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Designing of nonlinear functions using BJT Gilbert cells for nonlinear control applications

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Abstract In this article redesigning and simulating a precision multiplier block using BJT Gilbert cell to creating nonlinear functions like a sinusoidal function block is desirable. Nowadays nonlinear functions play a significant role in some fields like communication and signal processing. One of important problems that can be solved with them is designing nonlinear functions that are applicable in nonlinear control and effective control. Using discrete time nonlinear controllers cause some problems like instability that can be eliminated using continuous time nonlinear controllers. The most essential block for creating nonlinear functions is a precision multiplier.

Word -keys: Analog multiplier, Gilbert cell, four-quadrant multiplier

Introduction

In signal processing often, it is necessary to create an output signal using tow input signals that its magnitude is proportional to input magnitudes. This can be implemented using analog multiplier blocks.

Term of four-quadrant multipliers point to circuits that multiply two sign signals. Four quadrant multipliers are base of many structures such as phase detectors in phase locked loops [1] and frequency translators, especially have significant application in audio and video signal processing.

The next field of using nonlinear function is nonlinear control .Usage of desecrate time controllers creates some problems such as: quantization fault, no stability, more sensitivity, not ability to analyze the exact behavior of the systems, like nonzero being the amount of permanent state fault and delay [2].

These problems will be resonated when using nonlinear functions because the system dynamic of nonlinear control is richer than linear control [3].

It is recommended to use continuous time nonlinear controllers overcoming these problems [2]. Precision analog multipliers are the essential blocks to create nonlinear functions. Fig. 1 shows this multiplier. In this multiplier, assume:

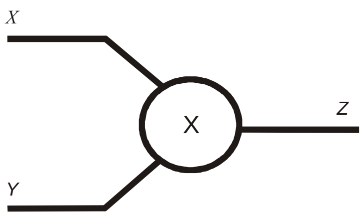

Fig.1

$$-10 < V_x, V_y < 10$$

And,

$$-25 < V_z < 25$$

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Gilbert four quadrant analog multiplier

1.1 Background

The first time Gilbert presented the first multiplier core in 1968 [4]. This cell consist of three transistors pairs (Fig. 2) and work on transconductance of transistors. Now this cell is the base of many integrated balanced multiplier systems, although during these years other improved multiplier structures have been presented [5], [6].

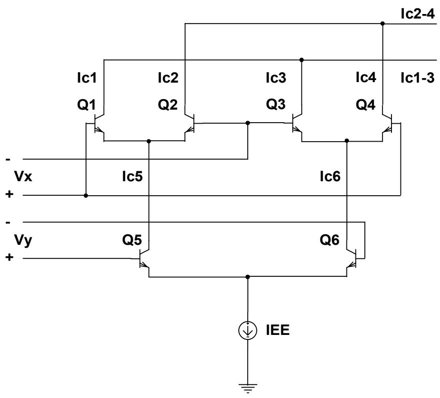

Fig.2 Gilbert cell

It has proved that differential output of Gilbert cell is:

$$\Delta I = I_{C1-3} - I_{C2-4} = (I_{C1} + I_{C3}) - (I_{C2} + I_{C4})$$

$$\Delta I = I_{EE}\left[\tanh\left(\frac{V_x}{2V_T}\right)\right]\left[\tanh\left(\frac{V_y}{2V_T}\right)\right] \quad (1)$$

When I_{C1} to I_{C4} be collector currents of Q1 to Q4.

According to equation (1), DC transfer characteristic of Gilbert cell is multiply of inputs tangent hyperbolic and where as, this output is merely linear near V_T, according to Fig. 3 it is recommended to use inverse tangent hyperbolic circuits in series with inputs.

Fig. 4 and Fig.5 illustrate the inverse tangent hyperbolic circuit.

In Fig.4, I_1 and I_2 are explained as:

$$I_1 = I_{o1} + K_1 V_x \quad (2)$$

$$I_2 = I_{o1} - K_1 V_x \quad (3)$$

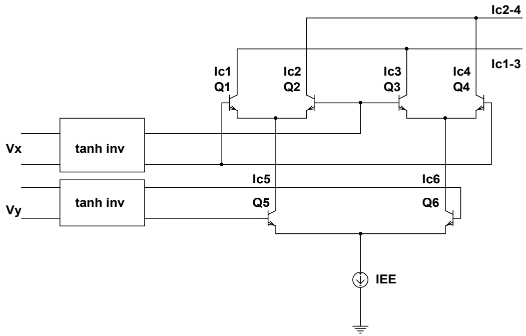

Fig.3 Gilbert cell with compensated inputs

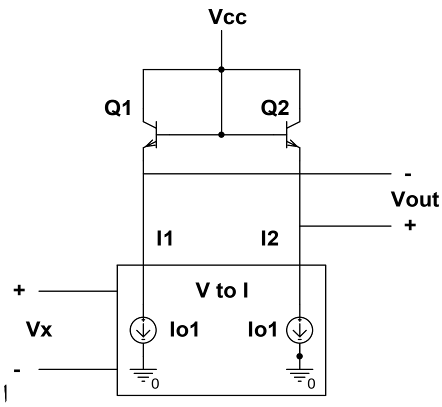

Fig.4 Tangent hyperbolic generator circuit

Where, I_{o1} are constant current source and K_1 is transfer conductance coefficient of voltage to current converter. Preventing transistors to being off in Fig.5 should assume:

$$-\frac{I_{o1}}{k_1} < V_x < \frac{I_{o1}}{k_1} \quad (4)$$

In Fig. 5, the differential output is expressed by:

$$\Delta V_{\mathrm{OUT}} = V_T Ln\left(\frac{I_{o1} + K_1 V_x}{I_S}\right) - V_T Ln\left(\frac{I_{o1} - K_1 V_x}{I_S}\right) =$$

$$= V_T Ln\left(\frac{I_{o1} + K_1 V_x}{I_{o1} - K_1 V_x}\right) \quad (5)$$

$$\tanh^{-1} X = \frac{1}{2} Ln\left(\frac{1 + X}{1 - X}\right) \quad (6)$$

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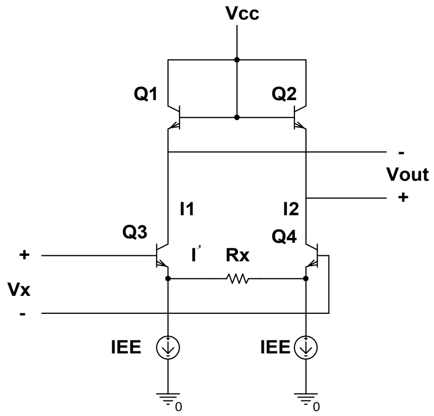

Fig 5 Tangent hyperbolic generator circuit

Considering equation (6), differential output voltage will obtain:

$$\Delta V_{OUT} = 2V_T \tanh^{-1}\left(\frac{K_1 V_x}{I_{o1}}\right) \quad (7)$$

It have been proved that without using tangent hyperbolic generator in series with second output, second output enters the equation directly and Differential output current will be as following:

$$\Delta I = I_{EE}\left(\frac{K_1 V_x}{I_{o1}}\right)\left(\frac{K_2 V_y}{I_{o2}}\right) \quad (8)$$

According to equation (8) output voltage expressed as:

$$V_{\text{out}} = \frac{2}{I_{o1} I_{o1}} R_L I_{EE} K_1 K_2 V_x \cdot V_y \quad (9)$$

1.2 Determining coefficients of full Gilbert multiplier cell

Fig.6 illustrates the Gilbert cell using compensated input series with first input. In Fig. 6, have been assumed that transistors are alike exactly.Additionaly in input loop, using voltage law can consider:

$$-V_x + I' R_x + V_{be7} - V_{be8} = 0 \quad (10)$$

$$\Rightarrow I' = V_x / R_x \quad (11)$$

Ignoring base currents of transistors, I_1 and I_2 are expressed as:

$$I_1 = I_{EE} + I' \Rightarrow I_1 = I_{EE} + V_x / R_x \quad (12)$$

$$\Rightarrow I_2 = I_{EE} - V_x / R_x \quad (13)$$

If assuming $I_{EE} = I_{o1} = I_{o2}$ and comparing equation (2) and (3) transfer, conductance coefficients are expressed as:

$$K_1 = 1/R_x \quad (14)$$

$$K_2 = 1/R_y \quad (15)$$

In addition, final voltage result is:

$$V_{\text{out}} = 2R_L \frac{V_x V_y}{R_x R_y I_{EE}} \quad (16)$$

Where, R_L is load of differential output to unique output converter. Equation (17) and (18) keep input transistors to be active:

$$V_y(max) < I_{EE} R_y \quad (17)$$

$$V_x(max) < I_{EE} R_x \quad (18)$$

To keep Q11, Q12, Q13and Q14 in active condition collector voltages of them must be higher than their base voltages also; collector voltages of Q9 and Q10 must be lower than collectors of Q11 and Q12 for two-diode voltage. In addition, base voltages of Q1 to Q4 must be lower than base voltages of Q9 and Q10 for one-diode voltage. Considering above conditions resistances of the circuit can be obtained. Fig.8 illustrates the simulation result for a squaring generator using this circuit when $-3.15 < V_{in} < 3.15$ using Orcad.

