

Error of Varhour Meter's Registration in the Presence of Harmonics

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Abstract: - The importance of metering reactive power is increasing with the development of electricity market. But there are obvious defects in reactive power metering in the presence of harmonics mainly due to mistake in phase shifting. This paper analyzes the phase-shift errors of common used varhour meters through sequence component analysis. The frequency response tests and harmonic reactive power tests are done to induction varhour meter and solid-state meter. The test data are analyzed by least square method. The analysis show that the error contribution of varhour meter's registration in the presence of harmonics are due to amplitude effects and phase effects through analysis of theory and experiments: the phase contribution will be large to both induction varhour meter and solid-state meter and may cause metering errors of double harmonic apparent power, the amplitude is decreased with increasing harmonic order and that of the induction varhour is decreased more dramatically.

Key-Words: - Harmonic analysis; Induction varhour meter; Solid-state varhour meter; Frequency response; Least squares methods

1 Introduction

The importance of reactive energy metering is increasing with the development of electricity market. Experimental and theoretical studies concerning the accuracies of induction watt-hour meters (IWHMs) for non-sinusoidal waveforms have been widely investigated and reported^[1-5]. The energy metering in the presence of harmonics have obtained considerable development through time-domain method and frequency-domain method, through considering the impedance of rotator, magnetic circuit and self-brake torsion. These studies conclude that most practical loads, errors of the conventional electrodynamic watt-hour meter are mainly due to harmonic power and harmonic current and are small within commercially acceptable limits. However, a little investigation appears to have been conducted concerning the registration errors of varhour meters for non-sinusoidal conditions. Both induction and solid-state varhour meters register the fundamental displacement volt-ampere hours only and do not respond to distortion volt-ampere hours for nonlinear load driven by sinusoidal voltages^[6]. Admittedly, one must be careful in discussing "reactive" power and energy in the case of distorted waveforms. However, since utilities often employ varhour meters in conjunction with watt-hour meters to determine power factors at various measurement points and because harmonic levels continue to increase in many power

systems, it is important that the performances of typical varhour meters under non-sinusoidal conditions be known. Although the reactive energy metering is different only from the phase between voltage and current, there is obvious defect in phase shifting.

In this paper, the basic principles of phase shifting of an induction varhour meter (IVHM) and a solid-state varhour meter (SVHM) are firstly analyzed through dividing the harmonics into positive-sequence, negative-sequence and zero-sequence. Secondly the frequency response tests and harmonic reactive power tests are done to IVHM and SVHM. In the end, the test data are analyzed by least squares method (LSM). The analysis show that the errors of varhour meter's registration in the presence of harmonics are due to amplitude effects and phase effects through analysis of theory and experiments: the phase errors will be large to both IVHM and SVHM and may cause metering errors of double harmonic apparent power, the amplitude is decreased with increasing harmonic order and that of the IVHM is decreased more dramatically. Characteristic of reactive energy metering of IVHMs are some nonlinear because of the nonlinear characteristic of magnetization, the variation of impedance of voltage coil and current coil, the nonlinear torque of friction and self-brake. So the test data is simulated synthetically in this paper. The test data are not properly simulated by FFT method for IVHM and SVHM are difficult to meter the reactive energy when the phase angle near 0° , So the least squares method (LSM) is applied in this paper. The

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analysis shows that there are obvious defects in reactive energy metering in the presence of harmonic power.

2 Analysis of phase shifting

The basic theory of reactive energy metering is similar to active energy metering. Varhour meters can be classified to sine and cosine type meters in terms of the connection types and operating theory. The cosine Varhour meters are also named cross-phase lined or artificial Varhour meters. Ordinary SVHMs are sine Varhour meters and IVHMs may be both type (cosine mainly). To meet phase angle shifting, SVHMs delay an interval of the voltage and make the voltage lag 90° (Exception the meters of using FFT method), sine IVHMs are series of a resistance in voltage circuit and parallel of a resistance in current circuit (So the energy loss is enlarged). Cosine IWHMs are series of a resistance in voltage circuit. But they may make phase shifting errors in the presence of harmonics. The detailed analysis of phase angle shifting is presented as follow.

