Proceedings of the 7th WSEAS International Conference on Automation & Information, Cavtat, Croatia, June 13-15, 2006 (pp71-76)

Open Loop Vibrational Control of a Recycled Bioprocess

DAN SELIŞTEANU[1], EMIL PETRE[1], HANI HAMDAN[2], DORIN POPESCU[1]
[1]Department of Automatic Control and Mechatronics
University of Craiova
A.I. Cuza Str. No. 13, RO-200585, Craiova
ROMANIA
[2]STMS, UMR CNRS 9912, IRCAM
1, place Igor Stravinsky, 75004 Paris
FRANCE

Abstract: - This paper deals with the vibrational control of a recycled bioprocess, which takes place into a Continuous Stirred Tank Bioreactor (CSTB). The classical control techniques for bioreactors necessitate measurements of states and/or disturbances. However, the bioprocesses are systems that require costly and difficult on-line implementation for sensors. Vibrational control is a non-classical open-loop control method, developed by Bellman, Bentsman and Meerkov, useful for the cases when the instrumentation is not available or is very expensive. The vibrational control is applied by means of zero mean parametric vibrations for shaping the response of a linear or nonlinear system. In this work, the design of a vibrational control for a CSTB with recycle stream is presented. Numerical simulations are considered in order to illustrate the performances of the vibrationally controlled bioprocess.

Key-Words: - Open-loop control, Vibrational control, Linear time lag systems, Biotechnology

1 Introduction

Feedback or feedforward control techniques can be used in order to achieve a desired change in the behaviour of a system. The problem is that these strategies require on-line measurements. In some cases, the instrumentation is not available or is very expensive. This is the case of the bioindustry, where the cheap and reliable instrumentation is missing. A possibility to overcome this problem is the application of the so-called vibrational control.

Vibrational control (VC) is a non-classical open-loop control method proposed by Bellman, Bentsman and Meerkov [2], [3], [9]. The "classical" theory of VC for linear systems was developed by Meerkov in [9] and extended to nonlinear systems by Bellman *et al.* [2], [3]. Others significant results are obtained by Lehman *et al.* [6], [7] for time lag systems with bounded delay. Applications of the vibrational control theory can be found for: stabilization of plasma, lasers [8], chemical reactors [5], biotechnological processes [10], [11], [12].

The VC technique is applied by oscillating an accessible system component at low amplitude and high frequency. The amplitude and the frequency of the control input are constants and independent of the state of the system, therefore this technique is a form of open-loop control.

The Section 2 of this paper deals with the presentation of the model of bioprocess that takes place into a CSTB with recycle stream. The dynamic model, the equilibrium points, the linearization around an equilibrium point and the phase portrait are analysed in this section. In Section 3, the VC theory for time-lag systems is presented. The vibrational control strategy is developed for the recycled bioprocess, using a linearized model. The existence and the choice of stabilizing vibrations, which ensure the desired behaviour for the bioprocess, are also analysed. Illustrative numerical simulations are included. Concluding remarks are collected in Section 4.

2 Model of the Recycled Bioprocess

A bioreactor is a tank in which several biological reactions occur simultaneously in a liquid medium. In industry, the bioreactors operate in three modes: the continuous mode, the fed-batch mode and the batch mode [1], [10]. Bioreactors that operate in the continuous mode are usually known as Continuous Stirred Tank Bioreactors. In a CSTB, the substrates (the nutrients) are fed to the bioreactor continuously and an effluent stream is continuously withdrawn from the CSTB such that the culture volume is constant. Often, a part of the biomass is recycled. To recycle, the biomass must be separated from the substrate and yield, then travel through pipes after separation. This time of recycle introduce delays in the states and complicates the dynamic. The benefits are that the recycle increases the overall conversion and reduces the costs.

