

Controlling Chaos in a Current Mode Controlled Boost Converter Using Ott-Grebogi-Yorke and Derivate Methods

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Abstract: - Chaotic behaviors of the current mode controlled boost converter were demonstrated by computer simulation studies. The study implied theoretical considerations and simulations for this kind of converter using for the control OGY method and OGY combined with Pyragas method. The methods tried to establish a periodic behavior from a chaotic one in the converter case.

Key-Words: - chaotic, converter, boost, OGY, Pyragas

1 Introduction

These years, the possibility of controlling nonlinear chaotic systems has been the subject of research. It was demonstrated that many unstable periodic orbits could transform in a periodic behavior using different control methods. The main class of strategies are based on the OGY control method. An alternative method was proposed by Pyragas and was called time-delayed autosynchronization. There is the third class of control strategies wich use state feedback controllers to solve the problem of controlling chaos[1-4].

capacity C. The control for the switch is assured by a flip-flop and a comparator circuit. The circuit has two modes for its functionality depend on the position of the switch ($Q=0$ open or $Q=1$ close). The position of the switch S is also dependent of the value of the main current i and of the period T of the clock impulse. So, in the case of switch 'close' the main current i arise and the clock hasn't any effect because the output of the flip-flop remains on '1'. The state of the switch is changed to 'open' when the current i is equal with reference current I_{ref} because in this case the flip-flop is reset. In this situation the current will fall.

2 Converter Operation

The circuit scheme for this converter is shown in figure 1:

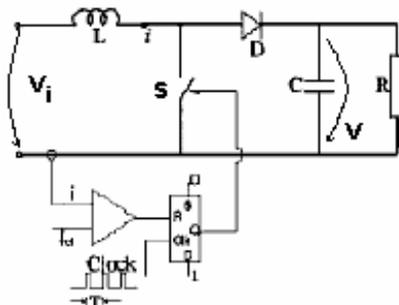


Fig. 1 The current mode controlled boost converter

The circuit consists from a controlled switch S, a diode D, an inductor, a capacitor and a load resistor characterized by inductance L, resistance R and

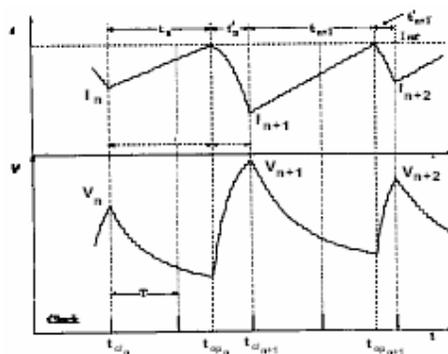


Fig. 2 The waveforms for current and voltage

From these waveforms is obvious that the circuit shows a variable structure and toggles the topology in relation with the states of the switch. The functionality of the circuit in the hypothesis of neglecting the voltage drop on D, can be described

by the following dynamical system with variable structure [1]:

$$\dot{x} = \begin{cases} A_1 x + B_1 V_i & \text{for } t_{cl,n} < t < t_{op,n} \\ A_2 x + B_2 V_i & \text{for } t_{op,n} \leq t < t_{cl,n+1} \end{cases}$$

$$\{t_{op,n}\} = \{t \mid i(t) = I_{ref}\}$$

$$\{t_{cl,n}\} \subset \{kT, k \in \mathbb{Z}\}$$
(1)

where $x=[v \ i]^T$ represent the state vector of the boost converter and depends on the state of the circuit:

$$A_1 = \begin{bmatrix} -1/RC & 0 \\ 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 1/L \end{bmatrix}$$
(2)

3 Theoretical Background

The OGY method assumes a few considerations. The dynamics of the system can be represented as arising from an n -dimensional nonlinear discrete-time function of the following form:

$$x_{k+1} = P(x_k, p)$$
(3)

where p is an accessible system parameter. In the (typically) case of continuous-time systems this map is given via some form of sampling.

There is a maximum small perturbation Δp^* in the parameter p by which it is acceptable to vary p from the nominal value p^* . For the value of p^* there is a chaotic attractor of the underlying system which contains a specific periodic trajectory around which one wishes to stabilize the dynamics.

The position of this periodic trajectory is a function of p , but the local dynamics do not vary much with the allowed small changes in parameter p .

While the dynamics is assumed to arise from a map, one needs no model for the global dynamics. These assumptions would seem to allow for the control of any chaotic system for which a faithful map can be constructed, e.g., from experimental data. For simplicity the presentation is restricted to a two-dimensional map P .