2.1 Analysis of Phase angle shifting of cosine IVHM

The phase of sine IVHMs in the presence of harmonics will change little with the variation of reactance of voltage and current circuits. The theoretical error analysis of cosine IVHMs is similar with the error analysis in the condition of asymmetry. The main cause is that the phase of negative sequence cannot be shifted correctly. However, the harmonics of $3k+2$ order (k is non-negative integer) are negative sequence. The zero sequence power is not metered for the zero sequence voltages cannot flow through. When neglecting the variation of impedance with harmonic order, the metered reactive power can analyze as follow:

The power of three phase circuits is calculated with the definition of plural power: The plural power \dot{S} can be described as follow:

$$\begin{aligned} \dot{S} &= \dot{U}_A \dot{I}_A + \dot{U}_B \dot{I}_B + \dot{U}_C \dot{I}_C \\ &= (\dot{U}_1 + \dot{U}_2 + \dot{U}_0)(\dot{I}_1 + \dot{I}_2 + \dot{I}_3) \\ &\quad + (\alpha^2 \dot{U}_1 + \alpha \dot{U}_2 + \dot{U}_0)(\alpha \dot{I}_1 + \alpha^2 \dot{I}_2 + \dot{I}_3) \\ &\quad + (\alpha \dot{U}_1 + \alpha^2 \dot{U}_2 + \dot{U}_0)(\alpha^2 \dot{I}_1 + \alpha \dot{I}_2 + \dot{I}_3) \\ &= 3\dot{U}_1 \dot{I}_1 + 3\dot{U}_2 \dot{I}_2 + 3\dot{U}_0 \dot{I}_0 \\ &= 3U_1 I_1 \cos \theta_1 + 3U_2 I_2 \cos \theta_2 + 3U_0 I_0 \cos \theta_0 \\ &\quad + j(3U_1 I_1 \sin \theta_1 + 3U_2 I_2 \sin \theta_2 + 3U_0 I_0 \sin \theta_0) \end{aligned} \quad (1)$$

where $\dot{U}_1, \dot{U}_2, \dot{U}_0$ is the positive, negative and zero sequence of \dot{U}_A respectively; $\dot{I}_1, \dot{I}_2, \dot{I}_0$ is the positive, negative and zero sequence of \dot{I}_A respectively; $\theta_1, \theta_2, \theta_0$ are the phase angle between positive, negative and zero sequence voltage and current.

The right part in the equation is the reactive:

$$Q = \text{img}(\dot{S}) = 3U_1 I_1 \sin \theta_1 + 3U_2 I_2 \sin \theta_2 + 3U_0 I_0 \sin \theta_0 \quad (2)$$

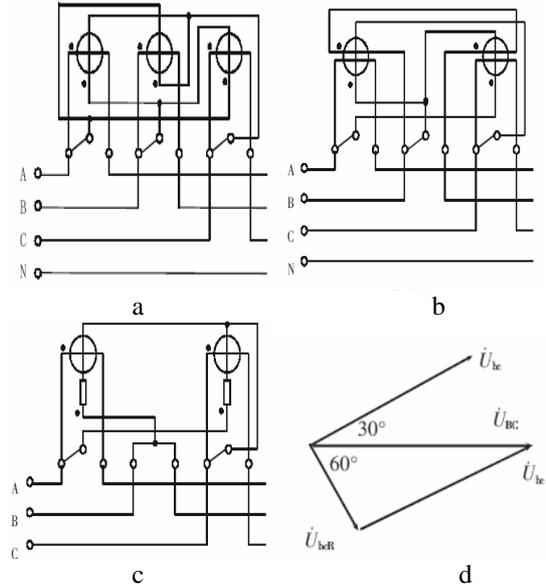


Fig. 1. IVHMs' connection method and the voltage phasor

There are some connection methods as follow: the three phase four lines 90° connection as shown in Fig. 1 a, the three phases four lines with additional windings connection as shown in Fig. 1 b, the three phase three lines 60° connection as shown in Fig. 1c. To the connection method of Fig. 1c, the voltage phasor is shown in Fig.1d, The voltage phase in the voltage winding leads the original 30° , and the amplitude is as the $\sqrt{3}/2$ of the original. So the reactive power shown in Fig. 1 can be described as follow:

$$\begin{aligned} Q_a &= \text{img}(\dot{S}) = \text{img}(\dot{U}_{BC} \dot{I}_A + \dot{U}_{CA} \dot{I}_B + \dot{U}_{AB} \dot{I}_C) \\ &= 3\sqrt{3}(U_1 I_1 \cos(\theta_1 - 90^\circ) + U_2 I_2 \cos(\theta_2 + 90^\circ)) \end{aligned} \quad (3)$$

$$\begin{aligned} Q_b &= \text{img}(\dot{S}) = \text{img}[\dot{U}_{BC}(\dot{I}_A - \dot{I}_B) + \dot{U}_{AB}(\dot{I}_C - \dot{I}_B)] \\ &= \text{img}(\dot{U}_{BC} \dot{I}_A + \dot{U}_{CA} \dot{I}_B + \dot{U}_{AB} \dot{I}_C) \\ &= 3\sqrt{3}(U_1 I_1 \cos(\theta_1 - 90^\circ) + U_2 I_2 \cos(\theta_2 + 90^\circ)) \end{aligned} \quad (4)$$

$$\begin{aligned} Q_c &= \text{img}\left[\frac{\sqrt{3}}{2}(\dot{U}_{BC} \dot{I}_A \angle 30^\circ + \dot{U}_{AC} \dot{I}_C \angle 30^\circ)\right] \\ &= \frac{3\sqrt{3}}{2}[U_1 I_1 \cos(\theta_1 - 90^\circ) + U_2 I_2 \cos(\theta_2 + 150^\circ)] \end{aligned} \quad (5)$$

Considered the phase angle of the negative sequence, it can be concluded in table 1.

Table 1 The phase shift statistics of IVHM

Type of varhour meter	Harmomic order	Shifted phase angle	description	Phase error	Metering error
2 component 60°	3*k+1	-90°	correct	0°	No error May reach
	3*k+2	+150°	fault	240°	$\sqrt{3}$ times of harmonic apparent power
	3*k+3	×	fault	×	Not metered
3 component 90°	3*k+1	-90°	correct	0°	No error
	3*k+2	+90°	fault	180°	Negative of the reactive power
	3*k+3	×	fault	×	Not metered

2.2 Analysis of Phase angle shifting of SVHMs

Difference with the active power metering, SVHMs meter reactive power through delaying the voltage an interval. The interval is based on the fundamental frequency. In the presence of harmonics, the nth harmonic phase angle will be shifted $n*90^\circ$, so that only $4*k+1(1, 5, 9, \dots)$ order harmonic can be shifted correctly. The results of analysis are listed in table 2.

Table 2 The phase shift statistics of SVHM

Harmonic order	Shifted angle	description	Phase error	Metering error
4*k+1	-90°	correct	0°	No error May reach
4*k+2	-180°	fault,	-90°	$\sqrt{2}$ times of harmonic apparent power
		counter with the active power		
4*k+3	-270°	fault,	-180°	2 times of harmonic apparent power
		counter with the reactive power		
4*k+4	0°	fault, same with the active power	90°	$\sqrt{2}$ times of harmonic apparent power

3 Experimental Procedure

The test circuit diagram, Fig.2, shows the principal components that were used in this study. They were the harmonic power generator (SW5250), the universal power analyzer (PM3000A), the waveform recorder (DF1024), the phase shifter, the rheostat and the IVHM (DSM9-1) and SVHM (DSSD22IV). The make and model of IVHM was the same as those commonly employed in the field.