The dynamical state-space model of a biotechnological process in a CSTB expresses the mass balance of the components in the bioreactor [1], [11]:

$$\frac{d\xi_1}{dt} = \mu(\xi_1, \xi_2) \cdot \xi_1 - D \cdot \xi_1 \quad (1)$$

$$\frac{d\xi_2}{dt} = -k_1 \mu(\xi_1, \xi_2) \cdot \xi_1 - D \cdot \xi_2 + D \cdot S_{in} \quad (2)$$

where ξ_1, ξ_2 represent the biomass and the limiting substrate concentrations [g/l]. S_{in} is the influent substrate concentration and D is so-called dilution rate [h^{-1}], i.e. the specific volumetric outflow rate. In (1), (2) μ is the specific growth rate and $k_1 > 0$ the yield coefficient. The bioprocess (1), (2) is in fact a fermentation process, which usually occurs in a bioreactor.

In the CSTB with recycle stream, a part of the biomass is recycled. If the recycle occurs, then the bioprocess model (1), (2) must be rewritten [12]:

$$\frac{d\xi_1}{dt} = \mu(\xi_1(t), \xi_2(t))\xi_1(t) - D\xi_1(t) + (1-q)D\xi_1(t-r) \quad (3)$$

$$\frac{d\xi_2}{dt} = -k_1 \mu(\xi_1(t), \xi_2(t))\xi_1(t) - D\xi_2(t) + F_{in} \quad (4)$$

The model (3), (4) is a prototype model for some depollution bioprocess, like the activated sludge bioprocess, which recycle a part of biomass [13].

In (3), $(1-q) \cdot D$ is the recycle flow rate. The constant q varies from 0 to 1, with zero corresponding to total recycle and 1 to no recycle. The constant r is the recycle delay time and F_{in} is the input flow. A compact representation of the state-space model (3), (4) is:

$$\dot{\xi} = f(\xi) \quad (5)$$

where $\xi = [\xi_1 \xi_2]^T$ is the state vector and the function $f(\cdot)$ is the nonlinear vector function $f(\xi) = [f_1(\xi_1, \xi_2), f_2(\xi_1, \xi_2)]^T$.

The equilibrium states of (3), (4) are of two types:

1. Wash-out equilibrium states (E3), defined by:

$$\text{(E1)} \quad \xi_s = [\xi_{s1} \xi_{s2}]^T = [0 \quad F_{in} / D]^T \quad (6)$$

This equilibrium is in fact a state when the bacterial life has disappeared.

2. Operational equilibrium states (E2), implicitly defined by:

$$\text{(E2)} \begin{cases} \mu(\xi_{s1}, \xi_{s2}) = qD \\ k_1 \mu(\xi_{s1}, \xi_{s2})\xi_{s1} + D\xi_{s2} = F_{in} \end{cases} \quad (7)$$

Equilibria (E1) correspond to the bioreactor wash-out, therefore only equilibria (E2) have a technological interest.

These equilibria can be attractive or repulsive depending on the particular form of $\mu(\xi_1, \xi_2)$. Only these equilibria have a practical interest. Let's assume that the specific growth rate is:

$$\mu(\xi_1, \xi_2) = \mu(\xi_2) = \mu_0 \frac{\xi_2}{K_M + \xi_2 + \xi_2^2 / K_i} \quad (8)$$

This is the Haldane kinetic model of the specific growth rate [1]. K_M is the Michaelis - Menten constant, K_i the inhibition constant and μ_0 the maxim specific growth rate. Next, we suppose that the form of the specific growth rate is the Haldane kinetic model (8) that takes into account substrate inhibition on the growth. Then, from (7) we have two possibilities for the equilibria (E2):

$$\text{(a)} \; \xi_{s1} = \frac{F_{in} - D\xi_{s2}}{k_1 qD} \overset{N}{=} \xi_{s1,1} = \frac{F_{in} - D\xi_{s2,1}}{k_1 qD}; \; \xi_{s2} \overset{N}{=} \xi_{s2,1} \quad (9)$$

$$\text{(b)} \; \xi_{s1} = \frac{F_{in} - D\xi_{s2}}{k_1 qD} \overset{N}{=} \xi_{s1,2} = \frac{F_{in} - D\xi_{s2,2}}{k_1 qD}; \; \xi_{s2} \overset{N}{=} \xi_{s2,2} \quad (10)$$

The case (a) corresponds to a stable equilibrium point (stable node). The case (b) leads to a saddle type for the equilibria (E2) [12].

