles contained between two adjacent impeller blades at time t=0, therefore, energy change is caused on those fluid particles. While, Tanaka, T. [2] considered that mechanical energy is supplied to the system. Then, hydraulic energy change has to be caused not only on the fluid particles contained between two adjacent impeller blades at time *t=0*, but also caused on the fluid particles entering the impeller blades from the leading edge of impeller inlet in time interval *dt*.

They applied the principle of conservation of angular moment of momentum to those masses of liquids in the rotating flow passage of impeller blades and got theoretical equations represent the pump head. However, they do not explain any interrelation with the mass weight flow rate at all. In this viewpoint, theoretical pump head is investigated.

In this paper, theoretical conservation low is introduced on Centrifugal force and it is applied to the fluid flow between rotating impeller blades and a reasonable interrelation, which represents the pump head and flow rate, is obtained. The source of pump head and mechanism of head rise are also discussed.

2 Introduction of Conservation Law on Centrifugal Force

Mechanical energy is supplied to impeller blade and transferred to fluid particle as hydraulic energy. Transferred hydraulic energy is constructed from mass weight flow rate ρgQ and pump head H [3]. Then, maximum pump head is obtained at zero mass weight flow rate in the practical operation of pump.

Now, let us consider two dimensional fluid flow in the rotating flow passage of Centrifugal pump at zero flow rate and image two arbitral circular lines A and B around center O. Fluid particles rotate together with impeller blades at constant angular velocity ω_O. If their rotational radiuses have the relation $r_{OA} < r_{OB}$, circular line B locates outside of circular line A.

Centrifugal force F_{OB} at outside circular line B is given by

$$F_{OB} = \rho(\frac{U_{OB}^2}{r_{OB}}) = \rho\omega_O^2 \cdot r_{OB} \quad (1)$$

Here, radius r_{OB} corresponds to discharge radius of a pump. If the leading edge of impeller blade starts at center O, the magnitude of Centrifugal force F_{OB} at radius r_{OB}, given by equation (1), which corresponds to pump head H_{Th}, may become the maximum.

If the Centrifugal pump is set at the depth H_{WX} measured vertical downward from the water surface W-W, the pressure head F_{WX} at the location of impeller discharge is given by

$$F_{WX} = \rho \cdot g \cdot H_{WX} \quad (2)$$

Pressure force F_{WX} of rested water at vertical depth H_{WX} due to gravitational acceleration g is balanced with the Centrifugal force F_{OB} of the fluids due to impeller blades rotational motion. While their acting directions are opposed. Then

$$F_{WX} = -F_{OB} \tag{3}$$

Theoretical pump head H_{OB} is given by

$$H_{OB} = -H_{WX} = -\frac{1}{g} \cdot \omega_O^2 \cdot r_{OB} \tag{4}$$

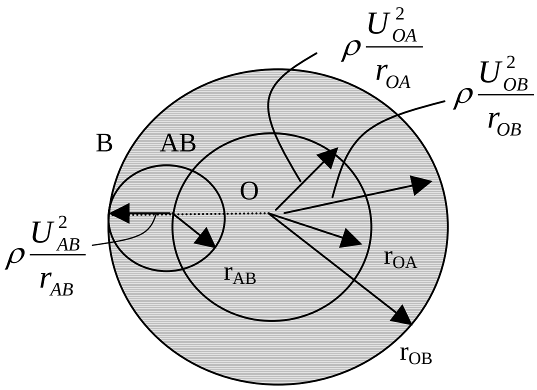

Fig.1 Illustration to show the conservation low on Centrifugal force caused at rotating flow passage around center O at constant angular velocity ω_O.

