Proceedings of the 10th WSEAS International Conference on CIRCUITS, Vouliagmeni, Athens, Greece, July 10-12, 2006 (pp217-222)

A Steady-state Analysis of PWM SEPIC Converter

ELENA NICULESCU, DORINA-MIOARA PURCARU and M. C. NICULESCU
Departments of Electronics and Instrumentation, and Mechatronics
University of Craiova
Al. I. Cuza Street, No. 13, Craiova, RO-200585
ROMANIA

Abstract: - The paper presents a unified steady-state analysis of PWM SEPIC converter with coupled inductors and separate inductors for both continuous and discontinuous conduction modes. The results of this simplified analysis allow to determine the operating point of converters and further the small-signal low-frequency parameters of linear model, and to design the converter too. The expressions of all steady-state properties of converter take the effects of inductor coupling into account. The converter with separate inductors is treated as a particular case of the more general coupled-inductor case.

Key-Words: - Coupled and non-coupled inductor PWM SEPIC Converter, Steady-state analysis

1 Introduction

As it is well known, the PWM converters are nonlinear dynamic systems with structural changes over an operation cycle: two for the continuous conduction mode (CCM) and three for the discontinuous conduction mode (DCM). Coupled inductor techniques can be applied to PWM SEPIC converter operating either with CCM or DCM to achieve low-ripple input current [1], [2], [3].

The paper provides a complete steady-state characterization for coupled-inductor PWM SEPIC converter with both CCM and DCM. The converter with separate inductors is treated as a particular case of the more general coupled-inductor case. In Section II, the steady-state analysis of PWM SEPIC converter with CCM is made. The expressions of average input and output currents, output voltage and voltage across energy storage capacitor, which take the effects of inductor coupling into account, are derived here. The same steady-state properties of PWM SEPIC converter with DCM are derived in Section III. The boundary equation is given in Section IV. Simplified mathematical models for the steady-state behavior of converter with CCM and DCM have been obtained as result of this analysis.

The analysis is based on the equations written on the circuit for each time interval corresponding to states of switches (transistor and diode) and the low-ripple capacitor voltages, and the waveforms of electrical quantities corresponding to operating mode of the converter. The analysis and modelling of SEPIC converter has surfaced in many forms over the years [1] – [5]. The purpose of this paper is to make a simplified analysis of coupled-inductor SEPIC converter avoiding magnetics theory.

Nomenclature
i_{L1} = current through L_1
i_1 = ripple component of i_{L1}
I_{L01} = initial and final condition of i_{L1}
I_{L1D} = average value of i_1
I_{L1} = average value of i_{L2}
i_{L2} = current through L_2
i_2 = ripple component of i_{L2}
I_{L02} = initial and final condition of i_{L2}
I_{L2D} = average value of i_2
I_{L2} = average value of i_{L2}
I_O = average output current
i_S = current through switch
i_D = current through diode
V_I = dc input voltage
v_O = output voltage
v_{L1} = voltage across L_1
v_{L2} = voltage across L_2
V_{C1} = average voltage across C_1
D_1 = switch-on duty cycle
D_2 = diode-on duty cycle
D_3 = switch and diode off ratio
f_s = switching frequency
L_1 = inductance of filtering inductor L_1

L_2 = inductance of filtering inductor L_2
L_M = mutual inductance
k_c = coupling coefficient
n = turn ratio
C_1 = capacity of energy storage capacitor C_1
C_2 = capacity of output filtering capacitor C_2
R = load resistance
η = conversion efficiency of converter
M = dc voltage conversion ratio

2 Steady-state Analysis of SEPIC Converter with CCM

The diagram of conventional PWM SEPIC converter with coupled inductors is given in Fig. 1a. The topology shown in Fig. 1b is another structure of PWM SEPIC converter with coupled inductors and identical operating behaviour as that in Fig. 1a. This one clearly highlights the PWM switch and the possibility to use the PWM switch model with all its benefits [6]. As compared to the conventional SEPIC topology, the control of the switch has to be realized potential free with respect to the potential reference in the second topology.