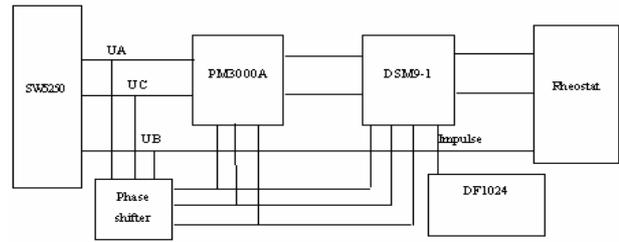


Fig. 2. Equipment Configuration for Experiment

The SW5250 provided harmonic voltage, while the phase shifter and rheostat adjusted the current phase angle and amplitude. The DF1024 was utilized to record the impulse from the meter. According to the average interval of impulses, the power could be gotten. The results recorded by the PM3000A provided the baseline for comparison of the accuracy of the IWHM over a wide range of frequencies.

The test waveforms included a square shape with a wave factor less than that of the sinusoid, a triangular shape with a wave factor larger than that of the sinusoid, f_3 which was the fundamental plus the 3rd harmonic with an amplitude 1/3 of the fundamental and the same phase as the fundamental, f_5 which was f_3 plus the 5th harmonic with an amplitude 1/5 of the fundamental, and the same phase as the fundamental, and f_7 which was f_5 plus the 7th harmonic with an amplitude 1/7 of the fundamental and the same phase as the fundamental. The waveforms of f_3 , f_5 , and f_7 were listed in table 3.

The test currents were 1A, 3A and 5A. The test current phase angles were $\pm 0^\circ$, $\pm 30^\circ$, $\pm 60^\circ$. The phase shifter was disabled during single-phase tests, and the voltage was connected directly from the SW5250 to the meter.

Table 3 Some test waveforms

Waveform name	Graph	Math description
f_3		$A[\sin(\omega t) + \frac{1}{3} \sin(3\omega t)]$
f_5		$A[\sin(\omega t) + \frac{1}{3} \sin(3\omega t) + \frac{1}{5} \sin(5\omega t)]$
f_7		$A[\sin(\omega t) + \frac{1}{3} \sin(3\omega t) + \frac{1}{5} \sin(5\omega t) + \frac{1}{7} \sin(7\omega t)]$

The processing procedure was performed as follow:

Step1: Preprocessing

Before the analysis, the metered power was multiplied by a parameter in terms of the apparent power, which

was measured by the PM3000A. The equation used was:

$$Q_m = Q \times S_{base} / S \tag{6}$$

where P_m is the modified power; P is the power metered by the IWHM; S_b is the base power; and S is the apparent power measured by the PM3000A.

Step 2: Development of Equations for Parameters with LSM

With the same frequency, voltage, current and varied phase angle, the follow equation is true:

$$Q = A \sin(x + \alpha) + B \tag{7}$$

where Q is reactive power; A is the amplitude of the power; x is the phase angle with which the voltage leads the current; α is the phase shift and B is the offset of the reactive power.

According to the LSM, the objective function is:

$$\min \sum_{i=1}^n [A \sin(x_i + \alpha) + B - Q_i]^2 \tag{8}$$

where i is the number of power data points with varied phase angle.

Step 3: Programming and Calculating

The test results were recorded in an Excel sheet, and a Visual Basic Application (VBA) was applied.

4 Experimental Results

Frequency response experiment is utilized to analyze the basis characteristic of varhour meters in the presence of harmonics and harmonic reactive power experiment is utilized to verify the characteristics.

There are only single-phase tests with SVHM for its symmetrical metering characteristics.

4.1 Frequency Response

On the basis of the metered data, the mathematical model $Q = A \sin(x + \alpha) + B$ was applied to simulate the harmonic power.

The maximum simulation error is under 0.3%, which can be negligible to the IVHM. The parameters are listed in Tables 4-8 and Figures 3-7. Notion: “-” in the first column denote negative sequence components; A% denotes percent to the fundamental.