Then, let us try to establish a new theory, which represents the case that impeller blade does not start its leading edge at center O, but starts at some distances away from the center O. Now, consider a circular line A, which locates some distances away from the rotational center O, which corresponds to the leading edge of impeller inlet. Its radius r_{OA} is therefore smaller than that of circular line B. Centrifugal force F_{OA} at rotational radius r_{OA} is given by

$$F_{OA} = \rho(\frac{U_{OA}^2}{r_{OA}}) = \rho\omega_O^2 \cdot r_{OA} \tag{5}$$

Then, difference of Centrifugal forces F_{OB} and F_{OA} is given by equations (1) and (5) as follow,

$$F_{OB} - F_{OA} = \rho(\frac{U_{OB}^2}{r_{OB}}) - \rho(\frac{U_{OA}^2}{r_{OA}}) = \rho\omega_O^2 \cdot (r_{OB} - r_{OA}) \tag{6}$$

Radiuses r_{OA} and r_{OB} have the relation

$$r_{OB} - r_{OA} = r_{AB} \tag{7}$$

Then, equation (6) can be rewritten as follow:

$$F_{OB} - F_{OA} = \rho(\frac{U_{OB}^2}{r_{OB}}) - \rho(\frac{U_{OA}^2}{r_{OA}}) = \rho\omega_O^2 \cdot r_{AB} \tag{8}$$

In other words,

$$F_{OB} - F_{OA} = \rho(\frac{U_{OB}^2}{r_{OB}}) - \rho(\frac{U_{OA}^2}{r_{OA}}) = \rho(\frac{U_{AB}^2}{r_{AB}}) \tag{9}$$

Where,

$$U_{AB} = \omega_O r_{AB} \tag{10}$$

From the physical parameters that organaze the equation (10), it could be recognized that U_{AB} may correspond to the peripheral velocity of rotational motion along the circular line AB, and the circular line AB may correspond to the locus of rotational motion at radius r_{AB} at angular velocity ω_O. Circular line AB touches internally with the outer circular line B and locates its center on the circler line A.

Radius r_{AB} of circular line AB is arbitrary between O and r_{OB}. It is due to the radius r_{OA} of inside circular line A regardless whether the radius r_{OB} of outside circular line B is fixed at constant or not if we consider unit radius $r_{OB}=1$. If the radius r_{OA} becomes large toward radial outward, radius r_{AB} becomes small. Circular line AB forms various kinds circular lines for the change in rotational radius r_{OA} between O and r_{OB}. However, all rotational motions caused along those circular lines are caused under constant angular velocity ω_O equivalent to those of circular lines A and B at center O. This is clear from equation (10).

Now, equation (9) can be represented as follow,

$$F_{OB} = F_{OA} + F_{AB} \tag{11}$$

Or

$$\rho(\frac{U_{OB}^2}{r_{OB}}) = \rho(\frac{U_{OA}^2}{r_{OA}}) + \rho(\frac{U_{AB}^2}{r_{AB}}) \tag{12}$$

In other words,

$$\rho\omega_O^2 \cdot r_{OB} = \rho\omega_O^2 \cdot [r_{OA} + r_{AB}] \quad (13)$$

Where

$$F_{AB} = \rho \cdot (\frac{U_{AB}^2}{r_{AB}}) = \rho\omega_O^2 \cdot r_{AB} \quad (14)$$

These state that the summation of magnitude of Centrifugal force F_{OA} due to rotational motion along the circular line A and that due to rotational motion along the circular line AB is always equivalent to that due to rotational motion along the circular line B. Rotational radius r_{OA} is arbitrary between O and r_{OB}. And above interrelation is always satisfied regardless whether r_{OB} is fixed at constant or not if unut radius r_{OB}=1. Then, these equations could be called *the Conservation Law of* Centrifugal *force due to rotational motion at constant angular velocity* ω_O [4].

In other viewpoints, magnitude of Centrifugal force due to rotational motion along the outside circular line B is constructed from two different kinds rotational motions. One is that of Centrifugal force due to rotational motion along the inside circular line A whose rotational axis locates at center O, which is equivalent to that of outside circular line B. And the other is that of Centrifugal force due to rotational motion along the circularline AB whose line touches internally with the outer circular line B and locates its rotational center on the inside circular line A.

In other expression, radius r_A of inside circular line A and radius r_{AB} of imaginary circular line AB construct radius r_B of outside circular line B and they all rotate at same angular velocity ω_O.

