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AN ESTIMATION OF THE COEFFICIENT OF DISPERSION IN ONE-DIMENSIONAL HEAT CONDUCTION EQUATION FOR QUADRATIC DIFFUSSION COEFFICIENT AND ITS APPLICATION TO THE GROWTH OF YEAST POPULATION

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1. Introduction

In Continuation of our Paper cf [11] we observe that the dispersal of the species in the biological context is an important phenomenon to study. The basics of the theory of random dispersion of Biological Population took shape after the pioneering work of Skellam[7].His method involved applying the analytical expression for molecular diffusion directly to ecological problems, relating it to the interaction among and between species. Accounting for differences in scale among ecological entities and processes has been suggested as a way to understand the hierarchical complexity of Natural systems(O'Neill et al;[4]O'Neill[5],Salthe[6]. Yet for all of its promise; examples of Mathematical or Stochastic development of the hierarchical approach are few Steele[8].Recently Timm and Okubo[9]observed (by considering an ecological model of Levin and Segal[3] for prey-predator Planktonic species)that diffusive instability is less likely to occur in system with time varying diffusivity,than those of constant diffusivity. Their model Parameter estimates are based on the data in Wroblewski and O'brien[10]. In the present paper we shall be studying the one dimensional diffusion equation with constant coefficient of diffusivity as well as, time varying diffusivity. Our main objective in this is to estimate the coefficient of dispersion in both the cases. We have shown in [11] that considering $D(t) = a + b\sin t$ in the Heat Conduction Equation the solution differs from the observed one but the difference between the two Cases namely constant D and time varying D is not that significant. For this a Statistical Estimation is being performed for the above coefficient upon considering D as a Polynomial function. We have then tested our Model over the same Data set as in [11].It is being observed that the difference in two cases is quite significant.Thus the Authors feel that a Polynomial function gives more accurate measure,in this Case. A further detailed study in this regard is under process.

2. Problem Formulation

The one dimensional diffusion equation is,

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} \quad (1)$$

Where D is the coefficient of diffusivity. The above equation is being Extensively used in Biology and other related areas. Diffusion is the spreading of particles ranging from molecules to bacteria, whose individual trajectories are regarded as random. An example to this situation is the dispersion of dye Particles that are released into a clear fluid from a needle. We are interested in the Statistical Estimation of the coefficient D in (1) called the coefficient of diffusivity.

3.Problem Solution

By a standard technique of separation of variables (1) is being solved to obtain,

$$u_n(x,t) = A_n \sin\left(\frac{n\pi x}{l}\right)\exp\left(-\frac{Dn\pi}{l}\right)^2 t$$

(2).

Where n =1,2,3,…. We shall refer this D as pure. If D is assumed to depend upon the time t

Then the solution of (1) is given by $u_n(x,t) = A_n \sin\left(\frac{n\pi x}{l}\right)\exp\left(-k\int D(t)dt\right)$ (3).

In the present work we shall study the case when $-k\int D(t)dt$ is a quadratic.

Case 1

The equation (2) can be written as,

$$u(t) = A_1' \exp\left(-A_2' t\right)$$

where (4)

$$A_1' = A_n \sin\left(\frac{n\pi x}{l}\right)$$

Upon considering

$$A_2' = \left(\frac{Dn\pi}{l}\right)^2,$$

We get

$\log u(t) = \log A_1' - A_2' t$,which upon further simplification results into;

$$y(t) = A_1 + A_2 t$$

where

$$y(t) = \log u(t)$$

$$A_1 = \log A_1'$$

$$A_2 = -A_2'$$

(5).

