Influence of averaging coefficients on weakly nonlinear stability of shallow water flows

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Abstract: - Stability of shallow flows is analyzed in the present paper. Momentum correction coefficients are introduced in the shallow water equations in order to take into account non-uniformity of the velocity distribution in the vertical direction. Linear stability of parallel base flow is governed by the modified Rayleigh equation. Methods of weakly nonlinear theory are used in order to derive the amplitude evolution equation for the most unstable mode. It is shown that the evolution equation is the complex Ginzburg-Landau equation.

Key-Words: - averaging coefficients, shallow flows, weakly nonlinear analysis, Ginzburg-Landau equation.

1 Introduction

Depth-averaged equations (the Saint-Venant equations) are often used to model large-scale turbulent motions in shallow water [1]. These equations are used when the transverse length scale of the flow is much larger than water depth. Shallow water equations have been recently used for linear stability analyses of transverse turbulent motions in shallow waters [2]-[8]. Experimental and theoretical analyses in [2]-[8] show that the development of instability in shallow water is different from the case of deep water. Bottom friction in shallow flows acts as a suppression mechanism of the transverse growth of perturbations. In addition, development of three-dimensional instabilities is prevented due to limited water depth.

One of the main assumptions in shallow water theory is the independence of the flow characteristics from the vertical coordinate since shallow water equations are depth-averaged equations. There are many cases, however, where this assumption is not valid. Changes in flow geometry, flow regimes or roughness of the bottom boundary can lead to large deviations from the above assumptions [9]. Averaging coefficients (momentum and pressure corrections coefficients) are introduced in [10]-[11] in order to take into account the non-uniformity of the velocity distribution in the vertical direction.

The linear stability theory can only predict when a particular flow becomes unstable. In particular, the critical values of the stability parameters (critical Reynolds number for viscous flows or critical bed friction number for shallow flows) can be calculated by means of the linear stability theory. However, the evolution of the unstable disturbance above the threshold cannot be predicted by the linear theory. Weakly nonlinear theories [12]-[13] are used to take into account the effect of nonlinearities analytically in the unstable region where the parameters are very close to the critical values. As a result, an amplitude evolution equation for the most unstable mode is derived. In particular, the methods of weakly nonlinear theory are used in [8] to derive the complex Ginzburg-Landau equation which can be used to describe the dynamics of shallow flows behind obstacles (such as islands) above the threshold.

Previous studies [14] indicated that the stability characteristics of shallow flows are quite sensitive to the relative magnitude of the averaging coefficients. In particular, it is shown that the averaging coefficients have significant impact on the stability domains of transverse flows in compound channels.

The present paper is devoted to weakly nonlinear stability analysis of shallow flows where the averaging coefficients are taken into account. The amplitude evolution equation is derived under the assumption that the bed friction number is slightly below the critical value. It is shown that the resulting equation has complex coefficients and is of Ginzburg-Landau type. Thus, the Ginzburg-Landau model may be used to analyze the dynamics of shallow flows in a weakly nonlinear regime.

2 Problem Formulation

The governing equations under the rigid-lid assumption are [10]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\begin{aligned}&\frac{\partial u}{\partial t}+(2\beta_1-1)u\frac{\partial u}{\partial x}+(\beta_2-1)u\frac{\partial v}{\partial y}+\beta_2 v\frac{\partial u}{\partial y}\\ &=-\frac{\partial p}{\partial x}-\frac{c_f}{2h}u\sqrt{u^2+v^2},\end{aligned} \tag{2}$$

$$\begin{aligned}&\frac{\partial v}{\partial t}+(\beta_2-1)v\frac{\partial u}{\partial x}+\beta_2 u\frac{\partial v}{\partial x}+(2\beta_3-1)v\frac{\partial v}{\partial y}\\ &=-\frac{\partial p}{\partial y}-\frac{c_f}{2h}v\sqrt{u^2+v^2},\end{aligned} \tag{3}$$

where x and y are the spatial coordinates, t is the time, u and v are the depth-averaged velocity components in the x and y directions, respectively, p is the pressure, h is water depth, c_f is the friction coefficient, β_1, β_2 and β_3 are the momentum correction coefficients. Note that the bottom shear stress is modeled by means of the Chezy formula. It is also assumed here that the coefficients β_1, β_2 and β_3 are independent on the spatial coordinates x and y.