3. If Circular Line X locates Outside of Circular lines A and B

Let us consider a circular line X in addition to circular lines A and B in the rotating flow passage. If their rotational radiuses have the relation $r_{OA}<r_{OB}<r_{OX}$, circular line X locates outside of circular lines A and B. Then, Centrifugal force F_{OX} of outside circular line X can be written as follow

$$F_{OX} = (F_{OA} + F_{AB}) + F_{BX} \quad (15)$$

Or

$$\rho(\frac{U_{OX}^2}{r_{OX}}) = \rho[(\frac{U_{OA}^2}{r_{OA}} + \frac{U_{AB}^2}{r_{AB}}) + \frac{U_{BX}^2}{r_{BX}}] \quad (16)$$

In other words,

$$\rho\omega_O^2 \cdot r_{OX} = \rho\omega_O^2 \cdot [(r_{OA} + r_{AB}) + r_{BX}] \quad (17)$$

In more general, let us consider infinitely large number of circular lines C, D, … V, W, X in addition to A and B. If their radiuses have the relation $r_{OA}<r_{OB}<r_{OC}<r_{OD}<\ldots<r_{OV}<r_{OW}<r_{OX}$, they also locate outside of circular lines A and B. Then Centrifugal force F_{OX} of outside circular line X can be written as follow

$$F_{OX} = (F_{OA} + F_{AB}) + F_{BC} + \cdots + F_{VW} + F_{WX} \quad (18)$$

Or

$$\rho(\frac{U_{OX}^2}{r_{OX}}) = \rho[(\frac{U_{OA}^2}{r_{OA}} + \frac{U_{AB}^2}{r_{AB}}) + \frac{U_{BC}^2}{r_{BC}} + \cdots + \frac{U_{VW}^2}{r_{VW}} + \frac{U_{WX}^2}{r_{WX}}] \quad (19)$$

In other words,

$$\rho\omega_O^2 \cdot r_{OX} = \rho\omega_O^2 \cdot [(r_{OA} + r_{AB}) + r_{BC} + \cdots + r_{VW} + r_{WX}] \quad (20)$$

These indicate that Centrifugal force F_{OX} of outside circular line X is a function only of summation of increased Centrifugal forces. If rotational radius r_{OA} of inside circular line A is fixed at constant, and if rotational radiuses of additionally added circular lines B, C, D, … V, W, X are larger than that of circular line A, total rotational radius increases toward outside. Then magnitude of total Centrifugal force also increases as much. Total magnitude of Centrifugal forces F_{OX} is explained as the summation of increased Centrifugal forces in addition to that of original Centrifugal force F_{OA} of circular line A.

If original rotational circular line A does not exist, its magnitude could be explained simply as the summation of increased Centrifugal forces due to increased rotational radiuses. See equations (18) and (19). In other words, if angular velocity ω_O is constant it might be a function only of summation of increased rotational radiuses. See equation (20).

4. If Circular Line Y locates between Circular lines A and B

Let us consider a circular line Y in addition to A and B again. If their rotational radiuses have the relation $r_{OA}<r_{OY}<r_{OB}$, circular line Y locates between circular lines A and B. Then, Centrifugal force F_{OB} of outside circular line B can be written as follow,

$$F_{OB} = F_{OA} + F_{AY} + F_{YB} \qquad (21)$$

Or

$$\rho(\frac{U_{OB}^2}{r_{OB}}) = \rho(\frac{U_{OA}^2}{r_{OA}} + \frac{U_{AY}^2}{r_{AY}} + \frac{U_{YB}^2}{r_{YB}}) \qquad (22)$$

In other words,

$$\rho\omega_O^2 \cdot r_{OB} = \rho\omega_O^2 \cdot (r_{OA} + r_{AY} + r_{YB}) \qquad (23)$$