CASE II

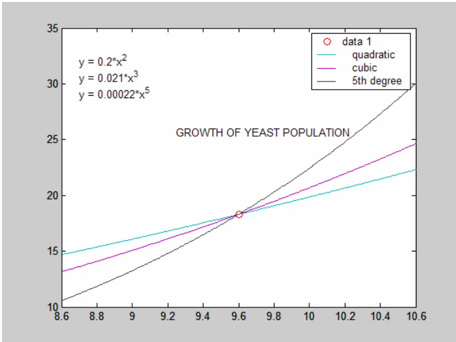
We deal with this case as in [11] to obtain the Regression model as

$$y(t) = \exp[-(2429.78 - 761.5217t + 37.64812752t^2)]$$

Now we apply our model to the following data from [1,2],

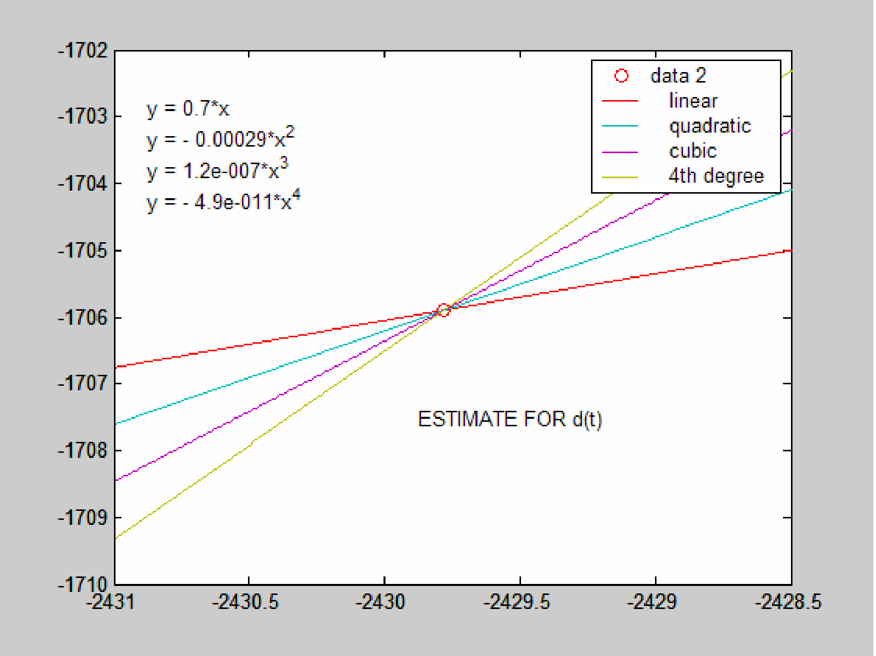
Table 1
Growth of Yeast Population

Hours (t)	Am.of Yeast (y(t))
0	9.6
1	18.3
2	29.0
3	47.2
4	71.1
5	119.1
6	174.6
7	257.3
8	350.7
9	441.0
10	513.3



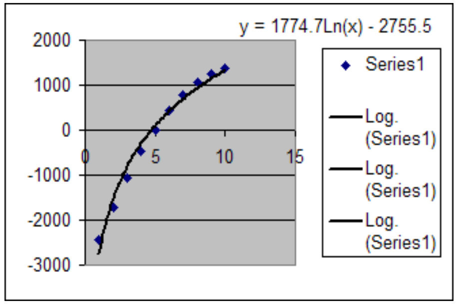
Thus if the function $v(t) = \ln y(t)$ is calculated for the above values of t we arrive at the following Table.

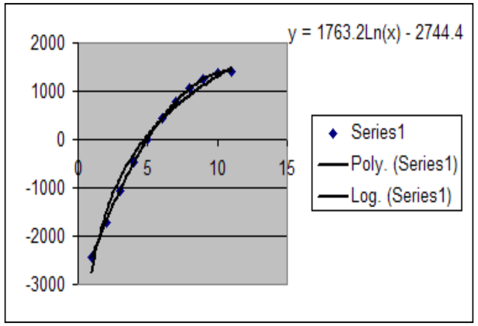
t	v(t)
0	-2429.78
1	-1705.905
2	-1057.328
3	-484.0467
4	13.9382
5	436.627
6	784.019
7	1056.12
8	1262.92
9	1374.42
10	1420.63