Introducing the stream function $\psi(x,y,t)$ by the relations

$$u=\psi_y,\ v=-\psi_x$$

and eliminating the pressure p we rewrite (1)-(3) in the form

$$\begin{aligned}&(\Delta\psi)_t+(2\beta_1-\beta_2)(\psi_y\psi_{xy})_y-\beta_2(\psi_x\psi_{yy})_y\\ &+(\beta_2-1)(\psi_x\psi_{xy})_x+\beta_2(\psi_{xx}\psi_y)_x\\ &-(2\beta_3-1)(\psi_x\psi_{xx})_y+\frac{c_f}{2h}\Delta\psi\sqrt{\psi_x^2+\psi_y^2}\\ &+\frac{c_f}{2h\sqrt{\psi_x^2+\psi_y^2}}\left(\psi_y^2\psi_{yy}+2\psi_x\psi_y\psi_{xy}+\psi_x^2\psi_{xx}\right)\\ &=0,\end{aligned} \tag{4}$$

where Δ is the Laplacian in two dimensions and the subscripts indicate the derivatives with respect to the variables x and y. Methods of weakly nonlinear theory are used in the next section to derive an amplitude evolution equation for the most unstable mode.

3 Derivation of the Ginzburg-Landau equation

Suppose that the base flow

$$\vec{U}=(U(y),0) \tag{5}$$

is perturbed, and the perturbed solution to (4) is assumed to be of the form

$$\psi=\psi_0+\varepsilon\psi_1+\varepsilon^2\psi_2+\varepsilon^3\psi_3+\ldots \tag{6}$$

where ε is a small parameter and $\psi_{0y}=U$. Substituting (5) and (6) into (4) and linearizing the resulting equation in the neighborhood of the base flow (5) we obtain

$$L\psi_1=0 \tag{7}$$

where L is the following linear operator:

$$\begin{aligned}L\varphi&=\varphi_{xxt}+\varphi_{yyt}+(2\beta_1-\beta_2)(U_y\varphi_{xy}+U\varphi_{xyy})\\ &-\beta_2(U_y\varphi_{xy}+U_{yy}\varphi_x)+\beta_2 U\varphi_{xxx}\\ &+\frac{c_f}{2h}\left(U\varphi_{xx}+2U_y\varphi_y+2U\varphi_{yy}\right)\end{aligned} \tag{8}$$

The perturbed component of the stream function is sought in the form

$$\psi_1(x,y,t)=\varphi_1(y)\exp[ik(x-ct)]+c.c. \tag{9}$$

where c.c means “complex conjugate”. Substituting (9) into (7) we obtain the linearized stability equation (the modified Rayleigh equation) in the form

$$\begin{aligned}&\varphi_1''\left[(2\beta_1-\beta_2)U-c+\frac{c_f}{ikh}U\right]\\ &+U_y\left(2\beta_1-2\beta_2+\frac{c_f}{ikh}\right)\varphi_1'\\ &+\left(k^2c-\beta_2U_{yy}-k^2\beta_2U-\frac{c_f}{2ih}kU\right)\varphi_1=0\end{aligned} \tag{10}$$

The boundary conditions are

$$\varphi_1(\pm\infty)=0. \tag{11}$$

Problem (10)-(11) is an eigenvalue problem. The eigenvalues, $c_m=c_{rm}+ic_{im}$, $m=1,2,\ldots$, determine the linear stability of the base flow (5). This base flow is said to be linearly stable if $c_{im}<0$ for all m and linearly unstable if $c_{im}>0$ at least for one value of m. Numerical methods [6] can be used to find the critical values of the parameter c_f which is defined as follows. The set of all points in the (k,c_f)-plane for which one eigenvalue satisfies the condition $c_i=0$ while all other eigenvalues have negative imaginary parts defines the neutral stability curve, $c_f^{(n)}(k)$. The critical value, $c_f^{(c)}$, of the parameter c_f is defined as follows:

$$c_f^{(c)} = \max_k c_f^{(n)}(k) \tag{12}$$

Stewartson and Stuart [12] developed a weakly nonlinear theory for stability of plane Poiseuille flow where the effect of nonlinearities can be taken into account analytically. The main idea behind the weakly nonlinear theory is the following. Consider the point in the parameter space where $k = k_c$, $c = c_c$ and $c_f = c_f^{(c)}$. Here the subscripts and the superscript c indicates the critical values of the parameters. Suppose that the stability parameter c_f is slightly below the critical value $c_f^{(c)}$, namely,