Table 4 Frequency response parameters of IVHM when RMS of current is 5A

frequency* 50	amplitude (var)	A%	phaseshift (degree)	offset (var)
1	877.47	100.00	0.42	2.36
2	756.62	86.23	1.55	7.47
3	639.03	72.83	6.32	7.56
5	556.46	63.42	12.21	-14.95
7	324.04	36.93	12.49	0.22
9	252.60	28.79	11.55	-2.11
11	201.53	22.97	9.35	-5.70
13	159.22	18.15	6.66	-7.37
25	67.62	7.71	-5.71	-8.12
-1	894.89	101.99	240.96	15.17
-2	789.80	90.01	242.79	19.41
-3	636.95	72.59	240.52	71.56
-5	522.93	59.60	243.13	10.60

Table 5 Frequency response parameters of IVHM when RMS of current is 3A

frequency* 50	amplitude (var)	A%	phaseshift (degree)	Offset (var)
1	523.59	100.00	0.83	1.38
2	448.37	85.63	1.82	4.57
3	371.32	70.92	5.68	0.89
5	251.79	48.09	9.79	-3.38
7	183.60	35.07	9.39	-3.34
9	135.39	25.86	8.71	-1.17
11	108.64	20.75	6.36	-3.09
13	87.61	16.73	3.91	-2.70
25				
-1	541.76	103.47	241.50	-3.24
-2	466.76	89.15	244.73	6.41
-3	382.84	73.12	249.50	5.50
-5	272.69	52.08	254.71	-1.42

Table 6 Frequency response parameters of IVHM when RMS of current is 1A

frequency* 50	amplitude (var)	A%	phaseshift (degree)	offset (var)
1	172.01	100.00	0.61	-4.40
2	144.33	83.91	1.85	-1.75
3	117.54	68.34	5.72	-0.96
5	79.21	46.05	9.40	-0.15
7	57.76	33.58	9.68	-1.13
9	43.00	25.00	8.07	-0.90
11	33.17	19.28	5.32	-0.26
13	28.55	16.60	3.85	-1.85
25	11.50	6.69	-11.95	-0.84
-1	176.40	102.55	242.23	-1.75
-2	150.54	87.52	244.73	0.39
-3	123.12	71.58	249.46	1.08
-5	83.06	48.29	254.14	1.84

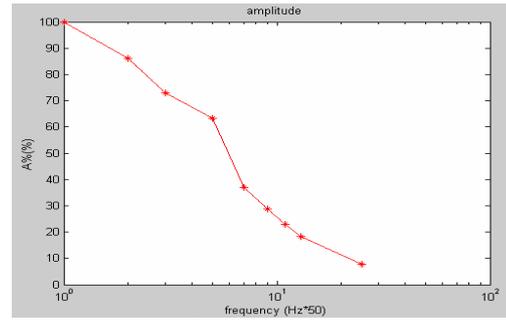


Fig. 3. Amplitude of IVHM with positive sequence harmonic in frequency response

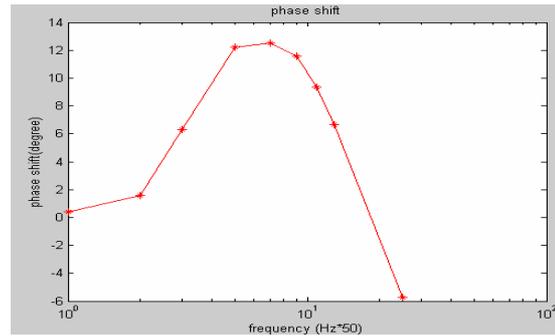


Fig. 4. Phase shift of IVHM with positive sequence harmonic in frequency response

Table 7 Frequency response parameters of SVHM when RMS of current is 5A

frequency* 50	amplitude (var)	A%	phaseshift (degree)	offset (var)
1	1101.79	100.00	0.03	0.16
2	1098.72	99.72	90.10	-1.97
3	1094.37	99.33	179.93	-4.74
4	1085.95	98.56	270.77	5.58
5	1080.37	98.06	0.07	2.90
6	1057.53	95.98	89.94	-9.61
7	1058.86	96.10	181.03	7.74

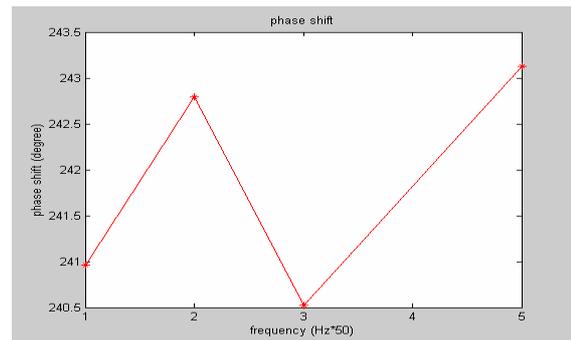


Fig. 5. Phase shift of IVHM with negative sequence harmonic in frequency response