In more general, let us consider infinitely large number of circular lines C, D, … X, and Y. If their rotational radiuses have the relation $r_{OA}<r_{OC}< r_{OD}<\ldots< r_{OX} <r_{OY} <r_{OB}$, they locate between circular lines A and B. Then, Centrifugal force F_{OB} of outside circular line B can be written as follow,

$$F_{OB} = F_{OA} + F_{AC} + F_{CD} + \cdots + F_{XY} + F_{YB} \qquad (24)$$

Or

$$\rho(\frac{U_{OB}^2}{r_{OB}}) = \rho(\frac{U_{OA}^2}{r_{OA}} + \frac{U_{AC}^2}{r_{AC}} + \frac{U_{CD}^2}{r_{CD}} + \cdots + \frac{U_{XY}^2}{r_{XY}} + \frac{U_{YB}^2}{r_{YB}}) \qquad (25)$$

In other words,

$$\rho\omega_O^2 \cdot r_{OB} = \rho\omega_O^2 \cdot (r_{OA} + r_{AC} + r_{CD} + \cdots r_{XY} + r_{YB}) \qquad (26)$$

These indicate that if outside rotational radius r_{OB} is fixed at constant or if we consider the unit radius $r_{OB}=1$, total magnitude of Centrifugal force F_{OB} at outside radius r_{OB} does not changes at all by the number of circular lines inside of it.

5. Physical Property of Circular line AB

One of previous results of theoretical discussion is that if outside rotational radius r_{OB} is fixed at constant or if we consider the unit radius $r_{OB}=1$, rotational radius r_{OB} does not change by the number of circular lines inside of it. This is very significant and meaningful. If we consider infinitely large number of circular lines between circular lines A and B, fluid particles geometrical rotational radiuses, which corresponds to radius AB in Fig.1, become infinitely small. Whilst, their summation of imaginary rotational motions, which correspond to Centrifugal force F_{AB} due to circular line AB, is remained constant. In other words, infinitely large number fluid particles rotate at infinitely small rotational radiuses in the rotating flow passage between r_{OA} and r_{OB} and resultsd to the Centrifugal force F_{AB} due to the imaginary rotational motion along the circular line AB at rotational radius r_{AB}, which is therefore, equivalent to summation of their Centrifugal forces. This might be clear from equations (24), (25) and (26).

In opposed viewpoints, the infinitely small but infinitely large number fluid particles vortex motions are caused at angular velocity ω_O by the impelling action of impeller blades and distribute in the rotating flow passage between r_{OA} and r_{OB}. And they construct the Centrifugal force F_{AB}. This indicates that infinitely small but infinitely large number fluid particles vortex motions cause substantial magnitude of Centrifugal force F_{AB} equivalent to that explained by imaginary circular line AB. In other words, the summation of magnitudes of Centrifugal forces caused by the infinitely small but infinitely large number fluid particles vortex motions caused in the impelling region between r_{OA} and r_{OB} is equivalent to the magnitude of Centrifugal force F_{AB} constructed by imaginary circular line AB at rotational radius r_{AB}.

From these viewpoints it could be said that fluid particles, which is forced to rotate together with impeller blades around the axis of rotation, may not only rotate around the center O, but also rotate around itself. In addition to these, both of those angular velocities are equivalent to those of circular lines A and B at center O of the flow passage.

6. Physical Meaning of Circular line AB

Here the most significant is the magnitude of Centrifugal force F_{AB} of rotational motion along the imaginary circular line AB at radius r_{AB} in Fig.1. It is obvious that fluid particle does not cause this kind real rotational movement in the practical rotational motion of fluid particles in the flow passage of impeller blades. This indicates that geometrical circular line AB due to imaginary rotational motion at radius r_{AB} could be recognized as that it might be the geometrical hypothetical description or the locus line appeared accidentally or following the fundamental energy transfer mechanism visible way to show the total amount of fluid particles substantial rotational vortex motion at angular velocity ω_O.

If the rotational center of circular line AB on the circular line A changes its location along the radius line from the location of radius r_{OB} of outside circular line toward the rotational center O, Centrifugal force F_{AB} changes its magnitude. Magnitude of Centrifugal force F_{AB} becomes large if the location of rotational center moves radial inward from the surface of circular line B toward the center O. If it locates on the surface of circular line B, both the rotational radius r_{AB} and the magnitude of Centrifugal force F_{AB} become the minimum (zero). If it becomes large and reaches the center O, both the rotational radius r_{AB} and the magnitude of Centrifugal force F_{AB} become the maximum, which are equivalent to those of rotational radius r_{OB} and Centrifugal force F_{OB}.