$$c_f = c_f^{(c)}(1-\varepsilon^2) \tag{13}$$

In accordance with the linear stability theory the most unstable mode is given by (9) where the function $\varphi_1(y)$ is the eigenfunction of the linear stability problem (10)-(11) calculated at $k = k_c$, $c = c_c$ and $c_f = c_f^{(c)}$. Since the problem is linear, $\varphi_1(y)$ can be replaced by $C\varphi_1(y)$, where C is an arbitrary constant which cannot be determined from the linear stability theory. Following Stewartson and Stuart [12] we restrict ourselves to the conditions around the critical point in order to study the nonlinear evolution of the most unstable mode. The constant C is replaced by a slowly varying function of the spatial coordinate and time. Thus, we introduce slow time τ and stretched longitudinal coordinate ξ which moves with a group velocity c_g:

$$\tau = \varepsilon^2 t, \quad \xi = \varepsilon(x - c_g t).$$

It follows from the chain rule that

$$\frac{\partial}{\partial t}\psi[x,t,\xi(x,t),\tau(t)] = \psi_t + \psi_\xi \xi_t + \psi_\tau \tau_t$$

$$= \psi_t - \varepsilon c_g \psi_\xi + \varepsilon^2 \psi_\tau$$

and

$$\frac{\partial}{\partial x}\psi[x,t,\xi(x,t),\tau(t)] = \psi_x + \psi_\xi \xi_x$$

$$= \psi_x + \varepsilon \psi_\xi$$

Thus, the differential operators $\partial/\partial t$ and $\partial/\partial x$ are replaced by

$$\frac{\partial}{\partial t} \to \frac{\partial}{\partial t} - \varepsilon c_g \frac{\partial}{\partial \xi} + \varepsilon^2 \frac{\partial}{\partial \tau} \tag{14}$$

and

$$\frac{\partial}{\partial x} \to \frac{\partial}{\partial x} + \varepsilon \frac{\partial}{\partial \xi} \tag{15}$$

The function ψ_1 in (9) is sought in the form

$$\psi_1 = A(\xi,\tau)\varphi_1(y)\exp[ik_c(x-ct)] + c.c \tag{16}$$

where c is the wave speed at $k = k_c$, $c_f = c_f^{(c)}$ and A is a slowly varying amplitude.

In order to find the equation which governs the evolution of the most unstable mode we consider higher terms of the perturbation expansion (6). Substituting (6) and (14)-(16) into (4) and collecting the terms of order ε, we recover equation (7), where the operator L is defined by (8). Collecting the terms of order ε^2 gives

$$\begin{aligned} L\psi_2 &= c_g(\psi_{1xx\xi} + \psi_{1yy\xi}) - 2\psi_{1x\xi t} \\ &- 2(\beta_1 - \beta_2)(U_y\psi_{1\xi y} + \psi_{1yy}\psi_{1xy} + U\psi_{1yy\xi} \\ &+ \psi_{1y}\psi_{1xyy}) + \beta_2(\psi_{1xy}\psi_{1yy} + U_y\psi_{1\xi y} + \psi_{1x}\psi_{1yyy} \\ &+ U_{yy}\psi_{1\xi}) - (\beta_2 - 1)(\psi_{1xx}\psi_{1xy} + \psi_{1x}\psi_{1xxy}) \\ &- \beta_2(\psi_{1y}\psi_{1xxx} + 3U\psi_{1xx\xi} + \psi_{1xy}\psi_{1xx}) \\ &+ (2\beta_3 - 1)(\psi_{1xx}\psi_{1xy} + \psi_{1x}\psi_{1xxy}) - \frac{c_f}{2h}(\psi_{1xx}\psi_{1y} \\ &+ 2U\psi_{1x\xi} + 2\psi_{1yy}\psi_{1y} - 2UU_y + 2\psi_{1x}\psi_{1xy}) \end{aligned} \tag{17}$$

