Proceedings of the 10th WSEAS International Conference on CIRCUITS, Vouliagmeni, Athens, Greece, July 10-12, 2006 (pp297-301)

All-Fiber Temporal Differentiators Operating at Terahertz Speeds

JOSÉ AZAÑA [1] and MYKOLA KULISHOV [2]

1. Institut National de la Recherche Scientifique (INRS), Montreal, Quebec, H5A 1K6 CANADA

2. Photonic Systems Group, Dept. of Electrical and Computer Engineering, McGill University, Montreal, Quebec, H3A 2A7 CANADA

Abstract- A simple and practical all-fiber approach for realizing ultrafast optical differentiators is proposed and numerically investigated. In particular, it is demonstrated that first-order and second-order temporal differentiation of an arbitrary optical waveform can be realized using a suitable uniform long-period grating photoinscribed in the core of an optical fiber (LPFG structure). Our results have confirmed the feasibility of the proposed approach for processing optical signals with temporal features in the sub-picosecond range (corresponding bandwidths of as much as a few terahertz).

Keywords: All-optical circuits; all-fiber technologies; ultrafast signal processing; all-optical computing; fiber optics communications.

1 Introduction

Recently, there has been an increasingly significant effort toward the realization of ultrafast all-optical signal processing devices for next generation fiber optics networks and all-optical computers. All-optical signal processors allow overcoming the typical bandwidth limitations that plague an electronic solution. One major step towards this goal has been the advent of optical fibers; in the low-loss telecommunications wavelength range (between 1.3 and 1.6 microns) in fibers, there is approximately 40 THz of available bandwidth. However, the need to convert an optical signal into the electronic domain for implementing fundamental processing and computing operations drastically limits the use to advantage of this available bandwidth (electronic solutions are fundamentally limited to a maximum speed processing of a few tens of gigahertz). Even though the progress towards all-optical solutions over the last few years have been impressive, following the realization of state-of-the-art laser sources, optical amplifiers, and multi-purpose ultra-compact optical filters [1], both in fiber and integrated technologies, several fundamental and basic devices (including *all-optical temporal differentiators*) need still to be designed and engineered, or require complicated, expensive and unpractical optical set-ups to be implemented [2]. In the electronic domain, practical, cost-effective temporal differentiators can be realized by using a simple combination of operational amplifiers, resistors and capacitors. However, there is still not a (simple) equivalent of these fundamental devices in the photonics domain.

In this work, we show that all-optical temporal differentiators can be implemented using a very well-known and mature all-fiber technology, i.e. long-period fiber gratings (LPFG's) [3], [4]. This finding opens important, new perspectives for implementing ultrafast first-order and high-order optical differentiators in a simple and practical fashion, using an all-fiber platform. Optical differentiators may find important applications as basic building blocks in ultrahigh-speed all-optical analog/digital signal processing circuits (in exact analogy with their electronic counterparts). Moreover, all-optical differentiators could be readily applied for ultrafast optical pulse shaping, or for control or sensing applications using optical signals.

A fiber phase grating is a periodic perturbation of the refractive index along the core of an optical fiber, which is permanently inscribed in the fiber by exposing the same to a spatially varying pattern of light intensity from an ultraviolet or an excimer laser source (the refractive index is changed in the fiber by a photosensitivity effect) [1]. Fiber gratings can be characterized into two types: fiber Bragg gratings (FBGs), in which coupling occurs between modes traveling in opposite directions; and transmission gratings, also called long-period fiber gratings (LPFGs). Specifically, an LPFG induces light coupling between the guided mode and cladding mode(s) in a single-mode fiber, thus resulting in a series of attenuation (resonant) bands centered at discrete wavelengths (phase-matching wavelengths) in the fiber transmission spectrum [5]. While there has been a considerable research effort to understand and exploit both the spectral and temporal properties of FBGs [1], [6] and a wide number of works have been also reported on spectral-domain analysis of LPFGs, time-domain analysis of LPFGs has received much less attention. This has similarly restricted the use of LPFG devices to the conventional filtering – type applications (band-pass and band-stop filters, mode converters, spectrum equalizers etc.). Very recently, we have carried out an extensive numerical study of optical pulse propagation through waveguide long-period gratings (including LPFGs) within the linear regime of operation, anticipating the potential of these devices for several picosecond/sub-picosecond optical pulse shaping and processing applications [7]. This potential is further evidenced by the work presented here.

