Synthesis of nonlinear dynamic systems using parameter optimization methods – a case study

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Abstract: - This paper discusses a parameter optimization methodology for the synthesis of nonlinear dynamic systems. The method can be interpreted as a special neural network (NN) technique with predefined structure and weights (parameters) to be optimized. As such, the paper focuses on the definition of an appropriate performance index and on the application of parameter optimization methods in view of the fact that the performance index possesses, due to the nonlinear character of the dynamic system, numerous local minima. Test cases illustrate the performance of the proposed method.

Key-Words: - Nonlinear dynamics; Control; Neural networks; Parameter optimization methods;

1 Introduction

The synthesis of nonlinear, passive or active (controlled), dynamic systems is of outstanding importance for numerous engineering applications.

The techniques that are proposed cover a wide range of methods, e.g. [1]-[7]. One of the most interesting approaches considered, is the approximation to nonlinear optimal control based on solving a Riccati equation at each point 'x', and this algorithm is often referred to as the "state-dependent Riccati equation" or SDRE feedback control. In a recent paper linear, time-varying (LTV) approximations which are arbitrarily close to the true system are introduced. The proposed algorithm uses the globally converged solution of an "approximating sequence of Riccati equations" (ASRE) to explicitly construct time-varying feedback controllers for the original control-affine nonlinear problem.

These and other methods provide solutions based on already existing algorithms, e.g. Ricatti equation. On the contrary, the application of neural networks opens this horizon, allowing more general synthesis procedures.

However, synthesis is generally not an easy task. Preceding numerical tests, experience and intuition are oft used in order to define the system's nonlinear structure. This situation does not change if neural networks are used, as no one knows *a priori* the optimal structure of the neural network (number of levels, nodes, etc.). In this context, the synthesis of nonlinear dynamic systems remains still a challenging problem.

This paper discusses a systematic synthesis design methodology based on the introduction of a number of parameters and nonlinear dynamic terms and on the application of parameter optimization methods. The method as such can be interpreted as a special neural network (NN) technique with predefined structure and weights (parameters) to be optimized. Thus, time simulation and parameter optimization methods are used for the computation of the optimal weights.

The paper focuses especially on features which can ensure the success of the method. This is necessary, in view of the fact that the problem possesses, due to the nonlinear character of the dynamic system, numerous local minima.

The above methodology has been already applied for a number of test cases, e.g. [8]-[10]. In this paper a new application is presented and numerical problems are discussed.

2 Problem Formulation

Let us assume that the system dynamics are described by the state space equation:

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{B}\mathbf{u} + \mathbf{f} + \mathbf{w} \tag{1}$$

In eq. (1) $\mathbf{z}$ denotes the $n\times 1$ state vector, $\mathbf{A}$ the generally nonlinear $n\times n$ system matrix, $\mathbf{B}$ the $n\times m$ control matrix, $\mathbf{u}$ the $m\times 1$ control and $\mathbf{f}$ and $\mathbf{w}$ the load and disturbance vectors respectively. If, now the control vector $\mathbf{u}$ is expressed as a function of $\mathbf{z}$ and of the desired state $\mathbf{z}_D$

$$\mathbf{u} = \mathbf{K} \cdot (\mathbf{z} - \mathbf{z}_D) \tag{2}$$

then eq. (1) is written:

$$\dot{\mathbf{z}} = (\mathbf{A} + \mathbf{BK}) \cdot \mathbf{z} + \mathbf{f} + \mathbf{w} \tag{3}$$

In the above equation **K** respectively **BK** can be designed to influence the system's original properties **A** and performance, independent of the realization technology (passive or active) of **BK.**

Consider e.g. a three-dimensional system described (Karagiannis [7]) by the equations of the form

$$\begin{aligned} \dot{x}_1 &= x_1 + x_2, \\ \dot{x}_2 &= x_2^2 + x_3 \\ \dot{x}_3 &= x_1(1 + x_3^2) + u \end{aligned} \tag{4}$$