Collecting the terms of order ε^3 we obtain

$$L\psi_3 = c_g(\psi_{2xx\xi} + \psi_{2yy\xi} + 2\psi_{1x\xi\xi}) - \psi_{1xx\tau}$$
$$- \psi_{1yy\tau} - 2\psi_{2x\xi t} - \psi_{1\xi\xi t} - (2\beta_1 - \beta_2)(U_y\psi_{2\xi y}$$
$$+ \psi_{1yy}\psi_{2xy} + \psi_{1yy}\psi_{1\xi y} + \psi_{2yy}\psi_{1xy} + U\psi_{2yy\xi}$$
$$+ \psi_{1y}\psi_{2xyy} + \psi_{1y}\psi_{1yy\xi} + \psi_{2y}\psi_{1xyy})$$
$$+ \beta_2(\psi_{1xy}\psi_{2yy} + \psi_{2xy}\psi_{1yy} + \psi_{1\xi y}\psi_{1yy} + U_y\psi_{2\xi y}$$
$$+ \psi_{1x}\psi_{2yyy} + \psi_{2x}\psi_{1yyy} + \psi_{1\xi}\psi_{1yyy} + U_{yy}\psi_{2\xi})$$
$$- (\beta_2 - 1)(\psi_{1xx}\psi_{2xy} + \psi_{1xx}\psi_{1\xi y} + \psi_{2xx}\psi_{1xy}$$
$$+ 2\psi_{1x\xi}\psi_{1xy} + \psi_{1x}\psi_{2xxy} + 2\psi_{1x}\psi_{1xy\xi} + \psi_{2x}\psi_{1xxy}$$
$$+ \psi_{1\xi}\psi_{1xxy} + \psi_{2\xi}\psi_{1xxy}) - \beta_2(\psi_{2y}\psi_{1xxx}$$
$$+ \psi_{1y}\psi_{2xxx} + 3\psi_{1y}\psi_{1xx\xi} + 3U\psi_{2xx\xi} + 3U\psi_{1x\xi\xi}$$
$$+ \psi_{1xx}\psi_{2xy} + \psi_{1xx}\psi_{1\xi y} + \psi_{2xx}\psi_{1xy} + 2\psi_{1x\xi}\psi_{1xy})$$
$$+ (2\beta_3 - 1)(\psi_{1xx}\psi_{2xy} + \psi_{1xx}\psi_{1\xi y} + \psi_{2xx}\psi_{1xy}$$
$$+ 2\psi_{1x\xi}\psi_{1xy} + \psi_{1x}\psi_{2xxy} + 2\psi_{1x}\psi_{1xy\xi} + \psi_{2x}\psi_{1xxy}$$
$$+ \psi_{1\xi}\psi_{1xxy} + \psi_{2\xi}\psi_{1xxy}) - \frac{c_f}{2h}B, \tag{18}$$

where

$$B = \psi_{1xx}\psi_{2y} + \frac{\psi_{1xx}\psi_{1x}^2}{2U} + \psi_{2xx}\psi_{1y} + 2\psi_{1x\xi}\psi_{1y}$$
$$+ 2U\psi_{2x\xi} + U\psi_{1\xi\xi} - \frac{U_y\psi_{1y}\psi_{1x}^2}{2U^2} + \frac{U_y\psi_{1x}\psi_{2x}}{U}$$
$$+ \frac{U_y\psi_{1x}\psi_{1\xi}}{U} + \psi_{1yy}\psi_{2y} + \psi_{1yy}\frac{\psi_{1x}^2}{2U} + \psi_{2yy}\psi_{1y}$$
$$- U\psi_{1xx} - U_y\psi_{1y} - U\psi_{1yy} + \frac{U_y\psi_{1y}\psi_{1x}^2}{2U^2}$$
$$- \frac{U_y\psi_{1x}\psi_{2x}}{U} - \frac{U_y\psi_{1x}\psi_{1\xi}}{U} + \psi_{1yy}\psi_{2y} - \frac{\psi_{1yy}\psi_{1x}^2}{2U}$$
$$+ \psi_{1y}\psi_{2yy} + 2\psi_{1x}\psi_{2xy} + 2\psi_{1x}\psi_{1\xi y} + 2\psi_{2x}\psi_{1xy}$$
$$+ 2\psi_{1\xi}\psi_{1xy} + \frac{\psi_{1x}^2\psi_{1xx}}{U} - U_y\psi_{1y} - U\psi_{1yy}. \tag{19}$$