The phase plane corresponding to the system (3), (4) for the values of the process parameters: $\mu_0 = 6h^{-1}$, $K_M = 10g/l$, $K_i = 100g/l$, $k_1 = 1$, $D = 3.6h^{-1}$, $S_{in} = 100g/l$, $F_{in} = 41\ h^{-1}\ g/l$, $q = 0.8$, $r = 0.25\ h$ and for different initial conditions is represented in Fig. 1. From this picture it can be seen that when the substrate inhibition appears, the process can exhibit unstable or, maybe worse, the evolution leads to wash-out steady-states, for which the microbial life has disappeared and the reactor is stopped. In these situations, the bioprocess requires control to stabilize the CSTB. Also, in many cases, the stable equilibrium point corresponding to (a) is not technological operable (requires a big initial amount of biomass). Furthermore, a bigger value for the delay time can induce a worst behaviour.

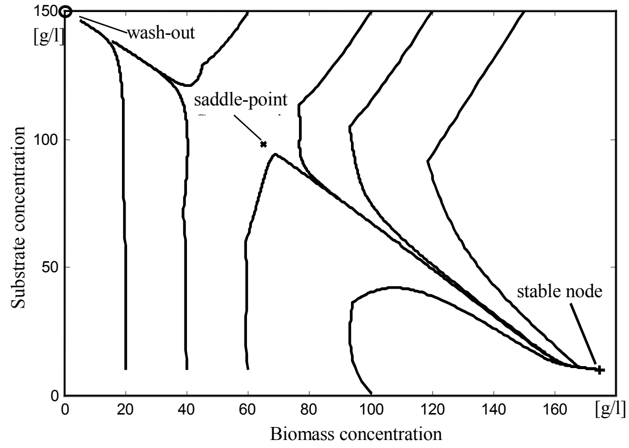

Fig. 1. Phase plane of the recycled bioprocess

The conclusion is that for the CSTB with recycle stream it is necessary to design a control strategy.

For control purposes, it is necessary to find the linear approximation of the system (3), (4) or equivalent (5) around the equilibrium point (E2). The linear approximation for given constant inputs D and F_{in} is:

$$\frac{d}{dt}\big(\xi(t)-\xi_s\big)=A_0(\xi_s)\big(\xi(t)-\xi_s\big)+A_1(\xi_s)\big(\xi(t-r)-\xi_s\big) \tag{11}$$

where the matrices A_0 and A_1 are obtained after straightforward calculations for the specific rate (8):

$$A_0(\xi_s)=A_0(\xi_{s1},\xi_{s2})=\begin{bmatrix}\mu(\xi_{s2})-D & \gamma\\ -k_1\mu(\xi_{s2}) & -k_1\gamma-D\end{bmatrix} \tag{12}$$

$$A_1(\xi_s)=A_1(\xi_{s1},\xi_{s2})=\begin{bmatrix}(1-q)\cdot D & 0\\ 0 & 0\end{bmatrix} \tag{13}$$

with

$$\gamma\overset{N}{=}\xi_{s1}\left[\frac{d\mu(\xi_2)}{d\xi_2}\right]_{\xi_2=\xi_{s2}}=\xi_{s1}\mu_0\frac{K_M-\xi_{s2}^2/K_i}{(K_M+\xi_{s2}+\xi_{s2}^2/K_i)^2}$$