The remainder of this paper is structured as follows. In Section 2, we present the principle of operation of ultrafast optical differentiators based on uniform LPFGs. Our theoretical predictions are confirmed by means of numerical simulations (Section 3). In Section 4, we summarize and conclude our work.

2 Principle of Operation

The temporal operation of a first-order optical differentiator can be mathematically described as $v(t) \propto \partial u(t)/\partial t$, where $u(t)$ and $v(t)$ are the temporal optical waveforms (complex envelopes) at the input and at the output of the system, respectively, and t is the time variable. In the frequency domain, $V(\omega) \propto -j\omega U(\omega)$, where $U(\omega)$ and $V(\omega)$ are the spectra of $u(t)$ and $v(t)$, respectively (ω is the base-band frequency, i.e. $\omega = \omega_{opt} - \omega_0$, where ω_{opt} is the optical frequency and ω_0 is the central optical frequency of the signals). Thus, first-order optical differentiation can be implemented using an optical filter with a transfer function $H_1(\omega) \propto -j\omega$, which depends linearly on frequency (along the whole bandwidth of the signal to be processed). A key feature of a first-order optical differentiator is that it must introduce a π phase shift exactly at the signal's central frequency, ω_0. We anticipate that these characteristics are provided "inherently" by the core's transfer function of a single uniform LPFG (around its resonance frequency, ω_0) working at full-coupling conditions, e.g. when $\kappa L = \pi/2$ (κ is the grating coupling coefficient, and L is the grating length).

In general, the transfer-function of an N-order optical differentiator is $H_N(\omega) = [H_1(\omega)]^N \propto (-j\omega)^N$. An N-order differentiator can thus be implemented by connecting in series N first-order optical differentiators, e.g. by concatenating N uniform LPFGs, each one working in full-coupling conditions, along the same optical fiber. In practice, some additional technique should be used in between the gratings in order to remove (or significantly attenuate) the energy coupled into the cladding mode by the LPFGs (e.g. by partial cladding removal). In particular, the transfer function of a second-order differentiator is a parabolic function of frequency, $H_2(\omega) = [H_1(\omega)]^2 \propto -\omega^2$. Here we also demonstrate that this filtering function is inherently achieved in the cladding mode of a LPFG providing full energy recoupling back from the cladding mode into the core mode, e.g. when $\kappa L = \pi$.

From coupled-mode theory, the core and cladding spectral transmission responses of a uniform LPFG (assuming that the input pulse is launched into the fiber core) are [1],

$$H_{co}(\omega_{opt}) = \left[\cos(\gamma L) + j\frac{\sigma}{\gamma}\sin(\gamma L)\right]\exp(j(\beta_{co} - \sigma)L) \quad (1)$$

$$H_{cl}(\omega_{opt}) = j\frac{\kappa}{\gamma}\sin(\gamma L)\exp(j(\beta_{cl} + \sigma)L) \quad (2)$$

where σ is the detuning factor, $\sigma(\omega_{opt}) = [\beta_{co}(\omega_{opt}) - \beta_{cl}(\omega_{opt})]/2 - (\pi/\Lambda)$, $\beta_{co}(\omega_{opt})$ and $\beta_{cl}(\omega_{opt})$ are the propagation constants for the core and cladding modes, respectively, Λ is the grating period, and $\gamma = \sqrt{\sigma^2 + \kappa^2}$. The transmission functions in Eqns. (1) and (2) can be approximated in the vicinity of the resonance frequency ω_0 (where $\sigma \to 0$) by the two first non-zero terms of the respective Taylor expansions:

$$H_{co}(\omega_{opt}) \approx \left[\cos(\kappa L) + j\frac{\sigma}{\kappa}\sin(\kappa L)\right]\exp(j\beta_{co,0}L) \quad (3)$$

$$H_{cl}(\omega_{opt}) \approx j\left(\sin(\kappa L) + \frac{\sigma^2}{2\kappa^2}\left[\kappa L\cos(\kappa L) - \sin(\kappa L)\right]\right) \times \times \exp(j\beta_{cl,0}L) \quad (4)$$

where $\beta_{co,0} = \beta_{co}(\omega_0)$ and $\beta_{cl,0} = \beta_{cl}(\omega_0)$. The detuning factor in the close vicinity of ω_0 can also be expanded in a Taylor series and approximated by $\sigma(\omega_{opt}) \approx \sigma_1(\omega_{opt} - \omega_0) = \sigma_1\omega$, where

$$\sigma_1 = (1/2)[\partial\beta_{co}(\omega_0)/\partial\omega - \partial\beta_{cl}(\omega_0)/\partial\omega].$$

