Hyperbolic temperature in two adjacent bodies without perfect thermal contact

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Abstract:- We study, from the point of view of the hyperbolic model of heat conduction, the temperature profiles of two semi-infinite bodies that initially are at different temperatures T_0^1 and T_0^2, respectively, and are placed together in contact at time $t = 0$, supposing that the contact is not perfect and there exists thermal contact resistance.

Key-words: - Hyperbolic heat equation, contact with resistance.

1 Introduction

The known parabolic model of heat conduction implies the unacceptable physically assumption that the speed of heat conduction is infinite. Indeed, from the practical point of view, this equation produces serious erroneous results in many processes of the modern industry and technology where great amounts of heat are applied to materials in very short times (high-intensity electromagnetic radiation, film applications, laser surgery, for instance). This fact has promoted the need of a new heat conduction model called hyperbolic model of heat conduction that gives rise to the hyperbolic heat equation (see [4])

$$\frac{\partial T}{\partial t} + \tau \, \frac{\partial^2 T}{\partial t^2} = \alpha \, \Delta T \qquad (1)$$

where τ is an assumed constant material characteristic called thermal relaxation time and the thermal diffusivity of the medium $\alpha = \frac{k}{\rho c}$, the thermal conductivity k, the specific heat c and the density ρ of the material are assumed to be constant.

In [3] we have studied, from the point of view of hyperbolic model, the heat conduction problem of two bodies that initially are at temperatures T_0^1 and T_0^2, respectively, and at time $t = 0$ are placed together in contact, assuming a perfect thermal contact between bodies.

In this paper we study the same problem supposing that the contact is not perfect and a finite thermal contact resistance there exists. We find a complete analytical solution of the problem and we compare it with the solution obtained in the case of direct contact in [3] and with the analogous ones obtained previously by Xin and Tao in [5] under the viewpoint of parabolic model of heat conduction.

2 Analytical development

Consider two semi-infinite isotropic bodies with physical properties τ_i, ρ_i, c_i and k_i where subscript $i = 1, 2$ refers each body, that are initially held at different but uniform temperatures, T_0^1 and T_0^2, respectively. At $t = 0$ the bodies are placed together in contact, and each one experiences transient heat conduction process. We assume that there is a finite value of thermal resistance between the two contacting surfaces. We denote by h the contacting conductance, which is the reciprocal of the contacting thermal resistance. The governing equations for the temperatures of the two bodies are

$$\alpha_i \frac{\partial^2 T_i}{\partial x^2} = \tau_i \frac{\partial^2 T_i}{\partial t^2} + \frac{\partial T_i}{\partial t} \quad (i = 1, 2), \qquad (2)$$

and the initial and boundary conditions are

$$\forall x < 0 \;\; T_1(x, 0) = T_0^1 \qquad \frac{\partial T_1}{\partial t}(x, 0) = 0 \qquad (3)$$

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$$\forall x>0\;\; T_2(x,0)=T_0^2 \qquad \frac{\partial T_2}{\partial t}(x,0)=0 \tag{4}$$

$$\forall t>0 \quad q_1(0,t)=q_2(0,t) \tag{5}$$

$$\forall t>0 \quad h(T_1(0,t)-T_2(0,t))=q_1(0,t) \tag{6}$$

$$\forall t>0\;\; T_1(-\infty,t)=\lim_{x\to-\infty}T_1(x,t)=T_0^1 \tag{7}$$

$$\forall t>0 \quad T_2(\infty,t)=\lim_{x\to\infty}T_2(x,t)=T_0^2\,. \tag{8}$$

We begin the resolution of this problem taking Laplace transform of (2) with respect to t. Using (3) and (4) in the standard way we get

$$\widehat{T}_i(x,s)=A_i\ e^{\beta_i\ x}+B_i\ e^{-\beta_i\ x}+\frac{T_0^i}{s},\;\;\beta_i=\sqrt{\frac{s+\tau_i s^2}{\alpha_i}}$$

and from (7) and (8) we obtain $B_1=0$, $A_2=0$.