Table 8 Frequency response parameters of SVHM when RMS of current is 1A

frequency* 50	amplitude (var)	A%	phaseshift (degree)	offset (var)
1	221.02	100.00	0.01	-0.06
2	220.35	99.70	90.28	-0.02
3	218.94	99.06	180.15	-0.22
4	216.70	98.05	270.37	3.74
5	215.28	97.40	0.00	0.65
6	211.66	95.77	90.73	1.80
7	209.42	94.75	180.78	0.23

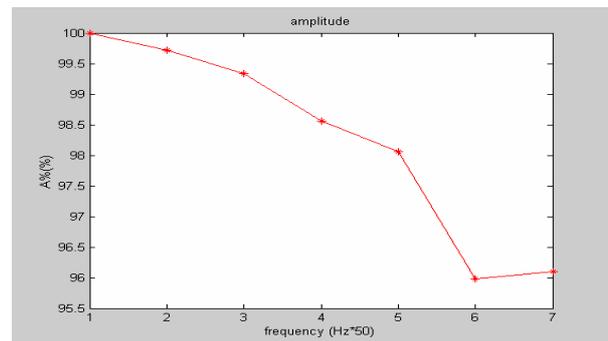


Fig. 6. Amplitude of SVHM in frequency response

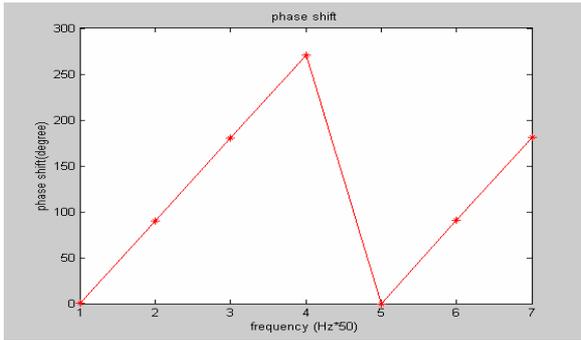


Fig. 7. Phase shift of SVHM in frequency response

From Tables 4-8 and Figures 3-7, it can be concluded that:

- The IVHM's phase shift in the presence of positive sequence harmonics increases with the order of the harmonic from 0° for the fundamental to maximum 9.79° for the 5th harmonic and then decrease with the harmonic order and varied 1° with the varied current from 1A to 5A. The IVHM's phase shift in the presence of negative sequence harmonics are about 240°, as verified the analysis of phase shifting described in section 1. The little difference between the analysis and experiment is caused mainly by the variation of impedance of coils and magnetic circuits.
- The amplitude of IVHM decreases acutely with the order of harmonic (e.g. the metered 3rd harmonic power is 72.8% of the fundamental; the 5th is 63.4% of the fundamental).
- The amplitude of SVHM decreases a little with the order of harmonic (e.g. the metered 3rd harmonic power is 99.3% of the fundamental; the 5th is 98.1% of the fundamental).
- The proportions of amplitude and phase shift of harmonic to fundamental frequency remain constant, and their offsets are negligible, when the current amplitude and phase varies.

4.2 Harmonic Reactive Power Test

The metered reactive power by induction varhour meter was simulated by equations as follow:

$$Q = Q_1 + \sum_i K_i U_i I_i \sin(\theta_i + \alpha_i) \quad (9)$$

where Q is the simulated reactive power; Q_1 is the metered reactive power of fundamental frequency; i is the harmonic order; K_i is the amplitude rate for i^{th} harmonic; U_i is the RMS of i^{th} harmonic voltages; I_i is the RMS of i^{th} harmonic currents; θ_i is the phase angle between i^{th} harmonic voltage and current; α_i is the phase shift of i^{th} harmonic current.

The results are listed in Tables 9, 10 as follow and descriptions of wave form are shown in table 3. Theta column denote the phase angle between fundamental voltage and current. Svalue column denote the simulated value.