7. Interrelation between Pump Head and Mass Weight Flow Rate

From the above discussion, it is clear that Centrifugal force F_{OB} corresponds to pump head H. Circular line AB, which produces the Centrifugal force F_{AB}, is formed at radius r_{AB} and its rotational center locates on the circular line A. This indicates that magnitude of Centrifugal force F_{AB} is the largest at circular line B because its circular line touches internally with the circular line B and the minimum (zero) at the inside circular line A because it is the location of rotational center. In other words, pump head H is the largest at the surface of circular line B at the trailing edge of impeller outlet and the minimum (zero) at the circular line A at the leading edge of impeller inlet.

Pump head H is zero at radius r_{OA} means that flow rate Q is the maximum, therefore rectangular velocity triangle is formed at the impeller inlet. Because transferred hydraulic energy E_H is constructed from mass weight flow rate ρgQ and pump head H. This indicates that radius r_{AB} corresponds to pump head and radius r_{OA} corresponds to flow rate Q, and their summation r_{OB} corresponds to transferred hydraulic energy. If large radius r_{OA} is selected in the practical design, flow rate Q becomes large but pump head H becomes small. If small radius r_{OA} is selected, flow rate Q becomes small, but head H becomes large.

8. Overview of Theoretical Results

In the previous discussion, it became clear that two different kinds rotational motions are caused in case of fluid particles rotational motion. One is the rotational motion around the axis of rotation and the other is the fluid particles own rotational vortex motion around itself. From these viewpoints, it could be said that fluid particles may not only cause the rotational motion around the axis of rotation, but also causes the fluid particles own rotational vortex motion around itself in the rotating flow passage of impeller blades.

This theoretical result could be expanded to the idea that if the fluid particle turns its flow direction at a curved flow passage and causes a Centrifugal force, fluid particle may not only cause its flow directions change along the curved flow passage, but also causes a rotational vortex motion around itself. These indicate that pressure head raise due to the fluid particles rotational motion and the flow condition at the curved flow passage are not only determined by the fluid particles rotational motion around the axis of rotation or the flow directions change along the curved flow passage, but also determined by the fluid particles own rotational vortex motion around themselves.

This result of discussion indicates that fluid particles rotational motion around the axis of rotation and the flow directions change along the curved flow passage cannot be discussed without considering the infinitely small fluid particles rotational vortex motion around itself and without the application of vortex theory to the fluid particles own rotational vortex motion around itself.

9. Application to the Fluid Flow in Centrifugal Pump

In case of fluid particles rotational motion, rotational motion is not same to that of materials. It is very obvious that rotational motion does not always start at center O, but starts very often at an arbitrary radius point larger than zero. Fluid particle starts its rotational motion at some distances away from center O. This kind rotational motion is seen for example in the rotating flow passage formed by impeller blades in hydraulic machines, such as pumps, blowers, fans, compressors, water turbines, gas turbines, and so on.

Let us consider practical application of conservation low on Centrifugal force to the fluid flow in the rotating flow passage between impeller blades in Centrifugal pump, next. See Fig.2. In this case, rotational radius r_{OB} corresponds to the trailing edge of impeller outlet. Rotational radius r_{OA} corresponds to the leading edge of impeller inlet.

If the Centrifugal pump is set at the depth H_{WX} measured vertical downward from the water surface W-W, the pressure head F_{WX} is given at the location of impeller discharge by equation (2). And this pressure force F_{WX} that directs vertical downward is balanced with the Centrifugal force F_{AB} that directs radial outward given by equation (14). Then, theoretical pump head H_{AB} is given by

$$H_{AB} = -H_{WX} = -\frac{1}{g} \cdot \omega_O^2 \cdot r_{AB} \qquad (27)$$

Rotational radius r_{AB} of circular line AB varies between r_{OA} and r_{OB}, which corresponds to the distance between the leading edge of impeller inlet and the trailing edge of impeller outlet of the impeller blade. Then, equation (27) indicates that pump head H_{AB}, that is, the magnitude of Centrifugal force F_{AB} is the minimum (zero) at radius r_{OA} at the leading edge of impeller inlet and the maximum at radius r_{OB} at the trailing edge of impeller outlet.