The form of the right-hand side of (17) and (18) suggests that the function ψ_2 should be sought in the form

$$\psi_2 = AA^*\varphi_2^{(0)}(y) + A_\xi\varphi_2^{(1)}(y)\exp[ik(x-ct)]$$
$$+ A^2\varphi_2^{(2)}(y)\exp[ik(x-ct)] + c.c. \tag{20}$$

Substituting (16) for ψ_1 and (20) for ψ_2 into (17) and collecting the terms proportional to AA^* we obtain

$$\frac{2c_f}{h}\left(U\varphi_{2yy}^{(0)} + U_y\varphi_{2y}^{(0)}\right) = ik\beta_2(\varphi_{1y}\varphi_{1yy}^* - \varphi_{1y}^*\varphi_{1yy}$$
$$+ \varphi_1\varphi_{1yyy}^* - \varphi_1^*\varphi_{1yyy}) - \frac{c_f}{2h}(k^2\varphi_1\varphi_{1y}^* + k^2\varphi_1^*\varphi_{1y}$$
$$+ 2\varphi_{1yy}\varphi_{1y}^* + 2\varphi_{1yy}^*\varphi_{1y}) \tag{21}$$

with the boundary conditions

$$\varphi_2^{(0)}(\pm\infty) = 0. \tag{22}$$

Collecting the terms proportional to $A_\xi\exp[ik(x-ct)]$ gives

$$ik^3c\varphi_2^{(1)} - ikc\varphi_{2yy}^{(1)} + (2\beta_1 - \beta_2)ik(U_y\varphi_{2y}^{(1)}$$
$$+ U\varphi_{2yy}^{(1)}) - ik\beta_2(U_y\varphi_{2y}^{(1)} + U_{yy}\varphi_2^{(1)} + k^2U\varphi_2^{(1)})$$
$$+ \frac{c_f}{2h}(-k^2U\varphi_2^{(1)} + 2U_y\varphi_{2y}^{(1)} + 2U\varphi_{2yy}^{(1)}) = -2k^2c\varphi_1$$
$$- k^2c_g\varphi_1 + c_g\varphi_{1yy} - (2\beta_1 - \beta_2)(U_y\varphi_{1y} + U\varphi_{1yy})$$
$$+ \beta_2(U_y\varphi_{1y} + U_{yy}\varphi_1) + 3\beta_2k^2U\varphi_1 - ikc_fU\varphi_1/h \tag{23}$$

with the boundary conditions

$$\varphi_2^{(1)}(\pm\infty) = 0. \tag{24}$$

Finally, collecting the terms proportional to $A^2\exp[2ik(x-ct)]$ yields

$$8ik^3c\varphi_2^{(2)} - 2ikc\varphi_{2yy}^{(2)} + (2\beta_1 - \beta_2)(2ikU_y\varphi_{2y}^{(2)}$$
$$+ 2ikU\varphi_{2yy}^{(2)}) - \beta_2(2ikU_y\varphi_{2y}^{(2)} + 2ikU_{yy}\varphi_2^{(2)}$$
$$+ 8ik^3U\varphi_2^{(2)}) + \frac{c_f}{h}(-2k^2U\varphi_2^{(2)} + U_y\varphi_{2y}^{(2)}$$
$$+ U\varphi_{2yy}^{(2)}) = -4ik(\beta_1 - \beta_2)\varphi_{1y}\varphi_{1yy}$$
$$+ ik\beta_2(\varphi_{1y}\varphi_{1yy} + \varphi_1\varphi_{1yyy})$$
$$- 2ik^3(\beta_2 - 1)\varphi_1\varphi_{1y} + 2ik^3\beta_2\varphi_1\varphi_{1y}$$
$$- 2ik^3(2\beta_3 - 1)\varphi_1\varphi_{1y} - \frac{c_f}{2h}(2\varphi_{1y}\varphi_{1yy}$$
$$- 3k^2\varphi_1\varphi_{1y}) \tag{25}$$

with the boundary conditions

$$\varphi_2^{(2)}(\pm\infty) = 0. \tag{26}$$

Note that equation (23) is resonantly forced since the corresponding homogeneous equation has a nontrivial solution if $k = k_c$, $c = c_c$ and $c_f = c_f^{(c)}$. Thus, in accordance with the Fredholm's alternative [15] equation (23) has a solution if and only if its right-hand side is orthogonal to all eigenfunctions of the corresponding homogeneous adjoint problem.