An equilibrium point $x_F(p)$ of the map (3) of the system is defined by:

$$x_F(p) = P(x_F(p), p)$$
(4)

therefore it moves with the parameter p by:

$$\frac{dx_F(p)}{dp} = \frac{\partial P}{\partial x} \Big|_{x_F(p), p} \cdot \frac{dx_F(p)}{dp} + \frac{\partial P}{\partial p} \Big|_{x_F(p), p}$$
(5)

Let $x^*(p^*)$ denote the unstable equilibrium point of the

map P existing for the parameter value p^* and corresponding to that periodic trajectory on the attractor which one wants to stabilize. From (5) results:

$$\frac{\partial P}{\partial p} \Big|_{x^*, p^*} = \frac{dx_F}{dp} \Big|_{p^*} - \frac{\partial P}{\partial x} \Big|_{x^*, p^*} \cdot \frac{dx_F}{dp} \Big|_{p^*}$$
(6)

or

$$b = (I - A) \cdot g$$
(7)

$$\text{where } g = \frac{dx_F}{dp} \Big|_{p^*}, A = \frac{\partial P}{\partial x} \Big|_{x^*, p^*}, b = \frac{\partial P}{\partial p} \Big|_{x^*, p^*}$$
(8).

In the close neighborhood of the desired equilibrium point $x^*(p^*)$ we can assume with good accuracy that the dynamics of map P is linear and can be expressed by the first-order approximation of :

$$\Delta x_{k+1} = \frac{\partial P}{\partial x} \Big|_{x^*, p^*} \cdot \Delta x_k + \frac{\partial P}{\partial p} \Big|_{x^*, p^*} \cdot \Delta p_k =$$

$$= A \cdot \Delta x_k + b \cdot \Delta p_k$$
(9)

Substituting (7) in (9):

$$\Delta x_{k+1} = A \cdot \Delta x_k + (I - A) \cdot g \cdot \Delta p_k$$
(10)

Matrix A may be determined using a measured chaotic time series x_k with $p = p^*$ and analyzing its behavior close to the equilibrium point x^* . Furthermore, the stable and unstable eigenvalues λ_s, λ_u and corresponding eigenvectors e_s, e_u of this matrix can be found and they determine the stable and unstable manifolds in the neighborhood of the equilibrium point like in the figure 3[5,7].

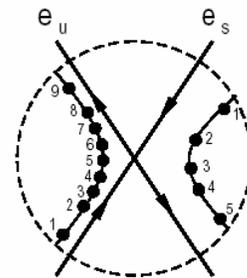


Fig.3. The values for determination for fixed point position and for the Jacobian of the matrix A

To control the chaos, the parameter p is adjusted at each iteration (so instead of p we have p_k) in such a way that the iterates of the map P are confined to a small neighborhood of the desired equilibrium point $x^*(p^*)$. When an iterate falls near the desired trajectory, parameter p is changed from its nominal value p^* by Δp_k , thereby changing the location of the trajectory and its stable manifold, such that the next iterate will be forced back toward the stable manifold of the original trajectory for $p = p^*$.

Assume that x_k falls near the desired equilibrium

point $x^*(p^*)$ so that (10) applies. The choice of Δp_k is attempted in such a way that Δx_{k+1} lies along the stable manifold of equilibrium point. Let $\Delta p_k = c^T \cdot \Delta x_k$ and (10) becomes:

$$\Delta x_{k+1} = A \cdot \Delta x_k + (I - A) \cdot g \cdot c^T \cdot \Delta x_k = [A + (I - A) \cdot g \cdot c^T] \cdot \Delta x_k \quad (11)$$

c^T must be picked so that Δx_{k+1} falls on the stable eigenvector e_s of matrix A , which can be rewritten as:

$$A = \lambda_u e_u f_u^T + \lambda_s e_s f_s^T \quad (12)$$

where f_u and f_s are the contravariant basis vectors, defined by $f_u^T e_u = f_s^T e_s = 1$ and $f_u^T e_s = f_s^T e_u = 0$.

Note that Δx_{k+1} lies along e_s if $f_u^T \Delta x_{k+1} = 0$. Thus, dotting (11) with f_u^T and expressing A with (12) the following equation was obtained:

$$f_u^T \cdot [\lambda_u e_u f_u^T + \lambda_s e_s f_s^T + (I - \lambda_u e_u f_u^T + \lambda_s e_s f_s^T) \cdot g \cdot c^T] \cdot \Delta x_k = 0, \forall \Delta x_k \quad (13)$$

Using the definition of the contravariant basis vectors, (13) results in:

$$\lambda_u \cdot f_u^T + (1 - \lambda_u) \cdot f_u^T \cdot g \cdot c^T = 0 \quad (14)$$

which leads to the following equation for c^T :

$$c^T = \frac{\lambda_u}{(\lambda_u - 1) \cdot f_u^T \cdot g} \cdot f_u^T \quad (15)$$

Therefore the *OGY control law* is:

$$\Delta p_k = \frac{\lambda_u \cdot f_u^T \cdot \Delta x_k}{(\lambda_u - 1) \cdot f_u^T \cdot g} \quad (16)$$

where $\Delta p_k = p_k - p^*$, $\Delta x_k = x_k - x^*$

The control (16) is only activated if the resulting change in the parameter Δp_k is less than the maximal allowed disturbance Δp^* ; otherwise Δp_k is set to zero.