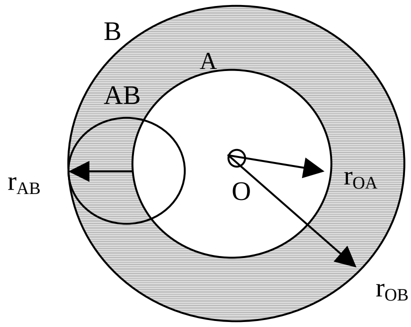

Fig.4 Illustration to show the example of application of conservation low of Centrifugal force caused in the flow field around the center O.

Here attention has to be paid to the facts that the fluid particle that locates on the inside circular line A rotates at peripheral velocity U_{OA} at rotational radius r_{OA} and that U_{OB} locates at the outside circular line B rotates at peripheral velocity U_{OB} at rotational radius r_{OB}. However, substantial practical magnitude of fluid particles Centrifugal force F_{OA} is not the value given by equation (5) but zero and that of Centrifugal force F_{OB} is not the value given by equation (1) but that given by equation (14). This indicates that substantial practical Centrifugal force F_{AB} indicated by circular line AB is supplied by the fluid particles rotational vortex motion. These indicate that essentially two different kinds fluid particles rotational motions: appearance and substantial rotational motions have to be considered in the rotating flow passage.

For example, magnitude of Centrifugal force F_{AB} obtained as a result of theoretical discussion is explained as the difference between that of Centrifugal force F_{OB} and that of Contrifugal force F_{OA} by applying the conservation law on Centrifugal force introduced in this investigation.

Resultant Centrifugal force F_{AB} corresponds to theoretical head H_{Th}. Magnitude of Centrifugal force F_{OB} corresponds to discharge pump head H_2 obtained at the trailing edge of impeller outlet. Magnitude of Centrifugal force F_{OA} corresponds to suction pump head H_1 obtained at the leading edge of impeller inlet.

From these it could be said that theoretical head H_{Th} is explained as the difference between discharge and suction heads, H_2 and H_1, by applying the conservation law on Centrifugal force.

In other words, substantial pump head H_{Pr} is given by $H_2 - H_1$ in the practical pump technology. This indicates that the discharge pump head H_2 is not the substantial pressure head but the appearance pressure head at the trailing edge of impeller outlet and the suction pump head H_1 is not the substantial pressure head but the appearance pressure head at the leading edge of impeller inlet.

10. Conclusions

- Conservation low of Centrifugal force indicates that the fluid particle, which is forced to rotate in the rotating flow passage of Centrifugal pump, causes two different kind rotational motion. One is the rotational motion around the axis of rotation together with impeller blades. And the other is the rotational vortex motion around itself. Head rise is caused by the infinitely small but infinitely large number of fluid particles rotational vortex motion around themselves caused by the impelling action in the rotating flow passage of impeller blades.

11. References

1. Stepanoff, A. J., *Centrifugal and Axial Flow Pumps, Theory, Design, and Application*, 2^{nd} Edition, John Wiley & Sons, Inc., 1967, p.76.
2. Tanaka, T., "An Investigation on Definition of Theoretical Head Produced by Impeller Blade in Centrifugal Pump", Proceedings of the 3rd International Conference on Heat Transfer, Fluid Mechanics, and Thermodynamics (HEFAT 2004), Cape Town, South Africa, June 2004
3. Tanaka, T., "An Investigation on Energy Transfer Mechanism Caused in Rotating Flow Passage of Turbomachinery: New Concept of Physical Parameters in Rectangular Coordinate System" *ASME Paper* No.FEDSM2005-77426, June 2005.
4. Tanaka, T. "Conservation Low of Centrifugal Force and Its Application to Fluid Flow in Turbomachinery", ASME Paper No.FEDSM2006-98504, July 2006.