If the LPFG is designed to provide full-coupling into the cladding mode, i.e. $\kappa L = m(\pi/2)$, with m = 1, 3, 5, …, then it follows from Eqn. (3) that $H_{co}(\omega_{opt}) \propto -j\sigma \propto -j\omega$, which corresponds to the spectral response of a first-order optical differentiator. Similarly, if the LPFG parameters are fixed to achieve full energy recoupling from the cladding into the core mode, i.e. $\kappa L = m\pi$, with m = 1, 2, 3,…, then it follows from Eqn. (4) that $H_{cl}(\omega_{opt}) \propto \sigma^2 \propto -\omega^2$, which corresponds to the spectral response of a second-order optical differentiator Notice that optical differentiation is achieved over a limited bandwidth (up to a few terahertz) around the LPFG resonance frequency ω_0 (the input optical signal is assumed to be centered at ω_0).

3 Numerical Simulations

Our theoretical predictions were confirmed by means of numerical simulations. For the results presented herein, we simulated a uniform LPFG written in the core of a SMF-28 optical fiber. In practice, a less standard fiber design, e.g. optical fiber with a cladding-inner layer, could be used to improve the device stability by minimizing the sensitivity of the LPFG spectrum to environmental fluctuations. The grating was designed to couple light between the core mode and the third cladding mode. Alternatively, the LPFGs could be designed to induce coupling into other higher-order cladding modes, just resulting in slightly different spectral features (e.g. in terms of bandwidth or energetic efficiency). The following wavelength dependence was assumed for the effective refractive indices corresponding to the two coupled modes: $n_0(\lambda)$=1.46410 – 0.00830λ –0.00185λ^2 for the core mode, and $n_3(\lambda)$= 1.45769 – 0.00507λ– 0.002479λ^2 for the cladding mode. These index approximations were obtained using full-vector simulation of the SMF-28 fiber. The grating period was fixed to Λ = 533.45 µm, which translates into a resonance wavelength of exactly λ_0 = 1555 nm (resonance frequency ω_0 = 2π×192.8 THz). Fig. 1(a) shows the amplitude of the core transmission response $H_{co}(\omega_{opt})$ (solid curve) of a 5-cm long uniform LPFG with κL= π/2. As expected, the core amplitude transmission response of this uniform LPFG has a nearly linear dependence with frequency along most of the transmission notch bandwidth, i.e. in the vicinity of the resonance frequency. Fig. 1(b) shows the amplitude of the cladding transmission response $H_{cl}(\omega_{opt})$ (solid curve) of a 5-cm long LPFG with κL= π. As expected, the cladding amplitude transmission response of this LPFG has a nearly quadratic dependence with frequency around its resonance frequency.

The operation of the simulated uniform LPFGs as optical differentiators was confirmed by calculating the temporal response of the gratings to different optical signals. In the examples shown in Fig. 2, the input signal was a transform-limited super-Gaussian optical pulse centred at the gratings′ resonance wavelength of 1555 nm and with unity peak intensity, that is $u(t) = \exp\left(-(\ln(2)/2)(2t/\tau_0)^{2Q}\right)$, of order Q = 3 and a Full-Width-half-Maximum (FWHM) time duration τ_0 = *3 ps*. The average optical intensity of this input signal, $|u(t)|^2$, is shown in Fig. 2(a) and Fig. 2(b) (dashed curves). Fig. 2(a) presents the results corresponding to the 5-cm long LPFG with κL= π/2 (spectrum shown in Fig. 1(a)), whereas Fig. 2(b) presents the results for the 5-cm long LPFG with κL= π (spectrum shown in Fig. 1(b)). In each case, the temporal waveform $v(t)$ at the output of the LPFG was calculated by taking the Inverse Fourier Transform (IFT) of the result of multiplying the input pulse spectrum $U(\omega)$ by the corresponding LPFG spectral transmission response [7]. The average optical intensity $|v(t)|^2$ obtained at the output of the simulated LPFGs is represented in Fig. 2(a) and Fig. 2(b) with solid curves.

