The Laminar Boundary Layer on a Continuously Moving Rough Plate Embedded in a Non- Darcian Porous Medium

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Abstract

The laminar boundary layer on a continously moving rough plate embedded in a non-Darcian porous medium is studied. The transformed non-linear differential equation describing the problem is solved numericaly using the shooting technique .The effects of permeability parameter (M) ,inertia coefficient parameter (N)and the roughness parameter (R_w) on the velocity are shown on graphs . Also the skin-friction coefficient are computed for values of M, N and R_w.

1.Introduction

A continuous moving plate through a porous medium has many applications such as geothermal reserviors and petroleum industries , and the aerodynamic extrusion of plastic sheets,etc.. Sakiadis (1961) was the first one to study the flow of fluid past a continuously moving plate.

The simultaneous effects of fluid inertia force and boundary viscous resistance upon flow and heat transfer in a constant porosity porous medium was analyzed by Vafai and Tien (1981). The boundary layer problems over a plate embedded in a non-Darcian porous medium were studied by many authors (see, e.g., Vafai and Tien (1982), Plumb and Huenefeld (1981), Yang and Shiang (1997), and Elbashbeshy and Bazid (2000)).In the previous studies, the plate was assumed to be smooth (i.e. no – slip condition).

Recent experiments and simulations show that the textbook assumption of "no-slip at the boundary" can fail greatly when wall are sufficiently smooth (Steve Granick, Yingxi Zhu and Hyunjung Lee (2003)). Consequently the flow past a rough plate is of as much practical interest (Schlichting 1960). A survey of literature reveals that it appears no attempt was made to study the boundary layer flow over a continuously moving rough plate embedded in a non-Darcian porous medium.

In this paper we investigate the effect of roughness parameter R_w, permeability parameter M and the inertia coefficient parameter N on the flow past a continuously moving plate embedded in a non- Darcian porous medium.

2. Formulation of the problem

Consider a steady, incompressible, two dimensional flow of a fluid past a continuously moving semi-infinite rough plate with constant velocity u_o immersed in fluid saturated porous medium. The origin is located at the spot through which the plate is drawn in the fluid medium. The x- axis is chosen along the plate and y-axis is taken normal to it. Upon treating the fluid saturated porous medium as a continuum, including the non-Darcian inertia effects, the boundary layer form of the governing equations can be written as (see Vafai and Tien 1981, 1982).

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\in \nu}{K}u-C\in^2 u^2 \qquad (2)$$

where u,v are the velocity components in the x and y directions, respectively. ν is the kinematic viscosity, C is the transport property related to the inertia effect, $\in$ and K are the porosity and permeability of the porous medium, respectively.

3.Boundary conditions

As for the surface roughness of the plate, we shall assume the relative tangential velocity at the wall to be proprotional to the gradient of the velocity u at the wall. The condition to be considered is

$$u_s = u_o + R\frac{\partial u}{\partial y} \tag{3}$$

at the plate (y=o).

The propotionality factor R is approximately the hight of the roughness element, and is taken to be of the same order of magnitude as the boundary layer thickness δ. In other words from the model assumed is equation (3), we can write

$$\frac{u_s}{u_o} \cong 1 + \frac{R}{\delta}$$

In our calculations we shall take $R \cong 0(\frac{\delta}{10})$ which seems to be practically reasonable even for extremely smooth surfaces.

The boundary conditions for our problem are then given by

$$\begin{aligned} y = 0 \quad &: u = u_o + R\frac{\partial u}{\partial y} \quad , \quad v = 0 \\ y \to \infty &: \quad u = 0 \end{aligned} \tag{4}$$

We now introduce the following transformation:

$$\eta = \sqrt{\frac{u_o}{2\nu\kappa}}\; y \quad , \quad \psi = \sqrt{2\nu u_o x}\; f(\eta) \tag{5}$$

where the stream function $\psi(x,y)$ satisfies the continuity equation (1). Next , by introducing equation (5) into equations (1)-(3), one arrives at the following equation

$$f^{///} + f\, f^{//} - (1/M)\, f^{/} - N\, f^{/2} = o \tag{6}$$

where

$M = \frac{K u_o}{2 \in \nu x}$ (permeability parameter)

$N = 2C \in {}^2 x$ (inertia coefficient parameter)

The boundary conditions, equation (4), are transformed as follows:

$$y = 0 \ : \quad f^{/} = 1 + R_w f^{//} \ , \quad f = o$$

(7)

$$y \to \infty : \ f^{/} = o$$

where $R_w = \frac{R(x)}{u_o}\sqrt{\frac{uo}{2\nu x}}$ (roughness parameter) is constant, (hence $R(x) \approx x^{-1/2}$ for similarity solution).

4. Numerical results and discussion

The transformed nonlinear ordinary differential equation (6), with the corresponding boundary conditions (7) are solved numerically by means of the fourth- order Runge- Kutta method.

The local wall shear stress τ_w is of great interest in technological application. The definition of this shear stress is always taken as

$$\tau_w = -\mu(\frac{\partial u}{\partial y})_{y=o} \qquad (8)$$

At the plate the coefficient of skin- friction C_f is given by

$$C_f = \frac{\tau_w}{{}^1/_2\, \rho u_o{}^2} = -\sqrt{2} R_{ex}{}^{-1/2} f^{//}(o) \qquad (9)$$

where $R_{ex} = \frac{\rho u_o x}{\mu}$

Figure 1 illustrates the effect of the permeability parameter M on the distribution of velocity within the boundary layer. From this figure we see that the velocity decreases as M increases.

The effects of the inertia coefficient parameter N on the velocity distributions is displayed in figure 2. From this figure we note that the velocity decreases with the increase in N.