If e.g. u is set equal to

$$u = \overbrace{-x_1 x_3^2}^{u1} + \underbrace{a_1 x_1 + a_2 x_2 + a_3 x_3 + b_1 x_1^2 + b_2 x_2^2 + \ldots.}_{u2} \tag{5}$$

then eq. (4) can be written:

$$\begin{aligned} \dot{x}_1 &= x_1 + x_2, \\ \dot{x}_2 &= x_2^2 + x_3 \\ \dot{x}_3 &= (1 + a_1) x_1 + a_2 x_2 + a_3 x_3 + b_1 x_1^2 + b_2 x_2^2 + \ldots \end{aligned} \tag{6}$$

The nonlinear system (6) is the result of intuition or previous analyses (term $u1$) and of a systematic {polynomial type) synthesis procedure (term $u2$). The performance of (6) depends now on the choice of the polynomial coefficients a_i, b_i etc.

3 Problem Solution

The solution of the synthesis problem depends significantly on the appropriate definition of the performance index J. The structure of the proposed performance index is explained using Fig. 1.

From the typical time response of a state variable, e.g. of z_i, one may deduct the following characteristics:

– The maximum overshooting S_{ij} of the variable $z_i(t)$ with respect to a given desired state z_D and initial conditions $\mathbf{z}(t=0) = \mathbf{z}_0$

$$S_i \equiv \max(z_i) / z_D \tag{7}$$

– The end-position $Z_i(T)$ and -velocity $V_i(T)$ of the variable $z_i(t)$ at prescribed simulation time T with respect to a given desired state z_D.

$$\begin{aligned} Z_i(T) &= z_i - z_D \\ V_i(T) &= \dot{z}_i(t) - \dot{z}_D \end{aligned} \tag{8}$$

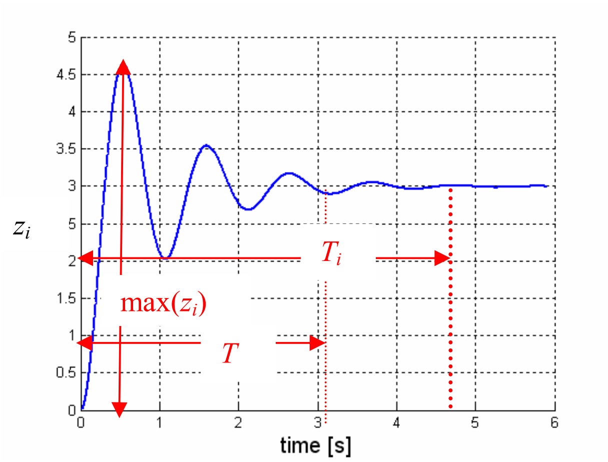

Fig. 1 Typical response of a state variable

This way, the following performance index J_i referring to the variable z_i, to the desired states $\mathbf{z}_D$ and to the initial conditions $\mathbf{z}_0$ is defined:

$$J_i = \lambda_1 \cdot S_i^+ + \lambda_2 \cdot Z_i^+(T) + \lambda_3 \cdot V_i^+(T) + t \tag{9}$$

In (9) the upper indices "+" denote that the functions are defined only for positive arguments (otherwise =0) while λ= weighting factors

If within a given observation period T S_i^+, $Z_i^+(T)$ and $V_i^+(T)$ become zero, the computation is stopped immediately and the performance index J_i is then equal to $t=T_i$. J_i represents in this case the response time of the system with respect to the variable z_i, the desired states $\mathbf{z}_D$ and the initial conditions $\mathbf{z}_0$.

Thus J_j is equal to:

$$J_i = \begin{cases} T_i \ \text{ if } S_i^+ = Z_i^+(T) = V_i^+ = 0 \\ \quad \textit{otherwise} = J_i[\mathbf{z}_0, \mathbf{z}_D, T] \end{cases} \tag{10}$$

The total performance index J of the dynamic system within the observation time period T is then equal to the sum:

$$J[\mathbf{z}_0, \mathbf{z}_D, T] = \sum_i J_i[\mathbf{z}_0, \mathbf{z}_D, T] \tag{11}$$

The most important parameter is the observation time T. In this context it has to be noticed, that if T is initially chosen small and is gradually increased, this helps avoiding local minima from (11). This is a computing rule that has been derived from numerous numerical experiments.

