## A comparison between asymmetric and symmetric vortex mergers

Mei-Jiau Huang Mechanical Engineering Department National Taiwan University #1 Sec.4 Roosevelt Rd., Taipei Taiwan

*Abstract:* - The dynamics of two two-dimensional asymmetric and symmetric mergers is investigated. In use of a newly developed, purely Lagrangian, vortex method, the evolution of each of the two vortices becomes obtainable. Remarkable similarities between the asymmetric and symmetric mergers are found. In either case, the merger arises from the defeat of the self-rotation of one vortex by the background straining. In asymmetric vortex merger, the losing vortex gets elongated first due to straining induced by the winning vortex. The side pointing to the winning vortex extends further and further and eventually wraps counter-clockwise around the winning vortex. The core part of the losing vortex, on the other hand, is continuously strained and flattened. It turns to be part of the outer spiral filament and eventually wraps around the winning vortex clockwise. Similar dynamics is observed in symmetric merger except that now both vortices are deformed and each extends one sheet-like structure toward the other. These sheet-like structures induce velocities at the vortex cores, pushing them to move toward each other. Merger therefore occurs.

Key-Words: - Vortex merger, Self-rotation, Background straining, Vortex core, Vortex filament

## **1** Introduction

Vortex merger is a common phenomenon in the natural world, particularly in the two-dimensional flows and quasigeostrophic turbulence [1-5]. Previous investigations examined the way in which two vortices merge together and determined the critical separation distance theoretically [6-9], numerically [10-15], as well as experimentally [16-19].

As far as symmetric merger is concerned, Saffman and Szeto [6] and Dritschel [13] were able to predict the critical ratio, the ratio of the critical separation distance to the radius of two circular vortices, of 3.4 by examining the stability of steadily rotating equilibria (so-called "V-states"). Meunier et al [7] also established a merging criterion based on the stability of the metastable state, which arises from a rapid adaptation process of each vortex to the external strain field generated by the other vortex [20]. Melander, Zabusky, and McWilliams [8] approximated the two vortices by elliptical patches and deduced equations for the centroid positions, for the aspect ratio of the two vortices, and for their orientations from a "moment model." Employing a co-rotating reference frame, they emphasized the existence of an exchange band bounded by homoclinic and heteroclinic manifolds passing the hyperbolic fixed points. Velasco Fuentes [21] dealt with the advection of fluid particles in merger. He found that filamentation is not produced by the penetration of a stagnation point of the Eulerian field into the vortex, but by the penetration of a stable manifold of a Lagrangian hyperbolic trajectory. Moreover, it is suggested that the filamentation is not the cause of merger but one of its effects.

When the vortex interaction is not symmetric, the situation becomes more complicated. The vortex which is stretched into a filament and merged by the other is not necessarily the smaller or the weaker one but depends on both the circulation ratio and the size ratio [22]. Contour dynamics experiments, which are applicable only for inviscid flows, are usually employed to explore the merger process. Overman and Zabusky [10] gave some examples. Dritschel and Waugh [14] studied vortices with different areas. Yasuda and Flierl [15] included both area and circulation ratios. They suggested that the critical merger distance is primarily a function of the circulation ratio (linear in the square root of the circulation ratio). The best fitting parameters nonetheless are different for different ranges of circulation ratios. Above all, partial merger, in which only part of the losing vortex is transferred as a filament to the winning vortex and the remaining stably co-rotates with the merged vortex, is observed in a wide parameter range. Interactions between a point vortex and an elliptical vortex are employed to explain the various phenomena observed in asymmetric merger and predict the critical merger distance [9.23]. In use of this model, Yasuda and Flierl [9] suggested that merger takes place because the self-rotation of one vortex is overcome by the straining field induced by the other vortex. Conjectured is that a filament will be extracted from the losing, elongated vortex towards and wraps around the winning vortex.

In a recent work, the author [24] was able to capture the evolution of each vortex in symmetric merger in use of a newly developed, purely Lagrangian, vortex method [25]. It was found that symmetric merger is directly caused by sheet-like structures extended from one vortex toward the other due to the mutually induced straining. The formation of these sheet-like structures possesses a similar mechanism to that of the filament that wraps the winning vortex in asymmetric merger. It is thus conjectured that symmetric and asymmetric mergers possess a same merging mechanism. This must be true because these flows are qualitatively the same. In fact, symmetric merger can be viewed as a special case of asymmetric merger. In the present work, the newly developed vortex method is further modified to simulate asymmetric merger. The evolution of vorticity of each vortex in either symmetric or asymmetric merger is desired. In particular, the flows are viscous. Contour dynamics are invalid therefore. A comparison between asymmetric and symmetric mergers is aimed.

The rest of this paper is arranged as follows. The modified vortex method is introduced in Sec.2. The obtained evolution of vorticity of each vortex in asymmetric/symmetric merger is presented and compared in Sec.3. Similarities and differences are identified and discussed in Sec.4. Conclusions are given at last.