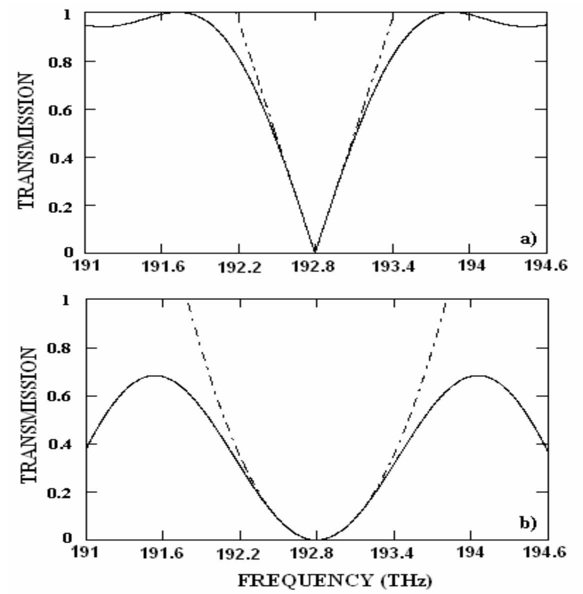

Fig.1. (a) Amplitude of the spectral transmission function corresponding to the core mode of a 5-cm long uniform LPFG with κL= π/2 (solid curve) and to an ideal first-order optical differentiator (dashed-dotted curve). **(b)** Amplitude of the spectral transmission function corresponding to the cladding mode of a 5-cm long uniform LPFG with κL= π (solid curve) and to an ideal second-order optical differentiator (dashed-dotted curve).

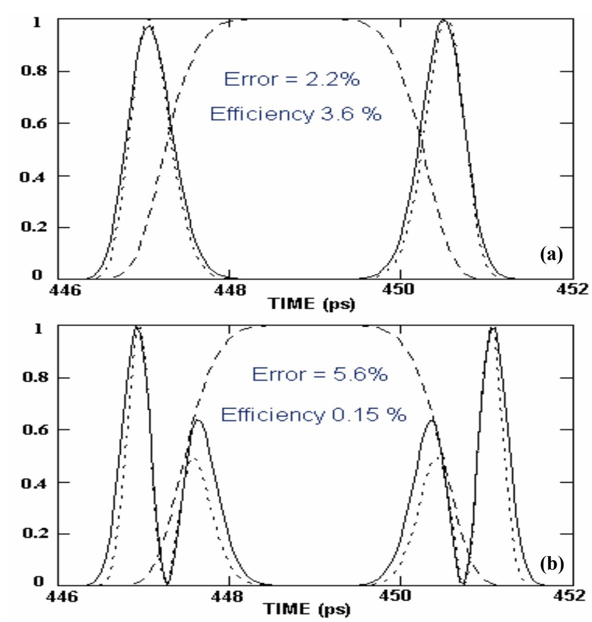

Fig. 2. Dashed curves: temporal waveform corresponding to the 3-ps super-Gaussian optical pulse launched at the input of the simulated LPFGs. Solid curves: output temporal waveforms **(a)** in the core of a 5-cm long uniform LPFG with κL= π/2 (spectrum in Fig. 1(a)) and **(b)** in the cladding of a 5-cm long uniform LPFG with κL= π (spectrum in Fig. 1(b)). Dotted curves: ideal (analytical) **(a)** first and **(b)** second time derivatives of the input pulse.

As predicted, the temporal waveform in the core mode at the output of the uniform LPFG with $\kappa L = \pi/2$ is very close to the ideal (analytical) first temporal derivative of the input pulse, which is also represented in Fig. 2(a) with a dotted curve. Specifically, the estimated error or deviation between the output temporal waveform from the LPFG and the ideal first-order time derivative of the input pulse in this first example is ≈ 2.2%. This deviation was estimated as the relative difference between the normalized optical intensities corresponding to the numerically obtained and the ideal (analytical) temporal derivatives [2], evaluated over a temporal window where the signals exhibit non-zero intensity. Similarly, the temporal waveform in the cladding mode at the output of the uniform LPFG with $\kappa L = \pi$ is very close to the ideal second temporal derivative of the input pulse (compare solid and dotted curves in Fig. 2(b)). In this case, the estimated error is ≈ 5.6%. Notice that in all the cases, the optical intensities are represented in normalized units. For completeness, the energetic efficiency of the processing operation was also estimated in each case by calculating the ratio of the output signal energy to the input signal energy. In all the cases, the energetic efficiency was significantly higher for first-order optical differentiation than for second-order optical differentiation.