Assume that the converter operates in continuous conduction mode and that the two inductors are magnetically coupled together to reduce the input current ripple and increase the power density. The equivalent circuit of two coupled inductors is given in Fig. 2a, and the waveforms of inductor currents and voltages are shown in Fig. 3, where $D_2 = 1 - D_1$.

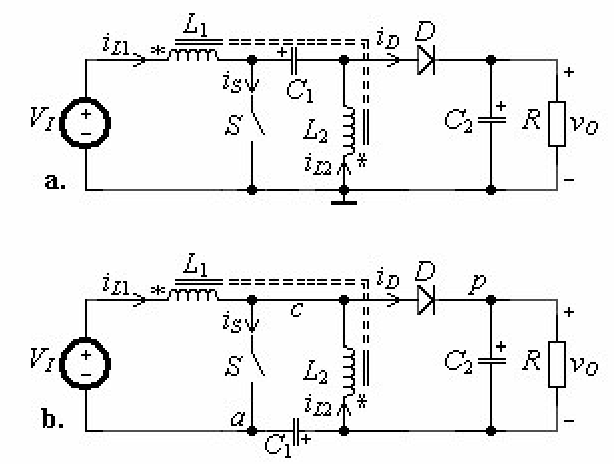

Fig.1 Two SEPIC PWM converter topologies with coupled inductors

The target of this analysis is that to find the expressions of dc voltage conversion ratio, average inductor currents, average currents through switch and diode, average voltage across the energy storage capacitor C_1.

As it can be seen from the inductor current waveforms shown in Fig. 3a and d, the ripple component i_1 respectively i_2 is superposed over the dc component I_{L01} respectively I_{L02} that represent the initial and final condition of inductor current in L_1 respectively L_2:

$$i_{L1} = i_1 + I_{L01} \tag{1}$$

$$i_{L2} = i_2 + I_{L02} \tag{2}$$

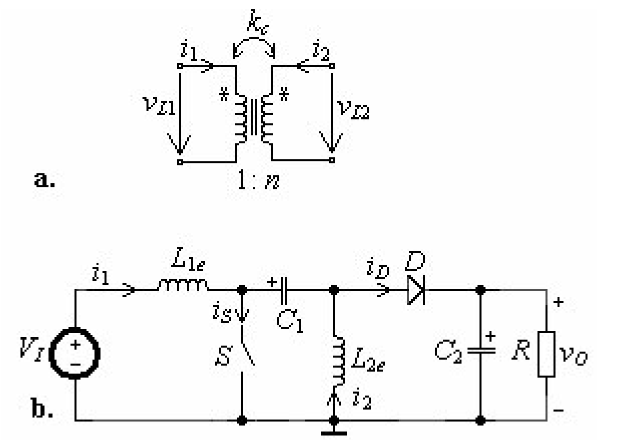

Fig.2 Equivalent circuits for: a. two coupled inductors; b. SEPIC topology with coupled inductors

In order to determine the average values of the current components, we need to find di_1/dt and di_2/dt firstly. The input and output voltages as well as the voltage across the energy storage capacitor C_1 are supposed without ripple. Based on the equivalent circuit of two coupled inductors in Fig. 2a, we have

$$v_{L1} = L_1 \frac{di_1}{dt} + L_M \frac{di_2}{dt} \tag{3}$$

$$v_{L2} = L_M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}. \tag{4}$$

Solving for the unknowns di_1/dt and di_2/dt, we find

$$\frac{di_1}{dt} = \frac{v_{L1}L_2}{L_1L_2 - L_M^2} - \frac{v_{L2}L_M}{L_1L_2 - L_M^2} \tag{5}$$

$$\frac{di_2}{dt} = \frac{v_{L2}L_1}{L_1L_2 - L_M^2} - \frac{v_{L1}L_M}{L_1L_2 - L_M^2} \tag{6}$$

After some algebra, the previous equations can be written as

$$\frac{di_1}{dt} = \frac{v_{L1} - k_c v_{L2}/n}{\left(1 - k_c^2\right)L_1} \tag{7}$$

$$\frac{di_2}{dt} = \frac{v_{L2} - k_c n v_{L1}}{\left(1 - k_c^2\right)L_2} \tag{8}$$