## 2 Numerical Method

The core-spreading vortex method developed by Leonard [26] is employed herein. The computational elements have Gaussian distributions which characteristic area grows linearly in time due to diffusion. The element splitting technique developed by Huang [25] based on a conservation of vorticity moments is used to control the core width below a maximum allowable one  $\sigma_{\mbox{\tiny cr}}.$  The computational amount on the other hand is reduced by combining similar and close-by elements into one, also based on a conservation of vorticity moments [24]. In order to capture the evolution of each vortex in merger, computational elements are separated into two groups, each constructing one vortex. When an element is split, its child elements are added to the group which the parent element originally belongs to. On the other hand, only elements belonging to a same group are allowed to be combined together. The evolution of the vorticity field contributed by a same group of computational elements therefore represents that of one vortex in merger.

In all simulations performed herein, the maximum allowable core width is 0.2 and the ratio of the core widths after and before splitting is 0.85. Five child elements are resulted in each splitting event. The error tolerance in combining similar and close-by elements is set to be 0.5% of the circulation of each vortex. The ratio of the minimum core width to the inter-element distance is thus maintained at about 1.2. Initially, the flow consists of two circular vortices, having radii of  $R_1$  and  $R_2$  as well as circulations of  $\Gamma_1$ and  $\Gamma_2$ . The initial separation distance is  $d_0$ . The vorticity is approximately uniformly distributed within the disk and zero outside the disk. The fluid viscosity is v=1 in all simulations. The two fluid particles that are initially located at two vortex centers are also tracked. Their instantaneous positions will be identified as the locations of the vortex centers at all times.

# 3 Merging Process

### 3.1 Asymmetric merger

The merging process of two asymmetric vortices is first simulated and shown in Fig.1. The two vortices have initially  $R_1 = R_2 = 1$ ,  $\alpha = \Gamma_2 / \Gamma_1 = 2$ ,  $\Gamma_1 = 1000$ , and  $d_0$ = 4. The co-rotating reference frame with the origin of the coordinates locating at the center of the weak (left) vortex has been adopted. Moreover, the simulation time is normalized by the co-rotating period  $4\pi^2 d_0^2 / (\Gamma_1 + \Gamma_2)$ . As seen, the weak vortex gets elongated first at early times due to the straining field caused by the strong vortex. The side pointing to the strong vortex extends further and further toward and wraps eventually around the strong vortex counter- clockwise. At the same time, this (first) filament induces a velocity at the center of the strong vortex, pushing it moving to the left. The separation distance is consequently reduced. The core region of the weak vortex on the other hand is continuously strained. The angle between the major axis and the line-of-center is maintained about  $-\pi/4$ for a while until the strength of the core structure has reduced by diffusion to certain extent, becomes part of the "tail" (the second filament lagging behind the vortex), and starts to wrap the strong vortex clockwise.

An interesting phenomenon also observed in Fig.1 is the ring-like vortex structure (cylindrical in the three-dimensional space) that is formed after the first filament mentioned above has made one turn around the strong vortex and become closed due to viscous diffusion. To highlight it, the vorticity surface plots of the weak vortex at two selected times are shown in Fig.2. Together shown are the contour plots of the total vorticity, which obviously are not sufficient to



FIG.1. The vorticity contours of asymmetric merger.

discover this ring-like structure. At later times, it is also found that the vorticity of the weak vortex continuously "diffuses" into the interior of the strong vortex. This "spurious" diffusion is due to the



FIG.2. The surface plots of vorticity distributions of the weaker vortex and the contours of total vorticity in asymmetric merger at two selected times.

spreading of computational elements through the splitting processes. The large gradients existing in the vorticity distribution of the weak vortex shown in Fig.1 and Fig.2 nonetheless suggest the observed evolution of vortex structure is physical. Similar phenomena have been observed also when the circulation ratio or the size ratio is changed (not shown herein).

The above merging process actually was qualitatively predicted by the point-vortex model proposed by Yasuda and Flierl [9], in which the strong vortex is approximated by a point vortex and the weak one is assumed to be elliptic at all times. To the leading-order terms, the background straining ( $\overline{e}$ ) and vorticity ( $\overline{\omega}$ ) fields induced by the point vortex under a co-rotating reference frame are found to be

$$\overline{e} = -\alpha/d_0^2 \tag{1}$$

$$\overline{\omega} = \left(\alpha + 1\right) / d_0^2 \tag{2}$$

The ratio of the aspect ratio  $\lambda$  (the ratio of the major and minor axes of the elliptic vortex) and the orientation angle  $\phi$  of the major axis satisfy, according to Kida [27],

$$\frac{d\lambda}{dt} = -\overline{e}\lambda\sin 2\phi \qquad (3)$$

$$\frac{d\phi}{dt} = \frac{\lambda}{\left(1+\lambda\right)^2} + \frac{\overline{\omega}}{2} + \frac{\overline{e}}{2}\frac{1+\lambda^2}{1-\lambda^2}\cos 2\phi \quad (4)$$