Let's consider:

$$x\overset{N}{=}\xi-\xi_s=\begin{bmatrix}x_1 & x_2\end{bmatrix}^T=\begin{bmatrix}\xi_1-\xi_{s1} & \xi_2-\xi_{s2}\end{bmatrix}^T$$

Then the linear model (21) becomes:

$$\frac{dx(t)}{dt}=A_0\cdot x(t)+A_1\cdot x(t-r) \tag{14}$$

The characteristic equation of the linearized system (14) around the equilibria (E2) is a transcendental equation:

$$\det\big(\lambda I-A_0-A_1e^{-\lambda r}\big)=0 \tag{15}$$

For a small delay time r, the equation (15) can be approximated with the equation:

$$\det\big(\lambda I-A_0-A_1\big)=0 \tag{16}$$

The roots of equation (15) decide the stability of the equilibria. For the operational equilibrium point (b), which is interesting from technological point of view, after some calculations, from (16) we obtain that this point is unstable (see Fig. 1) [12]. The delay time influences the stability properties; therefore, this equilibrium point of the initial system (5) is indeed unstable. The goal of the control strategy is to stabilize the equilibrium point (10).

3 Vibrational Control Design

3.1 Problem statement

A general theory of vibrational control was developed by Bellman *et. al.* [2], [3], who presented the criteria for vibrational stabilizability and vibrational controllability of linear and nonlinear systems.

Consider a nonlinear system given by the equation:

$$\dot{x}=f(x,\alpha) \tag{17}$$

with $f{:}R^n\times R^m\to R^n$, $x\in R^n$ is a state and $\alpha\in R^m$ is a parameter, in fact a vector that contains the system parameters. Suppose that for a fixed $\alpha=\alpha_0$ the system (17) has the equilibrium $x_s=x_s(\alpha)$. Let introduce now in (17) parametric vibrations according to the law:

$$\alpha(t)=\alpha_0+g(t) \tag{18}$$

where α_0 is a constant vector and *g(t)* is an almost periodic vector function with average equal to zero (APAZ vector). Then (17) becomes:

$$\dot{x}=f(x,\alpha_0+g(t)) \tag{19}$$

Definition 1 [2]: An equilibrium point $x_s(\alpha_0)$ of (17) is vibrationally stabilizable if for any $\delta>0$ there exists an APAZ vector *g(t)* such that (19) has an asymptotically stable almost periodic solution $x^s(t)$ characterized by:

$$\left\|\bar{x}^s-x_s(\alpha_0)\right\|<\delta \tag{20}$$

$$\bar{x}^s\overset{\Delta}{=}\overline{x^s(t)}=\lim_{T\to\infty}\frac{1}{T}\int_0^T x^s(\tau)d\tau \tag{21}$$

Definition 2 [2]: An equilibrium point $x_s(\alpha_0)$ of (17) is totally vibrationally stabilizable if it is vibrationally stabilizable and furthermore,

$x^s(t) = const = x_s(\alpha_0)\,,\ \forall t \in R$

Definition 3 [2]: An equilibrium $x_s(\alpha_0)$ is partially vibrationally stabilizable with respect to *i*-th component if for any $\delta > 0$ exists an APAZ vector *g(t)* and $\alpha_1 = const$ such the system

$$\dot{x} = f(x, \alpha_1 + g(t)) \tag{22}$$

has an asymptotically stable almost periodic solution $x^s(t)$, the *i*-th component of which is characterized by:

$$\left|x_i^s - x_{si}(\alpha_0)\right| < \delta \tag{23}$$

The vibrational stabilizability problem consists of finding conditions for existence of stabilizable vibrations. Meerkov has demonstrated since 1980 [9] that for linear systems vibrational stabilizability implies total stabilizability. If the matrix A of the linear system

$$\dot{x} = Ax \tag{24}$$

is a nonderogatory matrix, i.e. the minimal and characteristic polynomials coincide, a sufficient condition of vibrational stabilizability is:

$$\mathrm{tr}(A) < 0 \tag{25}$$

Parametric vibrations can stabilize the linear system only if they have a so-called multiplicative form:

$$g(t) = B(t)x \tag{26}$$

If $B(t)$ is periodic, then the condition (25) is also necessary [9].