If this and other rules are applied, then deterministic parameter optimization methods, e.g.

the Nelder-Mead algorithm (FMINSEARCH of MATLAB), can be successfully used. Else, evolution strategy methods have to be applied to localize the global optimum.

4 Test Problem

In this section, the proposed methodology is applied to a test problem, not yet included in [8]-[22].

The dynamic system is shown in Fig. 2. Mass m_2 is connected to m_1 through a spring k_1 and a modulated damper c_1. m_2 is excited through the spring k_2 and the disturbance $z_0(t)$. The equations of motion of the dynamic system and of the actuator (time constant T_{act} and limit $f_{\lim}$) are the following:

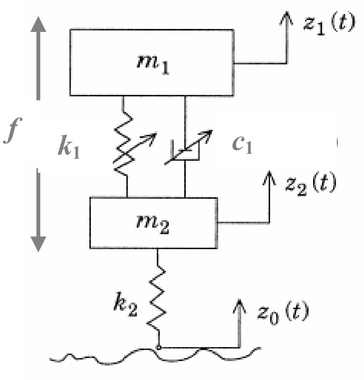

Fig. 2 Dynamic system

$$
\begin{aligned}
&m_1\ddot{z}_1 + c_1(\dot{z}_1 - \dot{z}_2) - f + k_1(z_1 - z_2) = 0\\
&m_2\ddot{z}_2 - c_1(\dot{z}_1 - \dot{z}_2) + f - k_1(z_1 - z_2)\\
&\qquad + k_2(z_2 - z_0) = 0\\
&T_{act}\cdot \dot{f} + f = u, \quad u \le u_{\lim}
\end{aligned}
\tag{12}
$$

u is the control law to be defined.

For a first application the (dimensionless) values are $m_1 = m_2 = 1$, $k_1 = 16$, $c_1 = k_2 = 0$ and the force f is applied only on mass m_1. The u-polynomial is considered to be equal to:

$$
\begin{aligned}
u = &\overbrace{[a_1 + b_1 abs(Z_1)]}^{kus1}\cdot Z_1 + \overbrace{[a_2 + b_2 abs(Z_2)]}^{kus2}\cdot Z_2\\
&+\underbrace{[a_3 + b_3 abs(Z_1)]}_{kus3}\cdot \dot{Z}_1 + \underbrace{[a_4 + b_4 abs(Z_2)]}_{kus4}\cdot \dot{Z}_2
\end{aligned}
\tag{13}
$$

with

$$
\begin{aligned}
Z_1 &= z_{1D} - z_1, \qquad & Z_2 &= z_{2D} - z_2\\
\dot{Z}_1 &= \dot{z}_{1D} - \dot{z}_1, \qquad & \dot{Z}_2 &= \dot{z}_{2D} - \dot{z}_2
\end{aligned}
\tag{14}
$$

For b_i=0 the dynamic system has only one nonlinearity ($f_c \le f_{\lim}$), else the nonlinearities (13) are also added. $f_{\lim}$ is supposed to be critical, therefore $f_{\lim}$<50.

5 Numerical Results

We demonstrate the proposed method, starting with b_i=0, $z_{1D} = z_{2D}$=10 and T=10 sec. The initial **a**-values are chosen arbitrary, e.g.

$$
\mathbf{a} = [a_1 ... a_4] = [100\ \ 0\ \ 0\ \ 0] \tag{15}
$$

Using the ‘fminsearch’ (Melder Nead) subroutine of MATLAB, the results shown in Fig. 3 are obtained.

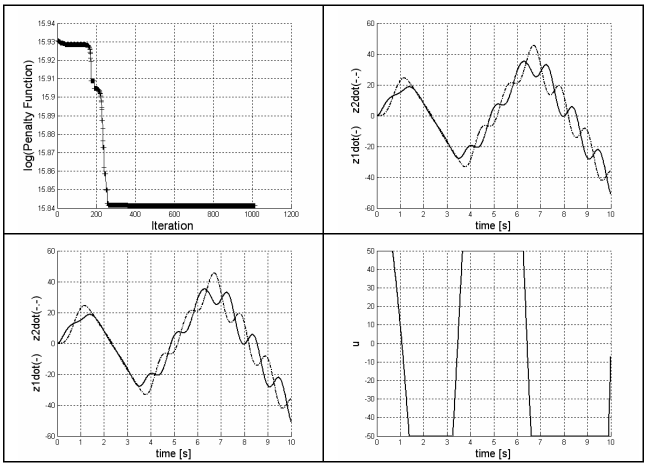

Fig. 3 Results for T=10 sec
$\mathbf{a} = [8.8604\ \ \ 0.1814\ \ -0.0070\ \ -0.0774]$

These results can not be further improved. The algorithm is trapped in a local minimum.