Table 9 Harmonic reactive power test results of induction varhour meter (three phases)

Wave shape	Theta (degree)	Metered Value (var)	Error to time domain (%)	Error to frequency domain (%)	Svalue (var)	SError (%)
f ₅	30	405.77	-6.40	2.78	412.08	1.56
f ₅	90	830.56	-3.10	2.96	840.31	1.17
f ₅	120	727.57	-2.10	3.55	732.02	0.61
f ₇	30	403.12	-7.27	-0.30	407.91	1.19
f ₇	90	822.97	-3.73	2.15	830.68	0.94
f ₇	120	721.62	-2.59	3.57	725.23	0.50

Table 10 Harmonic reactive power test results of solid-state varhour meter (single phase)

Wave shape	Theta (degree)	Metered Value (var)	Error to time domain (%)	Error to frequency domain (%)	Svalue (var)	SError (%)
f ₃	30	437.45	-42.54	-34.90	433.22	-0.97
f ₃	60	950.57	-16.99	-0.75	955.39	0.51
f ₃	270	-1222.49	-0.20	24.27	-1227.28	0.39
f ₃	300	-958.87	-16.42	0.44	-956.96	-0.20
f ₃	330	-433.94	-43.62	-35.53	-430.02	-0.90
f ₅	30	452.81	-47.74	-32.53	451.34	-0.32
f ₅	60	913.89	-22.76	-4.48	917.53	0.40
f ₅	270	-1264.76	-0.15	28.70	-1268.90	0.33
f ₅	300	-920.81	-22.10	-3.45	-917.89	-0.32
f ₅	330	-460.59	-47.44	-31.41	-456.21	-0.95
f ₇	30	460.83	-49.65	-32.39	461.47	0.14
f ₇	60	903.80	-24.55	-3.27	895.60	-0.91
f ₇	270	-1275.15	-0.70	27.45	-1286.80	0.91
f ₇	300	-891.80	-25.52	-4.36	-896.09	0.48
f ₇	330	-461.44	-49.90	-31.92	-459.94	-0.33

From Table 9 and Table 10, it can be concluded that: Equation (9) can express the characteristics of induction varhour meters and solid-state varhour meters. Additionally, the error based on time-domain and frequency-domain differs acutely.

5 Conclusion

This novel test-based procedure analyzes the causes of IWHM and SVHM's registration error in the presence

of harmonics. The procedure involves a frequency response test and a harmonic reactive power test with varied current and phase angle. Analysis of theory and experiment to IVHM and SVHM in the presence of harmonics allow classification of the cause by degree of contribution to the total error as amplitude and phase shifting, which are described as follow:

- The IVHM's phase shift in the presence of positive sequence harmonics increases with the order of the harmonic from 0° for the fundamental to maximum 9.79° for the 5th harmonic and then decrease with the harmonic order and varied 1° with the varied current from 1A to 5A. The IVHM's phase shift in the presence of negative sequence harmonics are about 240° , which may lead an absolute error to $\sqrt{3}$ times harmonic apparent power. Moreover the SVHM's phase shift in the presence of negative sequence harmonics may lead an absolute error to 2 time's harmonic apparent power.
- The amplitude of IVHM decreases acutely with the order of harmonic (e.g. the metered 3rd harmonic power is 72.8% of the fundamental; the 5th is 63.4% of the fundamental). The amplitude of SVHM decreases a little with the order of harmonic (e.g. the metered 3rd harmonic power is 99.3% of the fundamental; the 5th is 98.1% of the fundamental).
- The proportions of amplitude and phase shift of harmonic to fundamental frequency remain constant, and their offsets are negligible, when the current amplitude and phase varies.

This classification method is helpful in understanding the characteristics of an IVHM in the presence of harmonics, and simulates the reactive power with the data measured by power quality device, when the relevant parameters have been obtained by experiments.

It is strongly suggested that fundamental frequency reactive power and each order harmonic reactive power be metered individually or the apparent power be metered. Because there are defects presently in reactive power metering in the presence of harmonics.

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