Sinusoidal functions using analog multipliers.

Now it is time to create one of nonlinear functions like Cos(x).it can be expressed as:

$$Cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \ldots$$

This means that it is possible to create sinusoidal functions using analog multipliers and adders.

For this, four points must be considered:

1-Level of supply voltage to prevent saturation.

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2-CMRR of op-amps.

3-Power consumption.

4-Delay of each stage.

Fig.9 shows diagram of a sinusoidal function circuit and Fig. 10 is the result of sinusoidal function circuit simulation using Orcad software.

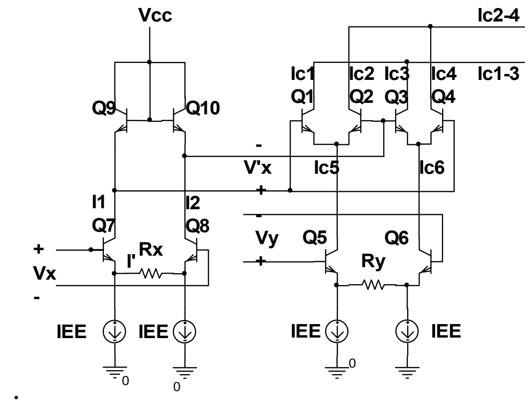

Fig.6 The second input inverse tangent hyperbolic circuit can be eliminated

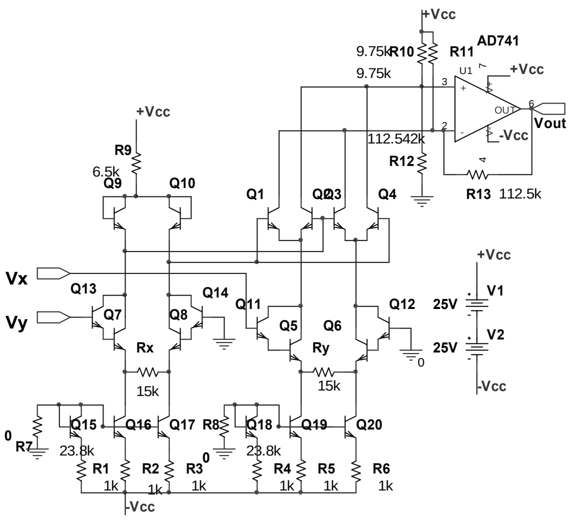

Fig.7 practical multiplier and details

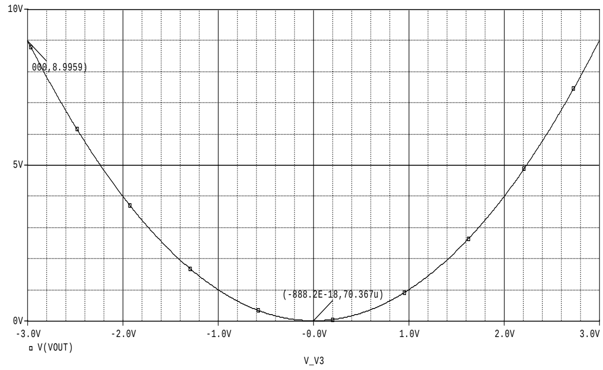

Fig.8 simulation result for squaring circuit

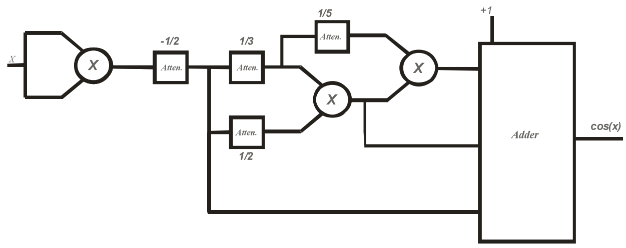

Fig.9 cos(x) function generator circuit

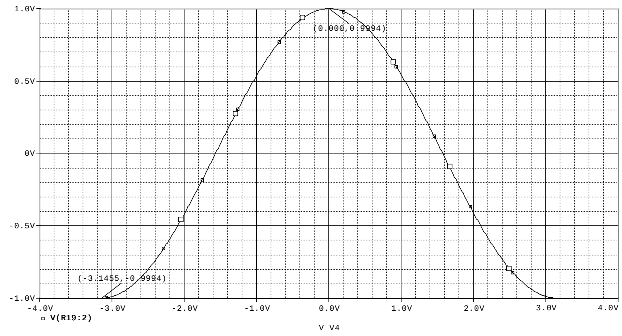

Fig.10 result of simulating for Fig.9

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