Once the conditions for existence of vibrational stabilizability are settled for a system, it is necessary to solve another important problem: finding the specific form of stabilizing vibrations. This problem is referred as vibrational controllability [3].

The VC theory was extended by Lehman *et al.* [6], [7] for time lag systems with bounded delay. The obtained results demonstrated the viability of VC technique as a possible alternative to feedback for time lag systems.

Let's consider the linear time lag system:

$$\frac{dx(t)}{dt} = A_0 \cdot x(t) + A_1 \cdot x(t-r) \tag{27}$$

where x is the state and r the time delay. Consider also the linear almost periodic system:

$$\frac{dy}{dt} = F\left(\frac{t}{\varepsilon}\right) y(t-r) \tag{28}$$

where $y : [-r, \infty) \to R^n$, $F : R^+ \to R^{nxn}$, $0 < \varepsilon < 1$.

Define the averaged equation corresponding to (28) as:

$$\frac{dz}{dt} = A \cdot z(t-r)\,, \tag{29}$$

$$z : [-r, \infty) \to R^n\,,\quad A = \lim_{T\to\infty} \frac{1}{T}\int_0^T F\left(\frac{t}{\varepsilon}\right) dt\,.$$

Lemma 1 [6]: Assume that all the roots of the $\det(\lambda I - Ae^{-\lambda r})$ have nonzero real parts. Then exists ε_0>0 such that for any $0 < \varepsilon \le \varepsilon_0$ the trivial solution of (28) has the same stability properties as that of (29).

Consider now the system with vibrations:

$$\frac{dx(t)}{dt} = A_0 \cdot x(t) + F(t,\varepsilon) \cdot x(t) + A_1 \cdot x(t-r) \tag{30}$$

with $F : R^+ \times R^+ \to R^{nxn}$ periodic in the first argument zero average matrix (PAZ matrix) and $0 < \varepsilon < 1$.

Definition 4: The trivial solution of the system (51) is said to be totally vibrationally stabilizable if there exists a PAZ matrix $F(t,\varepsilon)$ such that the trivial solution of the system with vibrations (30) is asymptotically stable.

Remark 1: The system with vibrations (30) can be interpreted as the result of introduction of a parametric excitation into the matrix A_0.

The problem of vibrational stabilization for system (27) consists of finding condition for existence of totally stabilization vibrations and finding the concrete stabilizing vibrations.

Theorem 1 [6]: Suppose that:
(i) there exists a PAZ matrix $B(t)$ such that the state transition matrix $\Phi(t,0)$ of

$$\frac{dx}{dt} = B(t) \cdot x(t) \tag{31}$$

is almost periodic;
(ii) the equation

$$\det(\lambda I - \overline{A_0} - \overline{A_1} e^{-\lambda r}) = 0\,, \tag{32}$$

$$\overline{A_i} \overset{\Delta}{=} \lim_{T\to\infty} \frac{1}{T}\int_0^T \Phi^{-1}(\tau,0) A_i \Phi(\tau,0) d\tau, \quad i = 1,2$$

has all the roots with the negative real parts. Then:
(a) there exists $\varepsilon_0 > 0$ such that for any $0 < \varepsilon \le \varepsilon_0$ the trivial solution of the equation

$$\begin{aligned}\frac{dy}{dt} &= \Phi^{-1}\left(\frac{t}{\varepsilon},0\right) A_0 \Phi\left(\frac{t}{\varepsilon},0\right) y(t) \\ &\quad + \Phi^{-1}\left(\frac{t}{\varepsilon},0\right) A_1 \Phi\left(\frac{t}{\varepsilon},0\right) y(t-r)\end{aligned} \tag{33}$$

is asymptotically stable.