The expression relating heat flux and temperature in the hyperbolic model is (see [4])

$$q_i(0,t)=\frac{-k_i}{\tau_i}\ e^{\frac{-t}{\tau_i}}\int_0^t e^{\frac{\eta}{\tau_i}}\ \frac{\partial T_i}{\partial x}(0,\eta)\ d\eta\,. \tag{9}$$

Taking into account the Laplace transform of expression (9) and condition (5), and from the Laplace transform of condition (6) we can compute A_1 and B_2. Finally, we get the Laplace transform of the temperature profiles

$$\widehat{T}_1(x,s)=\frac{T_0^1}{s}+\frac{(T_0^2-T_0^1)\ e^{x\sqrt{\frac{s+\tau_1 s^2}{\alpha_1}}}}{s\left(1+\frac{k_1\sqrt{\alpha_2}\sqrt{\tau_2 s+1}}{k_2\sqrt{\alpha_1}\sqrt{\tau_1 s+1}}+\frac{k_1\sqrt{s}}{h\sqrt{\tau_1 s+1}\sqrt{\alpha_1}}\right)}$$

$$\widehat{T}_2(x,s)=\frac{T_0^2}{s}+\frac{(T_0^1-T_0^2)}{s\sqrt{\alpha_1}\ k_2\ \sqrt{\tau_1 s+1}}\times$$

$$\times\frac{k_1\sqrt{\alpha_2}\sqrt{\tau_2 s+1}\ \ e^{-x\sqrt{\frac{s+\tau_2 s^2}{\alpha_2}}}}{\left(1+\frac{k_1\sqrt{\alpha_2}\sqrt{\tau_2 s+1}}{k_2\sqrt{\alpha_1}\sqrt{\tau_1 s+1}}+\frac{k_1\sqrt{s}}{h\sqrt{\tau_1 s+1}\sqrt{\alpha_1}}\right)}.$$

The hard mathematical problem is now the Laplace inversion of these functions.

2.1 Temperature profile of body 2.

We start by calculating the inverse Laplace transform of $\widehat{T}_2(x,s)$. As general rule in the sequel, the inverse Laplace transform of a given function $f(s)$ will be denoted with the same capital letter $F(t)$. Put

$$g_2(s):=e^{-x\sqrt{\frac{s+\tau_2 s^2}{\alpha_2}}} \qquad g_1(s):=\frac{k_1\sqrt{\alpha_2}}{k_2\sqrt{\tau_1 s+1}}\times$$

$$\times\frac{\sqrt{\tau_2 s+1}}{s\sqrt{\alpha_1}\left(1+\frac{k_1\sqrt{\alpha_2}\sqrt{\tau_2 s+1}}{k_2\sqrt{\alpha_1}\sqrt{\tau_1 s+1}}+\frac{k_1\sqrt{s}}{h\sqrt{\tau_1 s+1}\sqrt{\alpha_1}}\right)}$$

in order to find $T_2(x,t)$ by application of convolution theorem to $G_1(t)$ and $G_2(t)$.

We start finding the more easy inverse of $g_2(s)$. The inverse of $\frac{g_2(s)}{s}$ has been calculated in [1]

$$\mathcal{L}^{-1}\left[\frac{g_2(s)}{s}\right](t)=H\left(t-\frac{x}{v_2}\right)\left(e^{-\frac{x}{2\sqrt{\alpha_2\tau_2}}}+\right.$$

$$\left.+\frac{x\ v_2}{4\alpha_2\tau_2}\int_{\frac{x}{v_2}}^t e^{-\frac{u}{2\tau_2}}\frac{I_1\left(\sqrt{(\frac{u}{2\tau_2})^2-\frac{x^2}{4\alpha_2\tau_2}}\right)}{\sqrt{(\frac{u}{2\tau_2})^2-\frac{x^2}{4\alpha_2\tau_2}}}\ du\right),$$

where $H(u)$ is the Heaviside function and $v_i:=\sqrt{\frac{\alpha_i}{\tau_i}}$, $i=1,2$ is the speed of thermal propagation in every body. Now, the application of the known relation (see page 507 in [2] for instance)