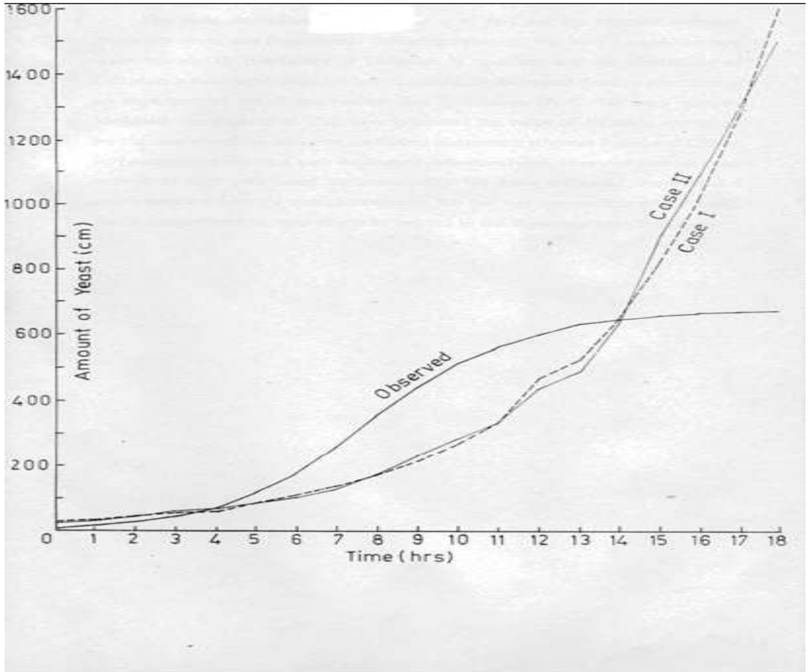


-
2429.78
-
1705.91

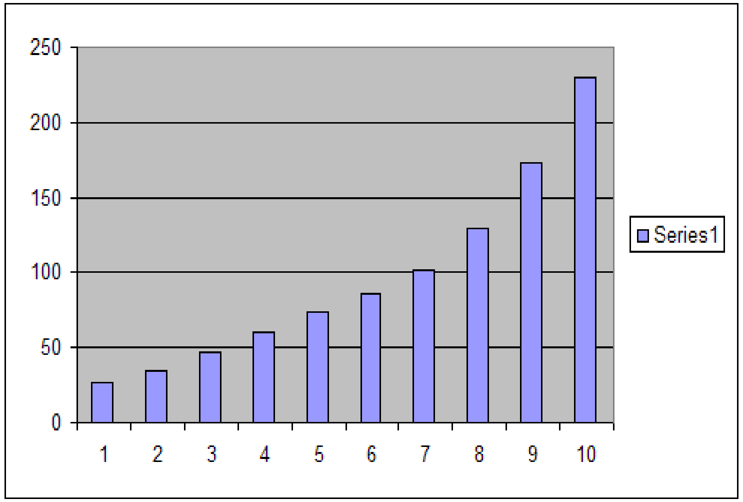
-
1057.33
-484.05
13.94
436.63
784.02
1056.12
1262.92
1374.42
1420.63

