Periodic perturbation on backward-facing step flows at moderate Reynolds numbers

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Abstract: In this study, we investigate two-dimensional backward-facing step flows at moderate Reynolds numbers. The investigation is done on both time-dependent and time-independent inflow. The time-dependent in flow investigated in this study is in sinusoidal form. We are mainly interested in the effect of amplitudes and the frequencies of the inflow on the characteristic of the flow fields. The main finding for this study is that the flow shows quasi-periodic behavior as we increase the forcing frequency to be at least 12 for all amplitudes studied.

Key-Words: DNS, periodic perturbation, backward-facing step flows.

1 Introduction

Numerical study of two-dimensional flow over a backward-facing step is considered. This problem is a well-know test case for studying separated flow at the step edge, producing a curved shear layer which separates at the reattachment region. One branch flows back creating a recirculation zone behind the step, another branch creates a new boundary layer on downstream. Because of these interesting phenomenon, numerous investigations have been carried out both numerical and experimental studied by many authors. The interest in such a flow was intensified with both experimental and numerical works of Armaly et. al, see [1]. They presented experimental works for the backwardfacing flows with expansion ratio H/h = 1.9423 and Reynolds numbers up to 8000. The flow appear to be three-dimensional as Reynolds number, Re_D , reaches 400 (definition of Re_D shown later in the next section). Also, around this Reynolds number a secondary recirculation zone was observed on the upper wall. Kaiktsis et. al, [3], studied the bifurcation of the twodimensional laminar flow to three-dimensional flow. They found that all unsteady states of flow are threedimensional when $Re_D \approx 700$. Moreover, when

 $Re_D \geq 700$, flow is periodic with characteristic frequency f_1 on upstream and frequency f_2 on downstream. Kim and Moin,[6] computed the flow over a backward-facing step using a second-order method in both space and time. They found a good agreement of the reattachment length with the experimental data of Armaly *et. al* up to $Re_D \approx 500$. When Re_D greater than 500, three-dimensionality effects needed to be included in the simulation.

In this paper, the numerical study of twodimensional flow over a backward-facing step is performed using the spectral-element method, [5]. We impose inlet flow condition as a standard parabolic profile. Flow fields are obtained by solving Navier-Stokes equation on a range of $33 < Re_D < 800$. Our numerical results agree well with experimental data of Armaly et. al up to $Re_D = 400$. As expected, for higher values of Re_D , the discrepancy between numerical and experimental data are found since above that Reynolds number the flow behavior appears to be three-dimensional.

We later focus our study to investigate the effects of inflow conditions on the recirculation zone. For the preliminary studies, the inflow condition is prescribed by a periodic forcing function with flow parameters amProceedings of the 4th WSEAS International Conference on Fluid Mechanics and Aerodynamics, Elounda, Greece, August 21-23, 2006 (pp227-231) plitude and frequency for $Re_D = 300$. At this Re_D , the flow solution is still steady and stable under nonforcing inflow condition. One interesting question is how amplitude and frequency affect on this steady and stable solution.

Some numerical methodology are described in the next section. Numerical results are shown in section 3. Finally, summary and discussions are given in section 4.

2 **Direct Numerical Simulation**



Figure 1: The computational domain.

We consider incompressible Newtonian fluid governed by the Navier-Stokes equations. The computational domain is shown in Figure 1. We set the nondimensional step height to be S = 0.9423, and channel inflow depth h = 1. Thus the expansion ratio is H/h = 1.9423, exactly as in the experiments of Armaly et. al, [1] and Biswas et. al, [2]. In our study, the Reynolds number, Re_D , is defined as

$$Re_D = \frac{\rho U_b D}{\mu},$$

where ρ is the density of fluid, μ is dynamic viscosity, U_b is bulk (average) velocity and D is 2h.

The boundary conditions are imposed at each boundary as follows. For $\Gamma_2 - \Gamma_4$ and Γ_6 , no-slip boundary condition is imposed. Γ_6 has out-flow boundary condition. For the inflow boundary, Γ_1 , the boundary conditions in form of velocity field U are set in two cases as follows:

1. Case 1: $\mathbf{U} = (1,0)$ for time-independent inflow boundary condition.

2. Case 2: U = $(u_0(1 + A\cos(\frac{2\pi u_0 ft}{H}), 0))$ for timedependent inflow boundary condition.

Here, u_0 is 1 A is amplitude and f is frequency of the excitation. Under these boundary conditions, the numerical solutions are obtained by solving the Navier-Stokes equations by the spectral/hp element method, $\mathcal{N}\varepsilon\kappa\mathcal{T}\alpha r$, [5]. This method has been successfully applied to simulate both two-dimensional and threedimensional flows for complex geometries, for example, [4, 7, 8]. The flow domain is decomposed into 1226 triangular elements. For each element, the Jacobi polynomial of order 10 is used as the expansion basis to achieve accurate numerical results.

3 **Numerical Results**

Steady inflow 3.1

In this section, we vary Re_D in the simulation from 33.33 to 800. At low Re_D , fluid flow behaves like creeping flow, see Figure 2, top. As shown in Figure 2, top, the reattachment point x_1 is defined as the point on the bottom wall where the streamwise velocity gradient $\partial u/\partial y$ becomes positive. Here, x_1 is about 1.263 for $Re_D = 33.33$.