$$\mathcal{L}^{-1}[f(s)]=\frac{d}{dt}\mathcal{L}^{-1}\left[\frac{f(s)}{s}\right], \tag{10}$$

gives us

$$G_2(t):=\delta\left(t-\frac{x}{v_2}\right)\left(e^{-\frac{x}{2\sqrt{\alpha_2\tau_2}}}+\right.$$

$$\left.+\ \frac{v_2 x}{4\alpha_2\tau_2}\int_{\frac{x}{v_2}}^t e^{\frac{-u}{2\tau_2}}\frac{I_1\left(\sqrt{(\frac{u}{2\tau_2})^2-\frac{x^2}{4\alpha_2\tau_2}}\right)}{\sqrt{(\frac{u}{2\tau_2})^2-\frac{x^2}{4\alpha_2\tau_2}}}\ du\right)+$$

$$+H\left(t-\frac{x}{v_2}\right)\ \frac{v_2\ x}{4\alpha_2\tau_2}\ e^{\frac{-t}{2\tau_2}}\ \frac{I_1\left(\sqrt{(\frac{t}{2\tau_2})^2-\frac{x^2}{4\alpha_2\tau_2}}\right)}{\sqrt{(\frac{t}{2\tau_2})^2-\frac{x^2}{4\alpha_2\tau_2}}},$$

where $\delta(u)$ is the Dirac's δ distribution .

We shall assume in the sequel $\tau_1\le\tau_2$. To find the more involved Laplace inverse of $g_1(s)$ we shall use the convolution theorem again. To do this we put

$$p_1(s):=\frac{\sqrt{\tau_2}\sqrt{s+\frac{1}{\tau_2}}}{s},\;\;B:=\frac{k_2\sqrt{\alpha_1}}{k_1\sqrt{\alpha_2}},\;\;A:=\frac{k_2}{h\sqrt{\alpha_2}}$$

$$p_2(s):=\frac{1}{B\sqrt{\tau_1}\sqrt{s+\frac{1}{\tau_1}}+\sqrt{\tau_2}\sqrt{s+\frac{1}{\tau_2}}+A\sqrt{s}}\,.$$

We try to use Bromwich's inversion formula. To get a convergent integral in this formula in order

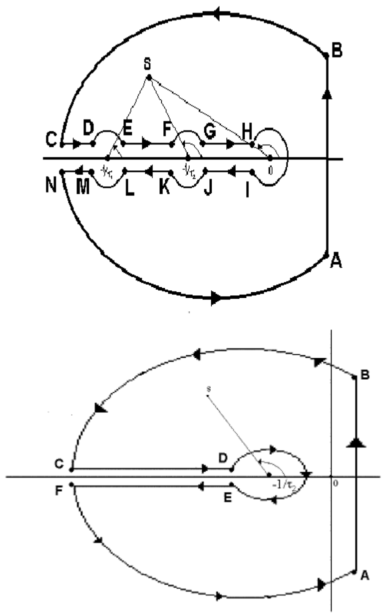

Figure 1: Bromwich's contours

to compute $\mathcal{L}^{-1}[p_1(s)]$ we have to consider $\frac{p_1(s)}{s}$. Applying Bromwich's formula

$$\mathcal{L}^{-1}\left[\frac{p_1(s)}{s}\right] = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{st}\,\frac{\sqrt{\tau_2}\sqrt{s+\frac{1}{\tau_2}}}{s^2}\,ds.$$

We have to deal with a branch point in $s = \frac{-1}{\tau_2}$ and a double pole in $s = 0$. Hence, we choose the down contour of figure 1 and by Bromwich's formula

$$\mathcal{L}^{-1}\left[\frac{p_1(s)}{s}\right] = t + \frac{\tau_2}{2} - \frac{1}{\pi}\int_{\infty}^{-\frac{1}{\tau_2}} e^{yt}\,\frac{\sqrt{\tau_2}\sqrt{-b}}{y^2}\,dy,$$

where $t + \frac{\tau_2}{2}$ is the residue in $s = 0$ and $b = y + \frac{1}{\tau_2}$. Once again according to (10)

$$P_1(t) = 1 - \frac{1}{\pi}\int_{-\infty}^{-\frac{1}{\tau_2}} e^{yt}\,\frac{\sqrt{\tau_2}\sqrt{-b}}{y}\,dy\ .$$