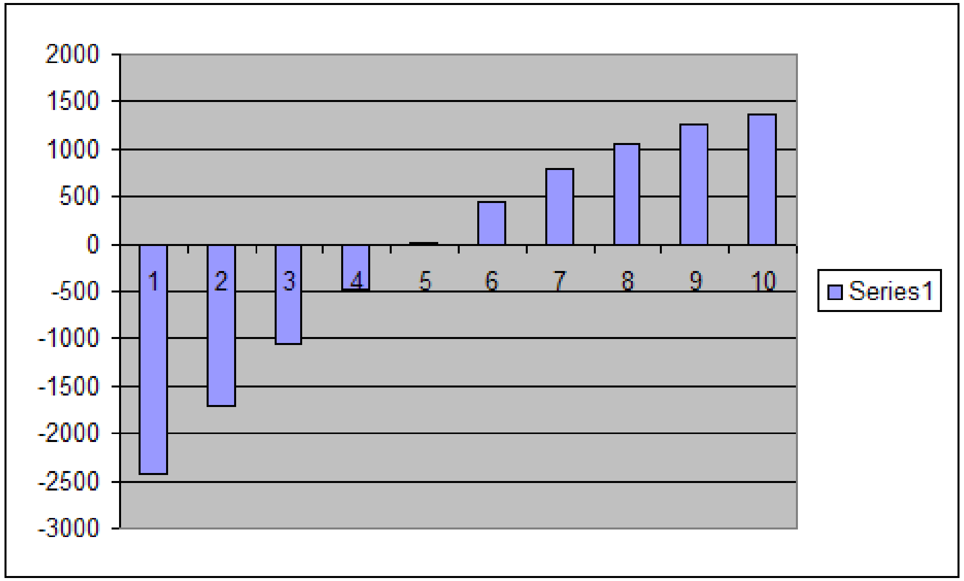






101.49	128.91
784.019	1056.12




2000
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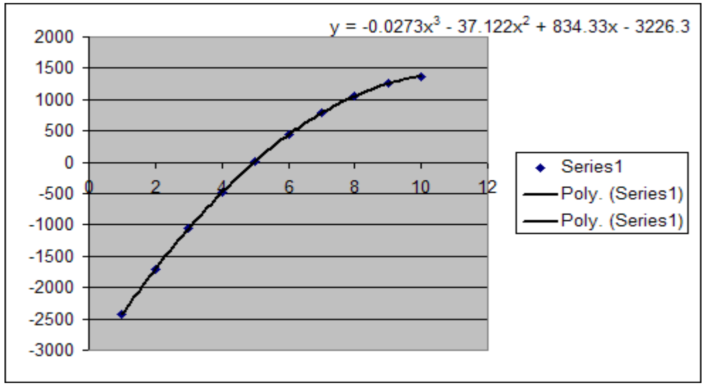
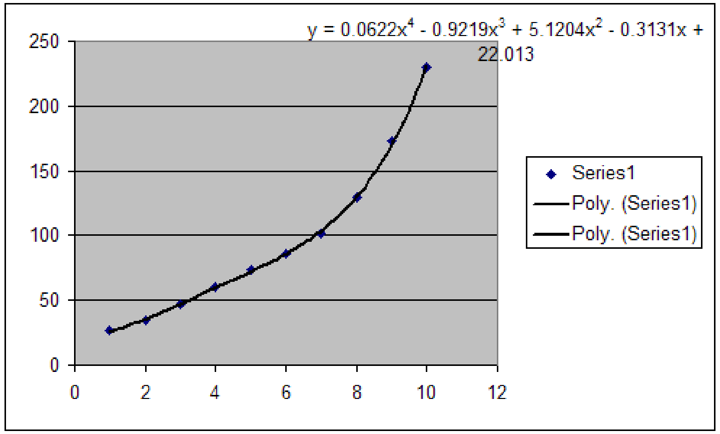
Series1

26.58	34.45	46.43	60.71	73.87	85.7	101.49	128.91	172.97	229.64
-2429.78	-1705.91	-1057.33	-484.04	13.9	436.627	784.019	1056.12	1262.92	1374.42

The Covariance of the two sets of data is 65836.4239



For the second set of data



OBSERVATIONS:

1. In this case we observe that the present regression model is more near to the observed values as shown in the above diagrame.
2. The values in the two Cases differ significantly, meaning thereby there is a marked difference in the above two cases,viz. pure D and D as a function of time.
3. Timm and Okubo[9] have estimated these Mathematically, however the above depicts a clearer picture.

SUGGESTIONS FOR FURTHER WORK IN THE AREA:

The above Estimation can be performed for other Biological Organisms and compare the two cases statistically.

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12.www.biol.sc.edu/~helmuthlab/classes/biol301L/yeast_lab.pdf Fleischmann's baking yeast

13.www.math.usu.edu/~powell/ysa-tml/node14.html

The class accounted for yeast population growth using a per-capita growth model, with growth rates inhibitted by the presence of alcohol