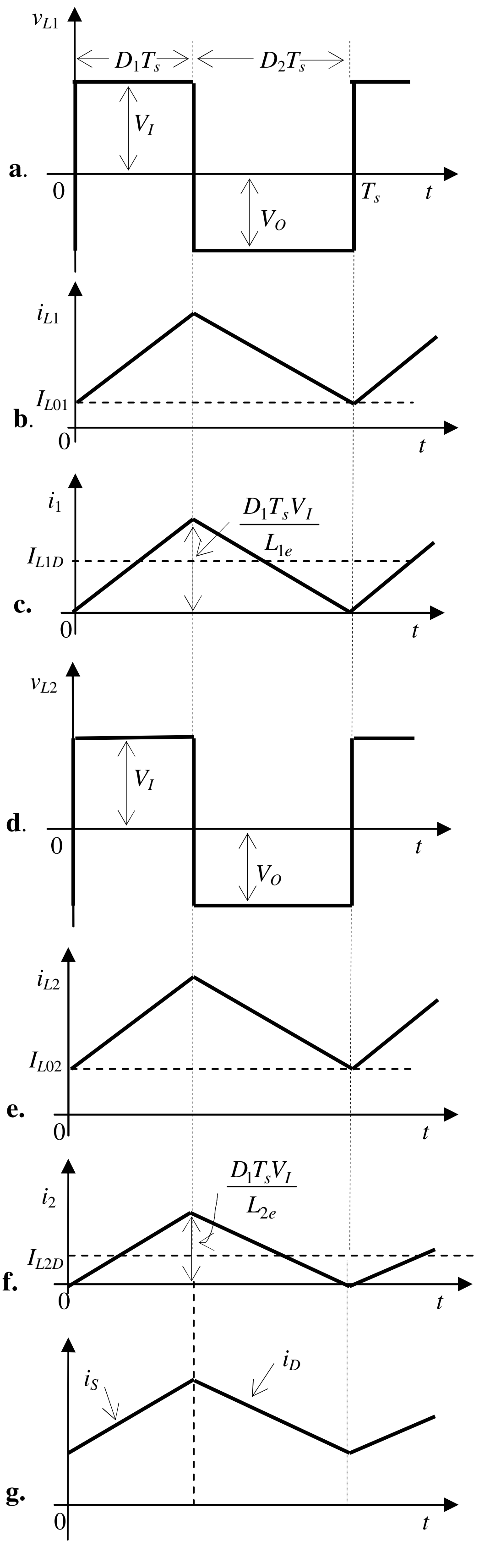

Fig.3 The waveforms of currents and voltages in PWM SEPIC converter with CCM

For $0 \le t \le D_1T_s$, substituting the inductor voltages $v_{L1} = V_I$ and $v_{L2} = V_{C1}$ into (7) and (8) yield:

$$\frac{di_1}{dt} = \frac{V_I - k_cV_{C1}/n}{\left(1-k_c^2\right)L_1} \tag{9}$$

$$\frac{di_2}{dt} = \frac{V_{C1} - k_cnV_I}{\left(1-k_c^2\right)L_2}. \tag{10}$$

We proceed similarly for $D_1T_s \le t \le T_s$ when $v_{L1} = V_I - V_{C1} - V_O$ and $v_{L2} = -V_O$, and we obtain:

$$\frac{di_1}{dt} = \frac{V_I - V_{C1} - V_O + k_cV_O/n}{\left(1-k_c^2\right)L_1} \tag{11}$$

$$\frac{di_2}{dt} = -\frac{V_O + k_cn\left(V_I - V_{C1} - V_O\right)}{\left(1-k_c^2\right)L_2}. \tag{12}$$