(b) the trivial solution of the system (27) is totally stabilizable by the vibrations:

$$F(t,\varepsilon)\cdot x(t) = \frac{1}{\varepsilon}\cdot B\left(\frac{t}{\varepsilon}\right)\cdot x(t) \quad (34)$$

if $\varepsilon \le \varepsilon_0$.

The Theorem 1 reduces the vibrational stabilization problem for linear time lag systems of form (27) to this strategy: first we induce an asymptotic stability of the system (33) by the correct choice of the PAZ matrix $B(t)$, and second we must find the value of ε_0 (analytically possible, but recommended via numerical simulation).

3.2 Design of VC for the recycled bioprocess

The basic idea of vibrational controlled CSTB is to vibrate the flow rates and in this way to operate the bioreactor at average conversion rates which were previously unstable. By using this technique is possible to eliminate significant expenses associated with feedback. Since the vibrations depend only on time and not on the value of states, there no longer was a need to take measurements of concentrations.

The design of VC for CSTB with recycle stream represented by the nonlinear model (3), (4) is based on the linearized time lag system (11), (12), (13) (or equivalent (14)) around the operational point (E2). From Fig. 1 it can be seen that the equilibrium (10) of (E2) is unstable and it is necessary control in order to stabilize this operational point.

The linear time lag system (14) is obviously of the form (27). For the implementation of the vibrational control following the methodology of the subsection 3.1, we choose the PAZ matrix B(t):

$$B(t) = \alpha\cdot\cos(t)\cdot\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix} \quad (35)$$

where $\alpha \in R$. The form of the matrix B suggests that the control action consist in vibrating the input flow rate, i.e. the inlet substrate rate. From (35), (12), (13), the state transition matrix of (31), the averaged matrix $\overline{A_0}$ and the averaged matrix $\overline{A_1}$ for our bioprocess are (see Theorem 1):

$$\Phi(t,0) = \begin{bmatrix}1 & \alpha\sin(t)\\ 0 & 1\end{bmatrix};\ \overline{A_1} = \begin{bmatrix}(1-q)D & 0\\ 0 & 0\end{bmatrix};$$

$$\overline{A_0} = \begin{bmatrix}\mu(\xi_{s2,2}) - D & \gamma + k_1\mu(\xi_{s2,2})\frac{1}{2}\alpha^2\\ -k_1\mu(\xi_{s2,2}) & -k_1\gamma - D\end{bmatrix}.$$

Therefore, if the matrix $B(t)$ is (35), the vibrational controlled bioreactor is described by:

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = A_0\cdot\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + A_1\cdot\begin{bmatrix}x_1(t-r)\\ x_2(t-r)\end{bmatrix} + \frac{1}{\varepsilon}\alpha\cos\left(\frac{t}{\varepsilon}\right)\cdot\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\cdot\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} \quad (36)$$

The system (36) is of the form (30) written in the terms of the CSTB with $B(t)$ from (35). The average controlled system is in this case the following:

$$\begin{bmatrix}\dot{z}_1(t)\\ \dot{z}_2(t)\end{bmatrix} = \overline{A_0}\cdot\begin{bmatrix}z_1(t)\\ z_2(t)\end{bmatrix} + \overline{A_1}\cdot\begin{bmatrix}z_1(t-r)\\ z_2(t-r)\end{bmatrix} \quad (37)$$

In conclusion, for α chosen such that all the roots of the characteristic equation (32):

$$\det(\lambda I - \overline{A_0} - \overline{A_1}e^{-\lambda r}) = 0$$

are negative real parts, the vibrations of the form (35) stabilize for ε sufficiently small the trivial equilibrium of the linearized system (14).

Remark 2: The vibrational control is applied in the case of the CSTB with recycle stream for a linearization around the operational point, therefore the unique equilibrium of the linearized time lag system (24) is trivial (the origin).