Figure 2, bottom, shows a increased zoom of the corner region revealing that the second corner eddy is indeed captured in the current simulation. The ratio of the first and second eddies found numerically in this current study is about 16.736 which is in good agreement with the analytical value. Similar considerations for $10^{-4} \leq Re_D \leq 1$ can be found in [2]. As the value of Re_D increases, the value of x_1 increases. We vary Re_D on the range of $33.33 < Re_D < 800$ and then measure the value of x_1 . The relationship between x_1 and Re_D is shown in Figure 3, top. We see that there is derivation between the experiment and numerical results since the flow appeared to be three-dimensional when value of Re_D closes to 400.

In addition to the primary recirculation zone, we found a secondary recirculation zone near the upper wall for $Re_D > 400$. This due to the effects of adverse pressure gradient at the edge of the step induces the separated flow. The secondary recirculation zone for $Re_D = 600$ is shown in Figure 3, bottom. Again, at moderated $Re_D = 600$, the second eddies near the corner of step is found in our numerical simulation. Thus



Figure 2: Top: Flow streamline in the primary recirculation zone at $Re_D = 33.33$. Bottom: Primary and secondary eddies near the corner of step for $Re_D = 33.33$.

Moffatt 's eddies exist not only for small Re_D but also for moderate Re_D .

3.2 Periodic forcing inflow

In this section, we modify the inflow boundary condition by including effects of small amplitude A and time-periodic excitation on the parabolic profile which can be expressed in case 2 of section 2.

In the preliminary studies, we perform the simulation with the amplitude of the excitation at A = 0.25, 0.5, 0.75 and 1.0 and for each value of A the flow is excited by five values of the frequency: f = 0.5, 6, 12, 18 and 24. The objective of the experiment is two folds: first, to assess the effect of frequency on the periodic oscillatory state; and second, to investigate the effects of amplitude and frequency to the steady flow solution.

We initiate our study first at $Re_D = 300$. Here, the flow solution of perturbed is steady and stable,[3]. For $Re_D = 300$, we found only primary recirculation zone



Figure 3: Top: Relationship between x_1 and Re_D . Bottom: Flow streamlines show primary and secondary recirculation zones for $Re_D = 600$.

Proceedings of the 4th WSEAS International Conference on Fluid Mechanics and Aerodynamics, Elounda, Greece, August 21-23, 2006 (pp227-231) with the length, $x_1 \approx 6.8$.

We plot the time history of the streamwise velocity u at the point P_1 (x = 7.53, y = 0.13) which nears the reattachment point of the recirculation zone. Figure 4, top, shows the time history of streamwise velocity component at point P_1 for various values of A. When the perturbed frequency f = 6, the flow is periodic for all values of A. As A increases, the amplitude of u increases with the same frequency. Moreover, we found that, for 0 < F < 6, the flow is still periodic. In contrast, when f = 12, the flow shows quasi-periodic behavior, see Figure 4, bottom. Moreover, for $f \ge 12$, we found numerically that the flow still shows quasi-periodic behavior for all values A studied.

We have also investigated the effects of A and f to the oscillatory flow solution over one period by measuring the averaged vorticity, ω_{avg} , defined as

$$\omega_{avg} = \int_{\Gamma_4} \omega(\mathbf{x}, t) d\mathbf{x},$$

where where $\omega(\mathbf{x}, t)$ is vorticity field. The results from variation of A as keeping f fixed at (f = 24) is shown in Figure 5, top. For each value of A, the sign of averaged vorticity changes between positive and negative.

From the figure, for each A, there are two points that the vorticity is zero and as A increases, the averaged vorticity increases. This behavior of flow is similar for other values of f.

The numerical results for fixing A and varying f over a period is shown in Figure 5, bottom. The averaged value of vorticity does increases when f increases.That is, the flow behavior approaches quasi-periodic when the period of flow is small.

4 Summary and Discussion

We have presented here two-dimensional backwardfacing step flows in moderate Reynolds numbers in both time-dependent and time-independent inflows. The DNS results from the time-independent inflow agree well with the experimental results until Re_D is beyond 400. And for the case of time-dependent inflow, we provide the results only for the case of Re_D = 300 first. However, we have observed the following conclusions:

- The flow behavior show quasi-periodic behavior as



Figure 4: Top: Time history of streamwise velocity at $Re_D = 300$ for various values of A with F = 6. Bottom: Same but for F = 24

we increase the forcing frequency to be at least 12 for every amplitudes studied.

- Amplitudes of the forcing inflow does indeed effect the amplitude of the average vorticity.

- In one period of oscillatory flow, the recirculation is broke down beyond the step when amplitude is relatively large, 0.75 and 1.0, for all frequencies studied but recirculation zone still exists for small values of amplitude.

The extension of our numerical study will be observed in cases of higher values of Re_D where we concentrate on two cases : Re_D are 600 and 900. Interesting phenomenon at $Re_D = 600$, Figure 3, bottom, is that the secondary recirculation zone appears on the upper wall and at $Re_D = 900$, is that the flow solution is convectively unstable, [3]. Effects of inflow excitation for various values of A and f to these solutions will be investigated and numerical results will be reported later.

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Figure 5: Top: Relationship between ω_{avg} and t over one period for various values of A and F = 24. Bottom: Same but for various values of F and A = 1.

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