Since the total rise of i_1 and respectively i_2 during $0 \le t \le D_1T_s$ must to be equal to the total fall of the same currents during $D_1T_s \le t \le T_s$, we have:

$$D_1T_s\left[\frac{di_1}{dt}\right]\Bigg|_{0\le t\le D_1T_s} = \left(1-D_1\right)T_s\left[-\frac{di_1}{dt}\right]\Bigg|_{D_1T_s\le t\le T_s} \tag{13}$$

$$D_1T_s\left[\frac{di_2}{dt}\right]\Bigg|_{0\le t\le D_1T_s} = \left(1-D_1\right)T_s\left[-\frac{di_2}{dt}\right]\Bigg|_{D_1T_s\le t\le T_s}. \tag{14}$$

The substitution of (9), (10) and (11), (12) into (13) and (14) yields the steady-state relationships between the dc input voltage and the average output voltage, and the average voltage across the energy storage capacitor C_1:

$$V_{C1} = V_I \text{ and } V_O = \frac{D_1V_I}{1-D_1}.$$

From these results, the dc voltage conversion ratio of coupled-inductor PWM SEPIC converter with CCM is obtained as

$$M = \frac{V_O}{V_I} = \frac{D_1}{1-D_1} \tag{15}$$

and

$$V_{C1} = V_I. \tag{16}$$

The above results have been obtained even for the separate inductor case. So, the magnetically coupling of inductors do not modifies the dc voltage conversion ratio that only depends on the duty cycle of PWM converter and also the relationship between the voltages V_{C1}, V_O and V_I. On the other hand, the relationship (16) shows that the voltages across the two inductors are equal during each distinct time interval that allows to couple the inductors in order to reduce the input or output current ripple [1], [2].

Using these new results, (9), (10) and (11), (12) become:

$$\frac{di_1}{dt} = \frac{V_I}{L_{1e}} \tag{17}$$

$$\frac{di_2}{dt} = \frac{V_I}{L_{2e}} \tag{18}$$

and

$$\frac{di_1}{dt} = -\frac{V_O}{L_{1e}} \tag{19}$$

$$\frac{di_2}{dt} = -\frac{V_O}{L_{2e}} \tag{20}$$

where we denoted two effective inductances:

$$L_{1e} = \frac{\left(1 - k_c^2\right)L_1}{1 - k_c / n} \tag{21}$$

$$L_{2e} = \frac{\left(1 - k_c^2\right)L_2}{1 - k_c n}. \tag{22}$$

The equations (17) to (20) show that coupled inductors fed by the same voltage waveforms will exhibit as two equivalent inductors, which only depend on the physical inductor structure. The same equations give the theoretical conditions to cancel the inductor current ripples that is $L_{1e} \to \infty$ or $L_{2e} \to \infty$. So, the analysis results from the coupled inductor SEPIC converter can be extended to non-coupled inductor SEPIC converter and vice versa. Furthermore, two coupled inductors exhibit as two equivalent inductors with the effective inductances given by (21) and (22), and shown in Fig. 2b. Therefore, Fig. 2b serves exclusively for a clear representation of (17) and (20).

The average values of the inductor current ripples can be directly found from the waveforms of the inductor currents given in the Fig. 3 as:

$$I_{L1D} = \frac{D_1 T_s V_I}{2L_{1e}} = \frac{D_1 V_I}{k_{1m} R} \tag{23}$$

$$I_{L2D} = \frac{D_1 T_s V_I}{2L_{2e}} = \frac{D_1 V_I}{k_{2m} R} \tag{24}$$

where we denoted two parameters of conduction through the inductors as follows:

$$K_{1m} = 2L_{1e} f_s / R \tag{25}$$

$$K_{2m} = 2L_{2e} f_s / R. \tag{26}$$

Next, we determine the components I_{L01} and I_{L02} of inductor currents. For this purpose, we use the equations of the dc current through the load and the efficiency as follows:

$$\frac{V_O}{R} = I_D = \left(1 - D_1\right)\left(I_{L1D} + I_{L01} + I_{L2D} + I_{L02}\right) \tag{27}$$

$$\frac{V_O^2}{R} = \eta V_I \left(I_{L1D} + I_{L01}\right). \tag{28}$$

Using the assumption of 100% efficiency, the following results are obtained:

$$I_{L01} = \frac{D_1 V_I}{R}\left[\frac{D_1}{\left(1 - D_1\right)^2} - \frac{1}{K_{1m}}\right] \tag{29}$$

$$I_{L02} = \frac{D_1 V_I}{R}\left[\frac{1}{1 - D_1} - \frac{1}{K_{2m}}\right]. \tag{30}$$