Simulation results. The simulation values are furnished in the previous section. The time evolution of the linearized CSTB around the unstable equilibrium is depicted in Fig. 2. The origin is unstable in this case. When the VC is implemented the simulation of (36) for $\alpha = 5, \varepsilon = 0.01$ leads to the time evolution depicted in Fig. 3. The state trajectories go to the unique equilibrium point in the case of linear systems - the origin. The average behaviour of the recycled bioprocess with vibrations is described by (37). Fig. 4 shows the averaged phase portrait of CSTB. It can be seen from the phase portrait that the saddle-type of equilibrium from Fig. 1 is stabilized.

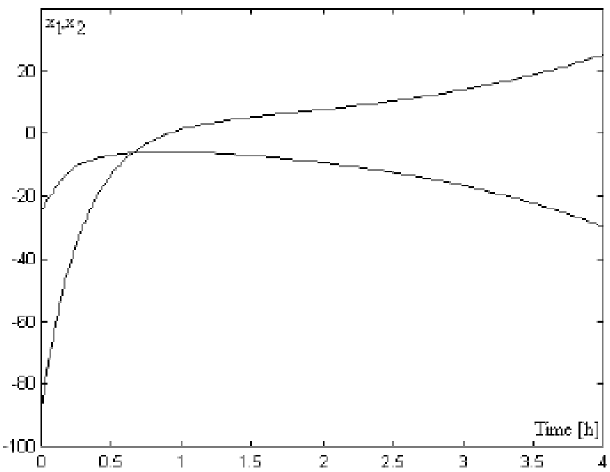

Fig. 2. State trajectories of initial unstable system

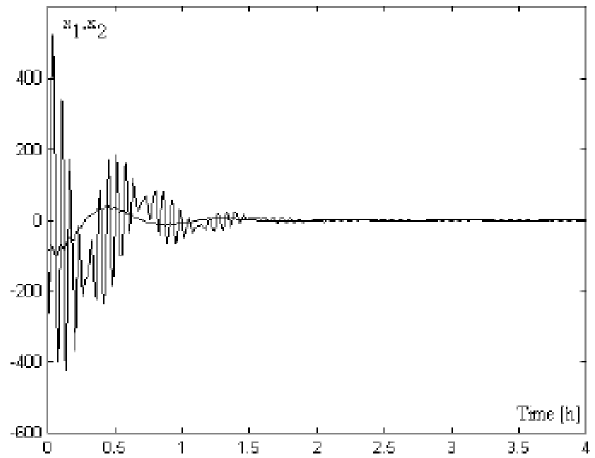

Fig. 3. Time evolution of the controlled bioprocess

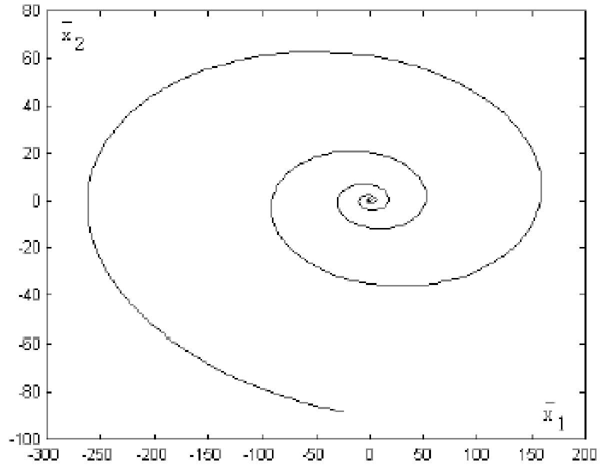

Fig. 4. Averaged phase portrait of the bioprocess

4 Conclusions

The results reported in this paper demonstrate that in some cases, the open-loop vibrational control can be successful used. VC is in fact a form of high-frequency control technique (high-frequency relative to the natural frequency of the system). A comparison between this technique and others' high-frequency methods - like sliding mode control and dithering - can be done. A main difference between VC and these methods is that in vibrational case, a component of the system is vibrated independent of the state; therefore the control is a function depending only on time.