If however T is reduced, e.g. T=5 sec then, although the starting vector (15) is kept the same, the results shown in Fig. 4 are obtained. The optimization algorithm leads to an acceptable solution.

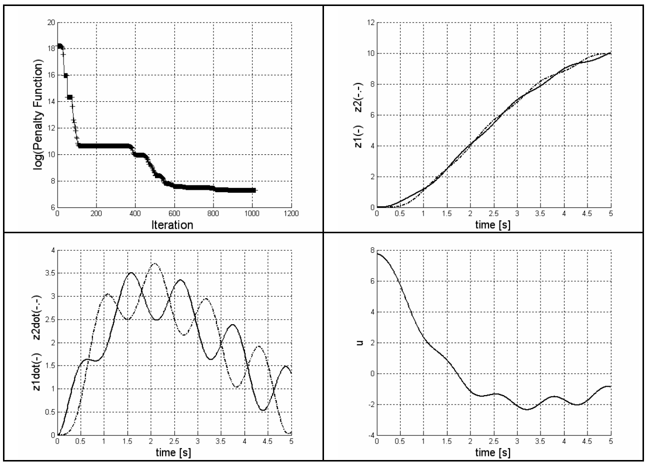

Fig. 4 Results for T=5 sec
$\mathbf{a} = [0.5020\ \ \ 0.2714\ \ \ 0.5996\ \ \ 1.1476]$

Noticing that the u-limit of 50 is not fully used by the controller of Fig. 4, we repeat the procedure, choosing now T=4, 3, 2.5 and 2 sec (Fig. 5-7)