We can now to find the average currents of transistor switch and diode as follows:

$$I_S = D_1\left(I_{L1} + I_{L2}\right) = \frac{D_1^2 V_I}{R\left(1 - D_1\right)^2} = \frac{M^2 V_I}{R} \tag{31}$$

$$I_D = \left(1 - D_1\right)\left(I_{L1} + I_{L2}\right) = \frac{D_1 V_I}{R\left(1 - D_1\right)} = \frac{M V_I}{R}. \tag{32}$$

The results of this simplified analysis completely characterise the steady-state behaviour of the coupled-inductor PWM SEPIC converter in continuous conduction mode. For the separate inductor case, it is enough to set k_c at zero into the above results. As consequence, $L_{1e} \to L_1$, $L_{2e} \to L_2$, $K_{1m} \to K_1$ and $K_{2m} \to K_2$.

3 Steady-state Analysis of SEPIC Converter with DCM

For SEPIC PWM converter as well as for Cuk PWM converter, a discontinuous conduction mode (DCM) means that the third time interval of operation cycle is nonzero, not that either inductor current is discontinuous. Coupled inductor techniques can be applied to PWM SEPIC converter with DCM to achieve low-ripple input current [1].

Assume that the output current is now further decreased to enable the converter to enter into discontinuous conduction mode. The waveforms of inductor currents and voltages corresponding to this operating mode are shown in Fig. 4.

Three distinct time intervals appears there, namely $D_1 T_s$, $D_2 T_s$ and $D_3 T_s$ with $D_1 + D_2 + D_3 = 1$ for a constant switching frequency. For DCM, besides of expressions of average inductor currents, average input and output currents, average voltage across the energy storage capacitor C_1 and average currents transistor and diode, we have to determine the parameter D_2 that fixes the decay interval of inductor currents and the dc voltage conversion ratio implicitly.

Concerning the waveforms of inductor currents, it can see that the shapes of the currents in Fig. 4 are similar to those shown in Fig. 3. Only the scales are different and the currents I_{L01} and I_{L02} are equal and have an inverse sense:

$$I_{L01} = -I_{L02} = I_{L0}. \tag{33}$$

The currents through inductors can be expressed as

$$i_{L1} = i_1 + I_{L0} \quad (34)$$
$$i_{L2} = i_2 - I_{L0}\,. \quad (35)$$

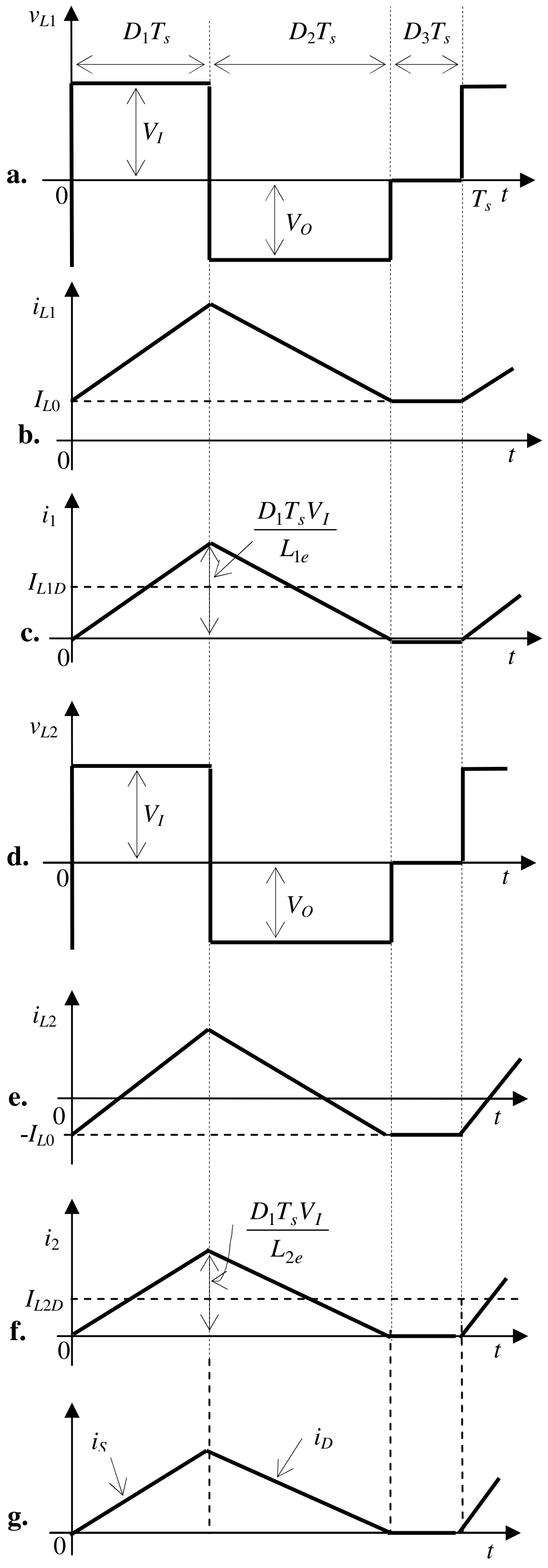

Fig.4 The waveforms of the currents and voltages in PWM SEPIC converter with DCM

For the first two time intervals D_1T_s and D_2T_s, (9), (10) and (11), (12) keep their validity. For the third time interval (D_3T_s), the voltages across inductors are zero: $v_{L1} = v_{L2} = 0$. Taking into account that the average voltages across the inductors over a switching period are zero, the following relationships result:

$$V_{C1} = V_I = \frac{D_2}{D_1} V_O\,. \quad (36)$$

The above relationships yield the dc voltage conversion ratio

$$M = \frac{V_O}{V_I} = \frac{D_1}{D_2} \quad (37)$$

and

$$V_{C1} = V_I\,. \quad (38)$$

The remark regarding the relationship (38) that is the same as (18) for the converter with CCM keeps its validity for the converter with DCM. The determination of the conversion ratio M needs the value of parameter D_2 too. It is obvious that the conditions of ripple cancellation from the inductor current found for CCM remain unchanged for DCM. Also, the two effective inductances and two parameters of conduction through the inductors hold their expressions.

Using the waveforms of inductor currents, the formula of average value of ripple component of inductor currents can be written as:

$$I_{L1D} = \frac{D_1(D_1 + D_2)V_I}{2L_{1e}f_s} = \frac{D_1(D_1 + D_2)V_I}{K_{1m}R} \quad (39)$$

$$I_{L2D} = \frac{D_1(D_1 + D_2)V_I}{2L_{2e}f_s} = \frac{D_1(D_1 + D_2)V_I}{K_{2m}R}\,. \quad (40)$$

The calculation of these components of inductor currents needs to find the parameter D_2 firstly. In order to find the expression of the component I_{L0}, we use again the relationship of the converter efficiency, that is

$$V_O I_O = \eta V_I (I_{L1D} + I_{L0})\,. \quad (41)$$

For assumption of 100% efficiency, the above equation yields

$$I_{L0} = \frac{D_1 V_I}{R}\left(\frac{D_1}{K_{2m}} - \frac{D_2}{K_{1m}}\right). \quad (42)$$

As it can be seen from the waveforms of currents in the transistor and diode, shown as i_S and respectively i_D in Fig. 4g, the total rise of i_S during $0 \le t \le D_1T_s$ should be equal to the total fall in i_D during $D_1T_s \le t \le (D_1 + D_2)T_s$. This means that we have

$$D_1 T_s \left[\frac{di_1}{dt} + \frac{di_2}{dt} \right]\Bigg|_{0 \le t \le D_1 \cdot T_s} = D_2 T_s \left[-\frac{di_1}{dt} - \frac{di_2}{dt} \right]\Bigg|_{D_1 \cdot T_s \le t \le (D_1 + D_2) \cdot T_s} \tag{43}$$

and after substituting the corresponding slopes of inductor currents and using (38), we find the dc voltage conversion ratio in (37) once more.