The phase portraits of equations (3) and (4) having  $\alpha=2$  and  $d_0=4$  are produced in Fig.3. As shown, an initially nearly circular vortex ( $\lambda \approx 1$ ) remains slightly elliptic for a while with its orientation angle quickly approaching  $-\pi/4$  (This may be illustrated by the flow at time=0.0475 in Fig.1). As time goes on, the elliptic vortex is continuously stretched and becomes more and more flat with the orientation angle gradually decreasing in



FIG.3. The phase plane of the point-vortex model.

magnitude (from time=0.095 till time 0.237 in Fig.1). The non-uniform shear and the rotating effect then cause one side of the weak vortex to rotate and wrap the strong vortex eventually. The remaining, elliptic, core structure is further stretched but the orientation angle is instead increasing in magnitude (from time=0.332 to time=0.712 in Fig.1). Probably due to the viscous diffusion and probably due to the reduced size, the evolution of the core structure of the weak vortex seemingly follows first the orbit that approaches the saddle point at  $\phi$ =0 in Fig.3 ( $\phi$  changes from  $-\pi/4$  to zero) and switches later to the orbit leaving the saddle point ( $\phi$  changes from zero and to  $-\pi/2$ ).

#### **3.2** Symmetric merger

The merging process of two identical vortices had been explored in [24] by enforcing the symmetry of the flow. The vorticity evolution of each vortex is re-obtained by employing the present scheme and shown in Fig.4 with  $\Gamma_1 = \Gamma_2 = 1000$  and  $d_0 = 4$ . The distributions are nearly but not perfectly symmetric because of the combining technique used for a reduction of the computational amount. As seen, the early-time evolution of the vortex structure is very similar to that of asymmetric merger found in Fig.1. The side of one vortex pointing to the other extends gradually toward and wraps slightly around the eroded core structure of the other vortex. No closed ring structure however is observed probably because the remaining, eroded, core structures are now too weak to draw the sheet-like structures. The sheet structures next induce significant velocities that cause two vortex centers moving toward each other [24]. Merger therefore occurs. For the same reason, most of the circulations of the sheet structures lag behind at later times, forming parts of the well-known spiral arms as shown in Fig.5. A careful observation finds that the spiral arm of the total



FIG.4. The vorticity contours of symmetric merger.



FIG.5. The surface plots of vorticity distributions of one vortex and the contours of total vorticity in

vorticity field beside one vortex core is contributed mainly by the sheet-like structure extracted from the other vortex. In fact, each vortex develops three "arms" (as indicated by the surface plot at time=0.665 for example) during the merging process. Two appear at the end of the sheet-like structure. The "arm" that slightly wraps the remaining core structure disappears quickly as the co-rotating core structure speeds up due to the reduction of separation distance and catches up with the "arm". A vortex core attached by two spiral arms is thus resulted for each vortex at last as shown, for illustration, by the plot at t=0.918 in Fig.5. surface The axisymmetrization process by viscous diffusion follows then [28].

#### **4** Similarity and Difference

The similarities between asymmetric and symmetric merger are obvious now. A comparison between Fig.1 and Fig.4 suggests that the involved merging mechanisms are really the same, namely a competition between the self-rotation and the mutual shearing. The early-time behaviors are similar: the losing vortex in asymmetric merger and both vortices in symmetric merger become elliptic with an orientation angle  $\phi \approx -\pi/4$ . At later times, the side pointing to the other vortex is extracted toward the other vortex, forming a sheet-like structure that pushes closer two vortex centers. The difference is, in the case studied herein ( $\alpha=2$  and  $R_2/R_1=1$ ) the stronger vortex in asymmetric vortex remains nearly unchanged and strong enough to draw the sheet-like structure making turns around it. A ring-like structure is therefore generated. In the symmetric merger, both vortices are deformed significantly and each develops a sheet-like structure. The co-rotating speed increases rapidly and complicated vortex structures (a core structure attached with two spiral arms for each vortex) are created consequently.

### 5 Conclusion

The result of the present simulations has surprising implications for the cause of vortex merger. It suggests a same mechanism for both asymmetric and symmetric mergers. When the self-rotation of one vortex is overcome by the straining induced by the other vortex, one sheet-like structure is developed, which is extracted from this vortex and extended to the other vortex. This is true for both asymmetric and symmetric mergers. Things are different thereafter however. In asymmetric merger, the winning vortex remains nearly circular (or elliptic) and is strong enough to make the deformed, flattened losing vortex wrapping around itself. In symmetric merger, both vortices are deformed and each develops a sheet-like structure that extends and stays beside the core region of the other vortex. Both vortex centers are thus pushed to move toward each other. A vortex core attached with two spiral filaments is finally developed for each vortex.

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