Note that the parameter value $p_k = p^* + \Delta p_k$ should be updated only when the trajectory crosses the surface of section that generates map P . However, the trajectory may not be brought to the equilibrium point

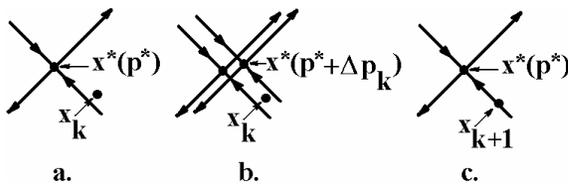


Figure 5. Schematic of the OGY control algorithm: a. The k th iterate x_k falls near the desired equilibrium point $x^*(p^*)$.

- b. Turn on the perturbation of p to move the equilibrium point.
- c. The next iterate is forced onto the stable manifold of $x^*(p^*)$.

Turn off the perturbation because of nonlinearities not included in (16). In this case the trajectory will move away and continue to move chaotically as if there was no control. Eventually (due to ergodicity of the uncontrolled attractor) the trajectory will fall near enough to the desired equilibrium point that attraction to it is obtained.

The OGY method combined with Pyragas takes from OGY the philosophy and the discrete framework and from the first method of Pyragas the technique based on state feedback. Considering the system modeled by the discrete map given by (3) and applying a proportional state feedback will force the eigenvalue to be in the interior of the unit circle.

4 Results

Using the current I as controlled measure, by simulating, (1-2) the results are shown in figure 4

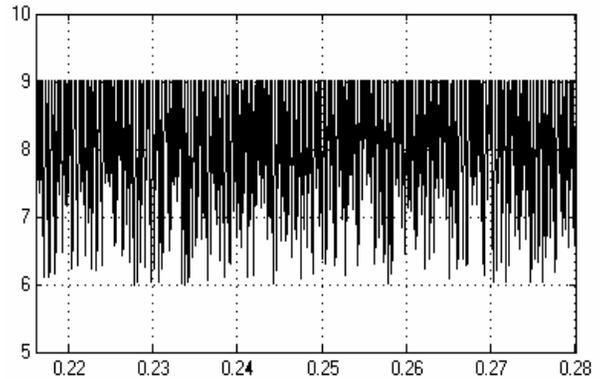
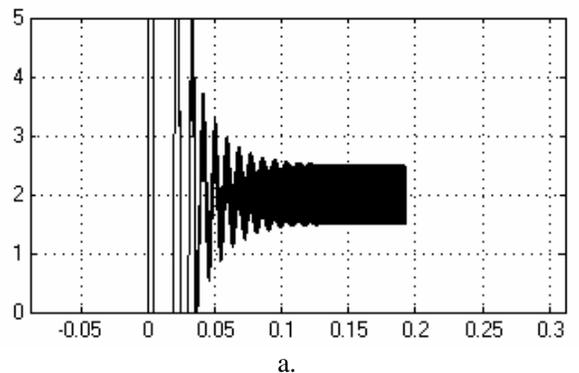
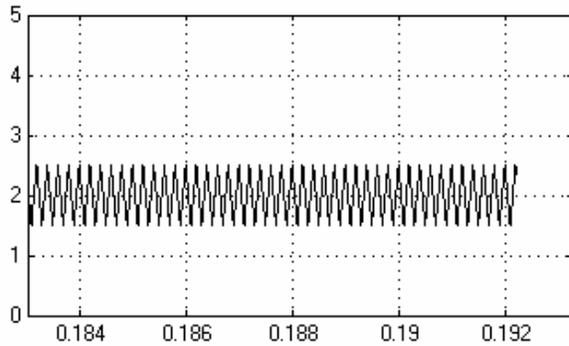


Fig. 4 The variation of I before control



a.



b.

Fig. 5 I variation after OGY control method(b is detailed for a)

The figure 5 shows the periodic variation after the OGY method was applied and figure 6 shows just the limitation of the current I after OGY combined with Pyragas method was applied.

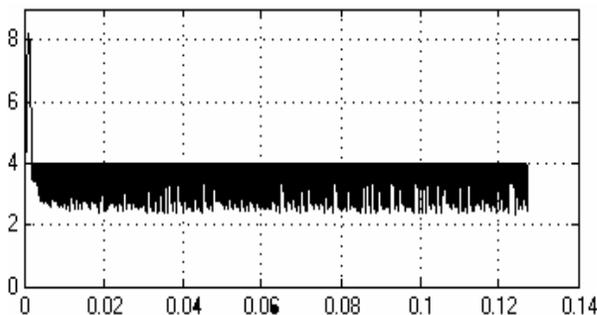


Fig. 6. I variation after OGY combined with Pyragas control method was applied

5 Conclusion

Two control strategies are proposed in order to avoid the unstable chaotic regimes of the behavior of a current mode controller boost converter and to ensure the stable periodic operation required by applications.

The first one, the OGY method, transform the attractor in a periodic evolution as shown in figures 3 b . The second one, combined from OGY and Pyragas methods just changes the basin of attraction to a small one but does not stabilize the current to a periodic regime.

The both methods change the chaotic regime of the controlled boost converter, with better results for the first.

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