Analogously, to find the inverse of $p_2(s)$ by Bromwich's formula it is necessary to consider $\frac{p_2(s)}{s}$. In this case there are three branch points $s = -\frac{1}{\tau_2}$, $s = -\frac{1}{\tau_1}$ and $s = 0$. Hence, we choose the up contour of figure 1. According to the value of τ_1, τ_2, A and B some poles can exist. For the materials used in subsequent numerical computations the poles are in the half-plane $Re(z) > 0$. In this way, in order to simplify we always suppose that the physical properties of the problem do not cause any pole inside the contour. Applying Bromwich's formula we obtain $\mathcal{L}^{-1}\left[\frac{p_2(s)}{s}\right]$ and using (10) we get by differentiating

$$P_2(t) = \int_{-\infty}^{-\frac{1}{\tau_1}} \frac{e^{yt}\frac{1}{\pi}}{B\sqrt{\tau_1}\sqrt{-a}+\sqrt{\tau_2}\sqrt{-b}+A\sqrt{-y}}dy +$$

$$+\int_{-\frac{1}{\tau_1}}^{-\frac{1}{\tau_2}} \frac{e^{yt}\,\frac{1}{\pi}\left(\sqrt{\tau_2}\sqrt{-b}+A\sqrt{-y}\right)}{B^2\tau_1 a-\tau_2 b-A^2 y+2A\sqrt{\tau_2}\sqrt{-y}\sqrt{-b}}dy +$$

$$+\int_{-\frac{1}{\tau_2}}^{0} \frac{e^{yt}\frac{1}{\pi}\,A\sqrt{-y}}{B^2\tau_1 a+\tau_2 b+2B\sqrt{\tau_1\tau_2}\sqrt{a}\sqrt{b}-A^2 y}dy$$

where $a = y + \frac{1}{\tau_1}$. Finally, by convolution theorem we obtain $G_1(t) = \int_0^t P_2(u)\ P_1(t-u)\ du$ and

$$T_2(x,t) = T_0^2 + (T_0^1 - T_0^2)H\left(t - \frac{x}{v_2}\right)\times$$

$$\times\left[e^{-\frac{x}{2\sqrt{\alpha_2\tau_2}}}\ G_1\left(t-\frac{x}{v_2}\right)\frac{v_2\ x}{4\alpha_2\tau_2}\times\right.$$

$$\left.\times\int_{\frac{x}{v_2}}^{t} G_1(t-q)\ e^{\frac{-q}{2\tau_2}}\frac{I_1\left(\sqrt{(\frac{q}{2\tau_2})^2-\frac{x^2}{4\alpha_2\tau_2}}\right)}{\sqrt{(\frac{q}{2\tau_2})^2-\frac{x^2}{4\alpha_2\tau_2}}}dq\right].$$

2.2 Temperature profile of body 1.

Computations for body 1 are completely similar. Putting

$$Q_1(t) = 1 - \frac{1}{\pi}\int_{-\infty}^{-\frac{1}{\tau_1}} e^{y\,t}\,\frac{\sqrt{\tau_1}\sqrt{-a}}{y}\,dy$$

and

$$Q_2(t) = \int_{-\infty}^{-\frac{1}{\tau_1}} \frac{e^{y\,t}\,\frac{1}{\pi}}{\sqrt{\tau_1}\sqrt{-a}+D\sqrt{\tau_2}\sqrt{-b}+C\sqrt{-y}}dy +$$

$$+\int_{-\frac{1}{\tau_1}}^{-\frac{1}{\tau_2}} \frac{e^{y\,t}\,\frac{1}{\pi}\ \left(D\sqrt{\tau_2}\sqrt{-b}+C\sqrt{-y}\right)}{\tau_1 a-D^2\tau_2 b-C^2 y+2CD\sqrt{\tau_2}\sqrt{-y}\sqrt{-b}}dy +$$