We define the adjoint operator, L^a, and adjoint eigenfucntion φ_1^a of L^a as follows:

$$\int_{-\infty}^{+\infty} \varphi_1^a L(\varphi_1) dy = \int_{-\infty}^{+\infty} \varphi_1 L^a(\varphi_1^a) dy \qquad (27)$$

Since φ_1 is an eigenfunction of the problem (23), (24), then the left-hand side of (27) is equal to zero. This also means that

$$L^a \varphi_1^a = 0 \qquad (28)$$

The boundary conditions are

$$\varphi_1^a(\pm\infty) = 0 \qquad (29)$$

The explicit form of the adjoint operator is obtained by integrating equation (10) by parts and using boundary conditions (11):

$$L^a \varphi_1^a \equiv \left(\varphi_1^a\right)'' \left[(2\beta_1 - \beta_2)U - c + \frac{c_f}{ikh} \right]$$
$$+ \left(\varphi_1^a\right)' \begin{bmatrix} 2U_y \left(2\beta_1 - \beta_2 + \dfrac{c_f}{ikh} \right) - (2\beta_1 - \beta_2)U \\ + c - \dfrac{c_f}{ikh} U \end{bmatrix}$$
$$+ \varphi_1^a \begin{bmatrix} -U_y \left(2\beta_1 - \beta_2 + \dfrac{c_f}{ikh} \right) + k^2 c - \beta_2 U_{yy} \\ -k^2 \beta_2 U - \dfrac{c_f}{2ikh} U \end{bmatrix}$$

Note that the eigenvalues of problems (10), (11) and (28), (29) coincide.

The solvability condition for equation (23) has the form

$$\int_{-\infty}^{+\infty} \varphi_1^a \begin{bmatrix} -k^2 c_g \varphi_1 + c_g \varphi_{1yy} - 2k^2 c \varphi_1 \\ -(2\beta_1 - \beta_2)(U_y \varphi_{1y} + U \varphi_{1yy}) \\ + \beta_2 (U_y \varphi_{1y} + U_{yy} \varphi_1) + 3\beta_2 k^2 U \varphi_1 \\ - ikc_f U \varphi_1 / h \end{bmatrix} dy = 0 \qquad (30)$$

Equation (30) determines the group velocity c_g.

The evolution equation for A is obtained from the solvability condition for equation (18). It is clear that (18) has a solution if and only if the right-hand side of (18) is orthogonal to all eigenfunctions φ_1^a of the adjoint problem (28), (29). It can be shown that the solvability condition for equation (18) is

$$A_\tau = \sigma A + \delta A_{\xi\xi} + \mu |A|^2 A \qquad (31)$$

where the complex coefficients σ, δ and μ of (31) are expressed in terms of integrals involving the functions φ_1, φ_1^a, $\varphi_2^{(0)}$, $\varphi_2^{(1)}$, $\varphi_2^{(2)}$ and their derivatives with respect to y and are not presented here. Equation (31) is the Ginzburg-Landau equation. In order to evaluate the coefficients of the Ginzburg-Landau equation one needs to calculate the critical values of the parameters k, c and c_f from the solution of the linear stability problem (10), (11), find the eigenfunction φ_1 of the problem (10), (11), calculate the eigenfunction φ_1^a of the adjoint problem (28), (29), solve three linear boundary value problems (21)-(26) and evaluate the integrals.

4 Conclusion

The complex Ginzburg-Landau equation for the evolution of the most unstable mode is derived in the present paper. The governing equations are the shallow water equations where the averaging coefficients are used in order to take into account non-uniformity of the velocity distribution in the vertical direction. The complex Ginzburg-Landau equation is much simpler than the original nonlinear shallow water equations. On the other hand, it is known that depending on the values of the coefficients the complex Ginzburg-Landau equation possesses a rich variety of solutions. This property of the equation stimulated researchers' interest to the Ginzburg-Landau model. In fact, the Ginzburg-Landau equation is used in the literature in two ways: first, as a phenomenological model (that is, as a model equation where the coefficients are determined from experimental data) and as an equation, which can be derived from the equations of hydrodynamics by means of the methods of weakly nonlinear theory. It is shown in the present paper that the complex Ginzburg-Landau equation does not have to be assumed. It is derived from the shallow water equations which contain the averaging coefficients.

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