In Fig. 3, we evaluated the maximum signal bandwidth that can be processed using uniform LPFGs of different lengths in order to ensure that the signal processing error is kept lower than 1%. This evaluation was made for both the core mode (first-order differentiation, solid-box curve in Fig. 3) and the cladding mode (second-order differentiation, dashed-diamond curve in Fig. 3), considering feasible grating lengths ranging from 2 cm to 10 cm.

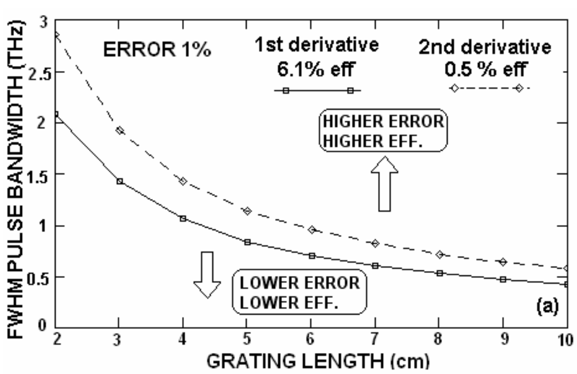

Fig. 3. Maximum signal bandwidth (FWHM bandwidth of input Gaussian pulse) which can be processed with a uniform LPFG operated as a first-order differentiator (solid-box curve) and as a second-order differentiator (dashed-diamond curve) to keep the processing error lower than 1%, evaluated for different grating lengths.

As expected, the shorter the LPFG, the larger the operation bandwidth it provides. It is important to note that ultrafast signals with temporal features in the sub-picosecond regime (corresponding bandwidths > 1THz) could be processed using uniform LPFGs of a few centimetres long. It should be also mentioned that for a given grating length, the processing of faster temporal features (i.e. larger bandwidths) is usually achieved with a higher energetic efficiency but always at the expense of a higher error.

4 Conclusions

In summary, we have shown that ultrafast first-order and higher-order temporal differentiation of an arbitrary optical waveform can be achieved by simply propagating this optical waveform along an optical fiber with a suitably designed long-period grating photoinscribed in its core (LPFG structure). Our numerical analysis has confirmed that signals with time features in the sub-picosecond regime (corresponding bandwidths up to a few terahertz) can be processed using readily feasible LPFG devices. All-fiber ultrafast optical differentiators are of interest as basic building blocks in all-optical circuits and should prove very useful for a wide range of applications, including sub-picosecond optical pulse shaping and processing, all-optical computing and networking and optical sensing and control.

Acknowledgements:

The authors are very grateful to Dr. Xavier Daxhelet for providing the numerical estimations of the SMF-28 fiber dispersion characteristics used in this work. This research was supported in part by the Natural Sciences and Engineering Research Council of Canada (NSERC).

References:

[1] R. Kashyap, *Fiber Bragg Gratings*, Academic Press, San Diego, (1999).

[2] N.Q. Ngo, S.F. Yu, S.C. Tjin, and C.H.Kam, "A new theoretical basis of higher-derivative optical differentiators", *Opt. Comm.*, vol. 230, pp. 115- 129, (2004).

[3] M. Kulishov and J. Azaña, "Long-period fiber gratings as ultrafast optical differentiators", *Opt. Lett.*, vol. 30, pp. 2700-2702, Oct. 2005.

[4] J. Azaña and M. Kulishov, "All-fiber ultrafast optical differentiator based on a π-phase-shifted long-period grating", *Electron. Lett.*, vol. 41, pp. 1368-1370, Dec 2005.

[5] A. M. Vengsarkar, J. R. Pedrazzani, J. B. Judkins, P. J. Lemaire, N. S. Bergano, and C. R. Davidson, "Long-period fiber-grating-based gain

equalizers", *Opt. Lett.*, vol. 21, pp. 336-338 (1996).

[6] M. Marano, S. Longhi, P. Laporta, M. Belmonte and B. Agogliati, "All-optical square-pulse generation and multiplication at 1.5 μm by use of a novel class of fiber Bragg gratings", Opt. Lett., vol. 26, pp. 1615-1617 (2001).

[7] M. Kulishov, J. Azaña, "Ultrashort pulse propagation in uniform and non-uniform waveguide long-period gratings", *J. Opt. Soc. Am. A* , vol. 22, pp. 1319-1333 (2005).