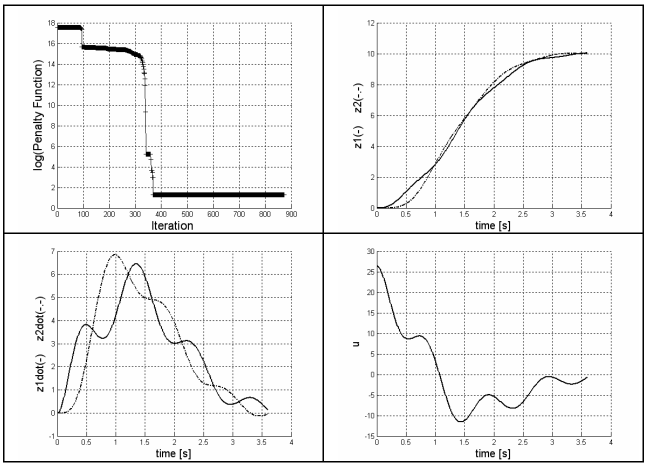

Fig. 5 Results for T=4 sec

$$\mathbf{a} = [2.7529 \;\; -0.1145 \;\; 3.9638 \;\; -0.1822]$$

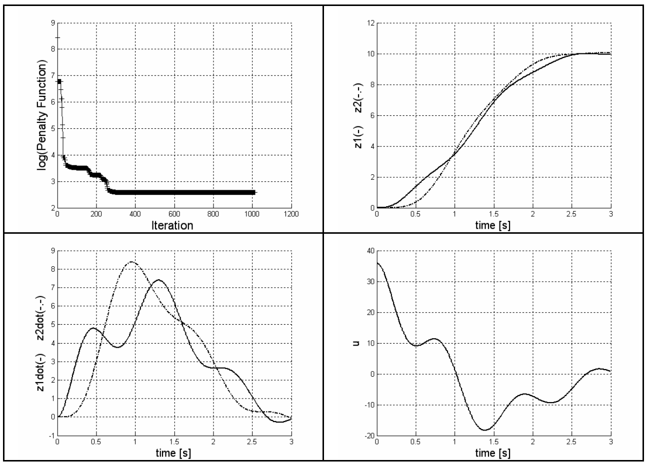

Fig. 6 Results for T=3 sec

$$\mathbf{a} = [3.6177 \;\; -0.0277 \;\; 4.7999 \;\; -0.3279]$$

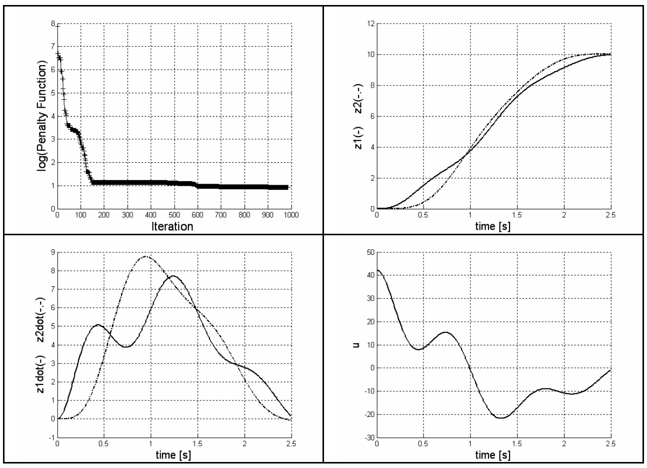

Fig. 7 Results for T=2.5 sec

$$\mathbf{a} = [4.7336 \;\; -0.5148 \;\; 6.1695 \;\; -1.0934]$$

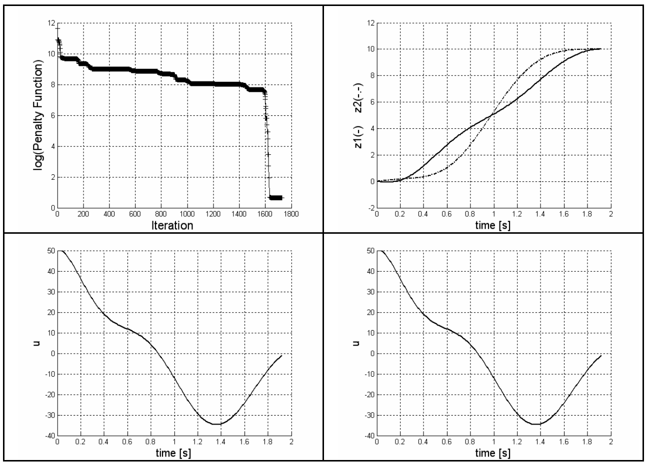

Fig. 8 Results for T=2 sec

$$\mathbf{a} = [-2.3771 \;\; 7.2369 \;\; 3.7099 \;\; 1.2233]$$

Having now established a basis for further optimization, the algorithm can be opened to include more desired states, e.g. z_D=10, 7.5, 5. 2.5 (see Figure 8). For the controller of Fig. 8 we obtain finally

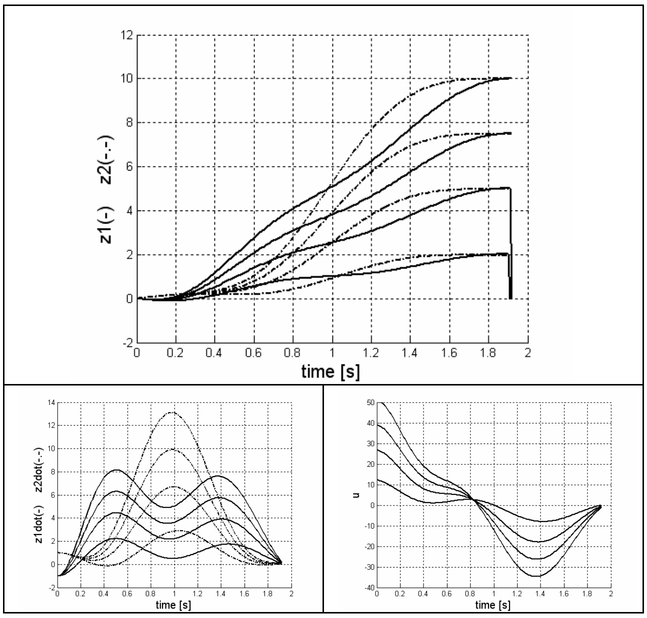

Fig. 9 Results for

$$\mathbf{a} = [-2.3771 \;\; 7.2369 \;\; 3.7099 \;\; 1.2233]$$

Exactly the same procedure is applied if both the a_i and b_i coefficients in eq.(13) are considered. In Fig. 10 we show analogous results.