$$+\int_{-\frac{1}{\tau_2}}^{0} \frac{e^{y\,t}\, C\sqrt{-y}}{\tau_1 a + D^2\tau_2 b + 2D\sqrt{\tau_1\tau_2}\sqrt{a}\sqrt{b} - C^2 y}\, dy$$

where $D = \frac{1}{B}$ and $C = \frac{k_1}{h\sqrt{\alpha_1}}$, we obtain by convolution $R_1(t) = \int_0^t Q_2(u)\, Q_1(t-u) du$ and

$$T_1(x,t) = T_0^1 + (T_0^2 - T_0^1) H\left(t + \frac{x}{v_1}\right) \times$$

$$\times \left[e^{\frac{x}{2\sqrt{\alpha_1\tau_1}}}\, R_1\left(t + \frac{x}{v_1}\right) - \frac{v_1\, x}{4\alpha_1\tau_1} \times \right.$$

$$\left. \times \int_{\frac{-x}{v_1}}^{t} R_1(t-q)\, e^{\frac{-q}{2\tau_1}} \frac{I_1\left(\sqrt{(\frac{q}{2\tau_1})^2 - \frac{x^2}{4\alpha_1\tau_1}}\right)}{\sqrt{(\frac{q}{2\tau_1})^2 - \frac{x^2}{4\alpha_1\tau_1}}} dq \right].$$

3 Solution analysis

To apply our theoretical results to some concrete problem we choose as materials uranium dioxide UO_2 for body 1 and liquid sodium Na for body 2 because they can give a good model to study an hypothetical accident condition in nuclear reactors (see [6]). The values of physical parameters used are taken from that paper: $\alpha_1 = 4.89\ 10^{-7}$, $\alpha_2 = 3.55\ 10^{-5}(\frac{m^2}{s})$, $k_1 = 0.5$, $k_2 = 9.15\ (\frac{cal}{m^\circ C})$, $T_0^1 = 3000$, $T_0^2 = 800\ (^\circ C)$, $\tau_1 = 1.69\ 10^{-13}$, $\tau_2 = 6.72\ 10^{-12}\ (s)$ and we assume $h = 10^9\ \frac{W}{m^{2\circ} C}$.

Perhaps the interface is the most interesting point to study the temperature distribution. We can compute easily the initial and limit temperature in left and right interfaces by using the initial and final value theorems of Laplace transforms. We get

$$\lim_{s\longrightarrow\infty} s\,\widehat{T}_2(0,s) = T_0^2 \qquad \lim_{s\longrightarrow\infty} s\,\widehat{T}_1(0,s) = T_0^1 \quad (11)$$

for the initial temperature in the interface and

$$\lim_{s\longrightarrow 0} s\,\widehat{T}_2(0,s) = \frac{T_0^1\ k_1\ \sqrt{\alpha_2} + T_0^2\ k_2\sqrt{\alpha_1}}{k_1\sqrt{\alpha_2} + k_2\sqrt{\alpha_1}} \quad (12)$$

$$\lim_{s\longrightarrow 0} s\,\widehat{T}_1(0,s) = \frac{T_0^1\ k_1\ \sqrt{\alpha_2} + T_0^2\ k_2\sqrt{\alpha_1}}{k_1\sqrt{\alpha_2} + k_2\sqrt{\alpha_1}} \quad (13)$$

for the limit temperature in the interface of bodies 2 and 1, respectively. According to (12) and (13) the equilibrium (limit) temperature is $1498^\circ C$ and is independent on the thermal resistance. The initial interface temperature is $3000^\circ C$ for body 1 and $800^\circ C$ for body 2 according to (11). We can see that the initial interface temperature is different according to the selected body to compute it. This fact is not surprising since equation (6) states that $T_1(0,t) \neq T_2(0,t)$ until the heat flux vanishes. On the other hand, under a physical viewpoint interface is a mixture of air and contact zones between the two bodies and the interface temperature evolution in each body is different. Figure 2 shows the temperature evolution at the interfaces of bodies 1 (up) and 2 (down) in the temporal interval $[0, 10^{-13}]$.