The practical engineering VC problem can be described as a three step technique: first it is necessary to find the conditions for existence of stabilizing vibrations, second to find which parameter or component is physically possible to vibrate and finally to find the parameters of vibrations that ensure the desired response. The study done in this work shows that vibrational control can stabilize a previously unstable steady state of a recycled bioprocess.

For the vibrational technique to be effective, one needs to have an accurate description of system dynamics. This fact together with physical limitation on the magnitude and the frequency of vibrations in some cases are the disadvantages of the technique. The major advantage is the ability to ensure a desired behaviour when the measurements are not on-line. The results can be extended in the future to the nonlinear time lag system case.

References:

[1] G. Bastin, D. Dochain, *On-line Estimation and Adaptive Control of Bioreactors*, Elsevier, 1990.

[2] R.E. Bellman, J. Bentsman, S.M. Meerkov, Vibrational Control of Nonlinear Systems: Vibrational Stabilizability, *IEEE Trans. A.C.*, vol. AC-31, no. 8, 1986, pp. 710-716.

[3] R.E. Bellman, J. Bentsman, S.M. Meerkov, Vibrational Control of Nonlinear Systems: Vibrational Controllability and Transient Behavior, *IEEE Tr. A.C.*, vol. 31, no. 8, 1986, pp. 717-724.

[4] J. Bentsman, Vibrational Control of a Class of Nonlinear Systems by Nonlinear Multiplicative Vibrations, *IEEE Trans. A.C.*, vol. AC-32, no. 8, 1987, pp. 711-716.

[5] A. Cinar, J. Deng, S.M. Meerkov, X. Shu, Vibrational Stabilization of a Chemical Reactor, *IEEE Tr. A.C.*, vol. 32, no. 4, 1987, pp. 348-352.

[6] B. Lehman, J. Bentsman, Vibrational Control of Linear Time Lag Systems with Arbitrarily Large but Bounded Delays, *IEEE Tr. on A.C.*, vol. 37, no. 10, 1992, pp. 576-582.

[7] B. Lehman, J. Bentsman, S.V. Lunel, E.I. Verriest, Vibrational Control of Nonlinear Time Lag Systems with Bounded Delay: Averaging Theory, Stabilizability, and Transient Behavior, *IEEE Tr. A.C.*, vol.39, no.5, 1994, pp.898-912.

[8] S.M. Meerkov, G.I. Shapiro, Method of Vibrational Control in the Problem of Stabilization of Ionization-Thermal Instability of a Powerful Continuous CO_2 Laser, *Autom. Remote Contr.*, vol. 37, 1976, pp. 821-830.

[9] S.M. Meerkov, Principle of Vibrational Control: Theory and Applications, *IEEE Trans. A.C.*, vol. AC-25, no. 4, 1980, pp. 755-762.

[10] D. Selişteanu, E. Petre, Vibrational Control of a Class of Bioprocesses, *Control Engineering and Applied Informatics*, vol.3, no.1, 2001, pp.39-50.

[11] D. Selişteanu, C. Ionete, D. Popescu, D. Şendrescu, On Vibrational Control of a Class of Nonlinear Systems, *5th Int. Carpathian Contr. Conf.*, Vol. I, 2004, Poland, pp. 245-250.

[12] D. Selişteanu, C. Ionete, Vibrational Control of a Bioreactor with Delay in the Recycle Stream, *Int. Conf. Q&A-R 2000 THETA12*, Cluj-Napoca, Romania, Vol. 2, 2000, pp. 113-118.

[13] D. Selişteanu, E. Petre, E. Bobaşu, D. Popescu, Modelling and On-line Estimation of Unknown Kinetics for an Activated Sludge Bioprocess, *WSEAS Trans. on Systems*, Issue 10, vol. 4, 2005, pp. 1650-1657.