Finally, the synthesis can consider also stability aspects. For example the case of Fig. 10 can be examined for disturbances of the initial conditions, e.g. for $\dot{z}_1 = -10, \dot{z}_2 = -10$ (Fig. 11)

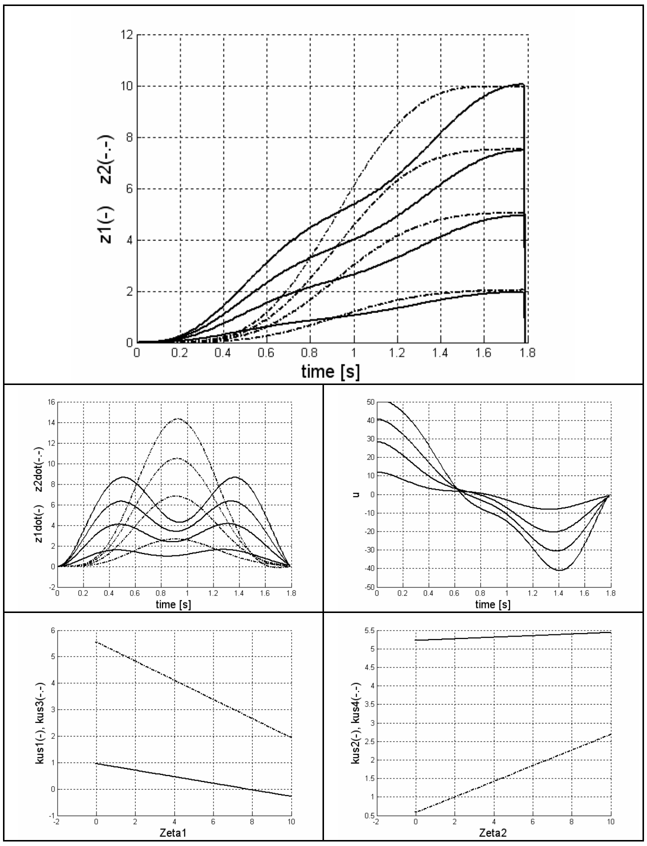

Fig.10 Results for

$$\mathbf{a} = [0.9595 \quad 5.2237 \quad 5.5610 \quad 0.5809]$$

$$\mathbf{b} = [-0.1239 \quad 0.0216 \quad -0.3619 \quad 0.2110]$$

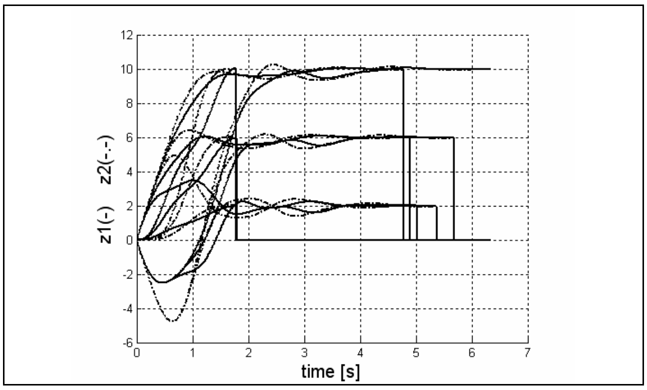

Fig.11 Stability analysis for Results of Fig. 10.

5 Conclusions

In this paper a methodology for the synthesis of nonlinear dynamic systems is presented. The method can be interpreted as a special neural network (NN) technique with predefined structure and weights (parameters) to be optimized.

The proposed performance index J depends on the observation time period T, which plays a significant role for the computation of the weights (parameters). If deterministic parameter optimization methods are used, as here (Nelder-Mead Algorithm), a relaxation of T, is absolutely necessary.

The numerical test presented in the paper succeeded in performing the synthesis of the dynamic system without difficulties. The results are generally encouraging so that the proposed method will be further investigated.

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