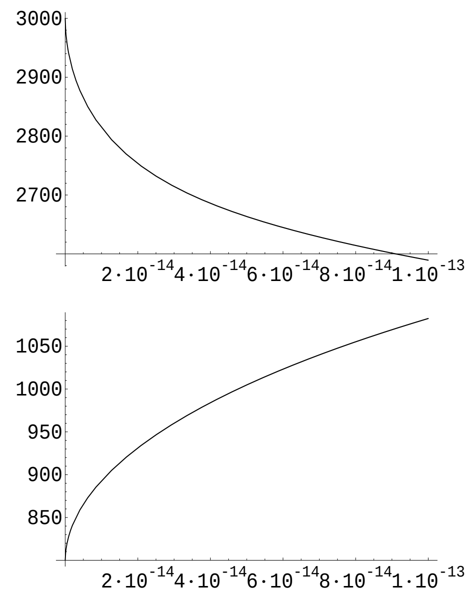

Figure 2: Hyperbolic interface temperature in body 1 (up) and 2 (down).

The presence of the Heaviside function in the obtained profiles of temperatures $T_1(x,t)$ and $T_2(x,t)$ provides them a wavy character because determines in every time t a zone in the body where the initial situation remains ($T_i(x,t) = T_0^i$) because the heat has not arrived and another zone where the temperature is different than the initial since the perturbation produced by the change of temperatures has arrived. Table 1 shows the temperature evolution of body 2 in a concrete point. We can see that in this point the change of temperatures is produced when $5\ 10^{-12}$ s have elapsed since the beginning of the heat conduction process.

x(m)	t(s)	Body 2 ($°C$)
10^{-8}	10^{-13}	800
10^{-8}	10^{-12}	800
10^{-8}	$5\ 10^{-12}$	1209
10^{-8}	10^{-11}	1301
10^{-8}	10^{-10}	1414
10^{-8}	10^{-8}	1489

Table 1: Temperature of body 2 in 10^{-8} m.

	Res.($°C$)		Dir. ($°C$)	
t(s)	$h = 10^9$	$h = 10^6$	$h = 10^9$	$h = 10^6$
10^{-5}	1537	1989	1539	1539
10^{-6}	1620	2052	1622	1622
10^{-7}	1876	2538	1879	1879
10^{-8}	2532	2937	2535	2535
10^{-9}	2998	3000	3000	3000
10^{-10}	3000	3000	3000	3000

Table 2: Temperature in $x = -10^{-7}$ m in different times for different h values in resistance and direct cases.

4 Differences with the case of perfect contact

In the case of perfect thermal contact, the solution of our problem has been obtained previously in [3]. The main difference between the two suppositions is found at the initial temperature values obtained at interface. In the case of direct contact the interface temperature varies from $2441°C$ until the equilibrium temperature $1498°C$. We obtain these values replacing $x = 0$ in one of the two temperature profiles (body 1 or 2), since in this case the interface is an "ideal" line ($T_1(0,t) = T_2(0,t)$). However, in the case of contact with resistance the interface is a complex region between two bodies and then, the temperature evolution of the two bodies at this point is different until the equilibrium temperature is reached.

In other points different from the interface the qualitative behavior of temperature is the same, but obviously in the case of direct contact, on a concrete point the equilibrium temperature is reached sooner than in the case of contact with resistance. Due to the fact that in the case of perfect contact there is no resistance, when the value of h increases (decreases the resistance value) the differences between the cases are smaller. Table 2 shows temperatures on a fixed point $x = -10^{-7}$ m and at different times in cases of direct contact and contact with resistance for the value of h that we have supposed ($h = 10^9 \frac{W}{m^2\, °C}$) and for $h = 10^6 \frac{W}{m^2\, °C}$.