The average currents of transistor and diode result as follows:

$$I_S = \frac{D_1}{D_1 + D_2}\left(I_{L1D} + I_{L2D}\right) = \frac{M^2 V_I}{R} \tag{44}$$

$$I_D = \frac{D_2}{D_1 + D_2}\left(I_{L1D} + I_{L2D}\right) = \frac{M V_I}{R}. \tag{45}$$

Now, we proceed to determine the formula for the parameter D_2. For this purpose, we use the dc load current combining (39), (40) with (45). As result, the parameter D_2 is given by the formula

$$D_2 = \sqrt{K_{em}} \tag{46}$$

where the quantity

$$K_{em} = 2 L_{em} f_s / R \tag{47}$$

represents the parameter of conduction through an equivalent inductor with the inductance $L_{em} = L_{1e} // L_{2e}$.

4 Boundary between the Continuous and Discontinuous Conduction Modes

For a complete characterization of the PWM SEPIC converter operating, we find now the equation of the boundary between CCM and DCM for the coupled-inductor case. It is well known that a converter will change the operating mode when the following equality is satisfied:

$$D_2 = 1 - D_1. \tag{48}$$

This means that the parameter K_{em} reached its critical value

$$K_{emcrt} = (1 - D_1)^2. \tag{49}$$

The same equation can be derived starting from (33). All these results characterize the PWM SEPIC converter with separate inductors, case in which we have to set k_c at zero.

5 Conclusion

A steady-state analysis that completely characterizes the behavior of PWM SEPIC converter with coupled or separate inductors and operating in CCM or DCM was made in this paper. Taking the effects of inductor coupling into account, the expressions of all steady-state currents such as the average input and output currents, and the average currents of transistor and diode are found. For the dc voltage conversion ratio and the average voltage across the energy storage capacitor, we found the same formula as in the separate inductor case because we neglected the equivalent series resistance of capacitors. The steady-state analysis developed here supplies the formula of equivalent inductances in the coupled-inductor case that are entailed by a dynamic analysis based on the small-signal equivalent circuit of converter. This simplified analysis yields the same results as one based on the large-signal PWM switch [6].

Formally, the dc conversion ratio of PWM SEPIC converter with coupled-inductors and operating in DCM has the same formula as in the separate-inductor case. But, the parameter D_2 takes the coupling effect into account, by means of the parameter of conduction through an equivalent inductor with the inductance L_{em}. The average input and output currents, and the average currents of transistor and diode as well as the boundary equation include the coupling effect too. So, the steady-state operating point and the parameters of small-signal low-frequency model of converter depend on the coupling parameter.

References:

[1] J. W. Kolar, H. Sree, N. Mohan and F. C. Zach, Novel Aspects of an Application of ‘Zero’-Ripple Techniques to Basic Converter Topologies, *IEEE PESC’97*, Vol. 1, pp. 796-803

[2] J. Chen and C. Chang, Analysis and Design of SEPIC Converter in Boundary Conduction Mode for Universal-line Power Factor Correction Applications, in *Proc. IEEE PESC’0,* 2001.

[3] S. Ben-Yaakov, D. Adar and G. Rahav, A SPICE compatible Behavioral Model of SEPIC Converters, *IEEE PESC’96,* Vol. 2, pp. 1668-1673

[4] W. M. Moussa, Modeling and performance evaluation of a DC/DC SEPIC converter, *APEC’95,* Vol. 2, pp. 702–706

[5] D. Adar, G. Rahav and S. Ben-Yaakov, A unified behavioral average model of SEPIC converters with coupled inductors, *IEEE PESC’97*, Vol. 1, pp. 441- 446

[6] V. Vorperian, Simplified analysis of PWM converters using model of PWM switch, Parts I (CCM) and II (DCM), *IEEE Trans. on Aerospace Electronic Systems*, Vol. 26, 1990, pp. 497-505