In figure 3 we have plotted the velocity distributions in the boundary layer showing the effect of R_w. It can be seen that the velocity distributions decrease as the roughness parameter R_w increase.

From the numerical results obtained in table 1, it is observed that:

(1) C_f increases as the permeability parameter M increases while N and R_w are kept constants.

(2) C_f increases with the increasing of the inertia coefficient parameter N while M and R_w are kept constants.

(3) C_f decreases as the roughness parameter R_w increases while N and M are kept constants.

Table (1): Numerical values of $-f^{//}(o)$

M	N	R_w	$-f^{//}(o)$
0.5	0.1	0.05	0.908843
0.5	0.1	0.1	0.861478
0.5	0.1	0.15	0.819313
0.5	0.1	0.2	0.781487
1	0.2	0.1	1.06787
1	0.4	0.1	1.10478
1	0.6	0.1	1.13993
0.5	0.2	0.1	0.885986
1	0.2	0.1	1.06787
1.5	0.2	0.1	1.2167

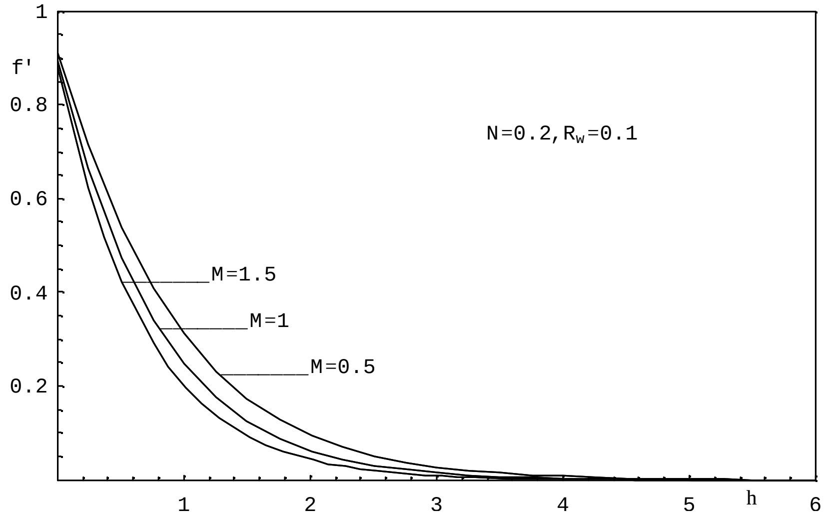

Figure 1. Velocity distribution for different values of M with N = 0.2 and R_w = 0.1

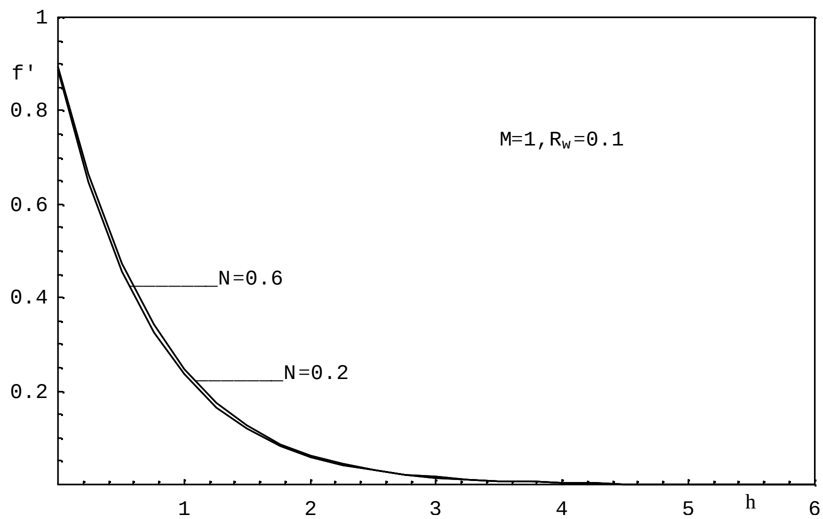

Figure 2. Velocity distribution for different values of N with M = 1 and R_w = 0.1

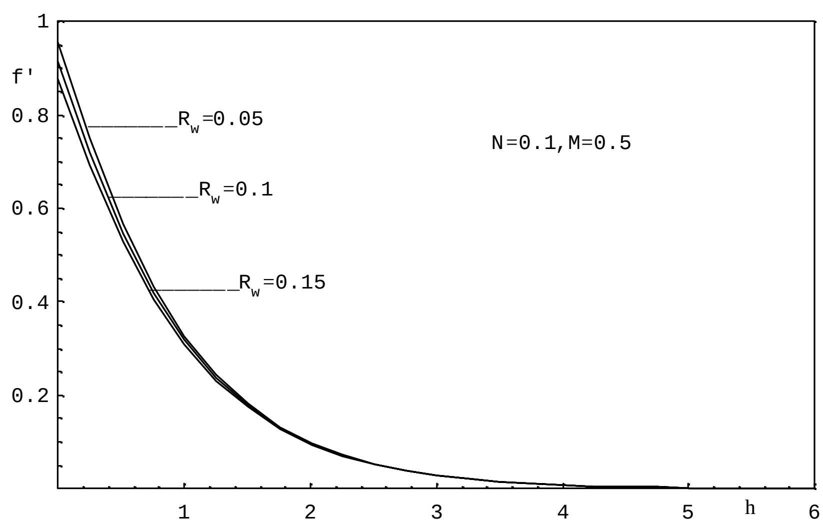

Figure 3. Velocity distribution for different values of R_w with M = 0.5 and N = 0.1

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