As we can observe at table 2, for $h = 10^9 \frac{W}{m^2\, °C}$ there are not relevant differences between both cases. However, when the value of h decreases ($h = 10^6 \frac{W}{m^2\, °C}$) these differences become more and more considerable. The value of h for which there are not important differences between both cases depends on the considered point. Closer to interface the point is, higher the value of h must be in order to decrease these differences. For instance, if we suppose $h = 10^9 \frac{W}{m^2\, °C}$ there are not differences between two cases in $x = -10^{-7}$ m, however, in $x = -10^{-8}$ m (nearer of the interface) there are differences for $h = 10^9 \frac{W}{m^2\, °C}$, but there are not for $h = 10^{10} \frac{W}{m^2\, °C}$.

5 Parabolic solution

The temperature distribution in body 1 and 2 from the point of view of parabolic model is found in [5].

The main difference with hyperbolic solution is the absence of the Heaviside function reflecting the fact of infinity speed of heat propagation in parabolic case. In fact parabolic solution shows that since the heat conduction process begins, the temperature of two bodies is always different from the initial.

We have found differences between both models at interface for short time intervals. We can also note that differences are irregular, this is to say, there are temporal intervals in which parabolic temperature is higher than hyperbolic one, and temporal intervals in which hyperbolic temperature is higher than parabolic one. This is due to a different temperature evolution in every model. In figure 3 we can observe the behavior of the temperature profile of body 2 at interface for parabolic model (dashed line) and hyperbolic model (continuous line). We have obtained that in our concrete case interface differences are present for $t < 10^{-10}\ s$.

We have obtained differences between both models in points different from interface too. Table 3 is made for $x = 10^{-8}$m at different times. Differences

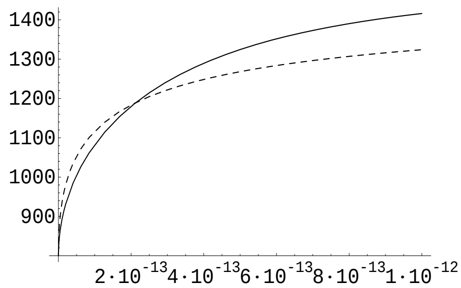

Figure 3: Parabolic (continuous) and hyperbolic (dashed) interface temperature of body 2.

x(m)	**t(s)**	**Par.($^\circ C$)**	**Hyper.($^\circ C$)**
10^{-8}	10^{-8}	1490	1489
10^{-8}	10^{-10}	1413	1414
10^{-8}	10^{-11}	1240	1301
10^{-8}	$8\ 10^{-12}$	1213	1281
10^{-8}	$5\ 10^{-12}$	1148	1209
10^{-8}	10^{-12}	900	800

Table 3: Parabolic and hyperbolic temperatures in $x = 10^{-8}$m at different times.

are found in short times again. When the value of h decreases (the resistance value increases) differences between the models are smaller. At point $x = 10^{-8}$ we see in the table that there are differences for $h = 10^9 \frac{W}{m^2\,^\circ C}$, but they vanish for $h = 10^6 \frac{W}{m^2\,^\circ C}$. This fact is easy to explain: The main difference between both models is the relaxation time, and then, when the resistance value is high the effect of thermal relaxation time is not significant in front of the effect of the resistance, which makes that in both cases we practically work with the same model.

6 Conclusions

In this paper we have obtained, from the point of view of hyperbolic model, the complete analytical solution for the problem of computation of the temperatures of two bodies that initially are at different temperatures and at $t = 0$ are placed together in contact, supposing that exists thermal resistance between them.

The obtained temperature profiles are compared with the solution in the case of direct contact. The differences between both cases are found at every point of the bodies and decrease when the value of h increases. The value for which we cannot observe differences depends on the considered point. Then, when the value of h is significatively small the temperature profiles obtained in the case of direct contact produce erroneous results and we have to use the temperature profiles obtained in the case of contact with resistance although the expressions are more involved.

Between parabolic and hyperbolic solutions we have found differences too. These differences can be observed at every point of bodies in short times, and decrease when the value of h decreases too. Hence, we cannot use parabolic solution in processes in which short times are important, this is in processes in which great amounts of heat are applied to material in very short times and hyperbolic model has to be used in this kind of processes.

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