

# A Multi-objective Optimization Model of Transmission Network Planning Based on Fuzzy Set Pair Analysis<sup>\*</sup>

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**Abstract:** - To solve the problems in multi-objective transmission network planning, a novel model based on Fuzzy Set Pair Analysis (FSPA) method is presented, in which economics, reliability and flexibility of the schemes are all taken into account. Economics and reliability are indicated by the investment cost and the outage cost respectively, and flexibility of the network is quantified by Available Transfer Capability (ATC) for the first time. The involvement of flexibility in the objective function can improve the adaptability of the optimal scheme to the uncertain future and avoid unnecessary compensation investment effectively. The planning schemes are assessed by two opposite pairs: certainty- uncertainty and identity-contrary, by FSPA method, which produce more comprehensive estimations. The model is then verified by an actual example.

**Key-Words:** - Transmission network planning; Multi-objective; Fuzzy Set Pair Analysis (FSPA); Outage cost; Available Transfer Capability (ATC)

## 1 Introduction

With the introduction of electricity market mechanism [1] and continuous expansion of electric power system, more and more factors are being involved into transmission network planning. So multiple objectives are required to be considered in the planning at the same time, which may have indexes with different importance, or even conflictive indexes. To solve the conflicts among the objectives rationally, an ideal way is to use multi-objective optimization method.

Most previous methods in multi-objective transmission network planning assembled multiple objectives into single objective to solve the problems. J.A Momoh, et al. achieved an optimal solution with the least total cost by involving investment cost, operating cost and outage cost of reliability index into objective function [2]. Jingdong Xie, et al. considered investment cost, outage cost and environment (area of hallway) as three important factors [3]. They found a compromised solution by objecting the dispersion from the optimal values of the above three factors. Hongbo Sun, et al. presented the fuzzy evaluation method to solve the multi-objective transmission planning problem by using the advantage of fuzzy set theory in the description of degree [4]. By seeking the satisfaction degree of each objective index, an optimal solution with the best total satisfaction degree is achieved.

For the above methods, the main difficulty is how to unify various objective dimensions and make choice of evaluation functions of various objectives. To deal with this problem, the paper proposes a novel method of Fuzzy Set Pair Analysis (FSPA) to establish the

multi-objective optimization model of the transmission network planning, with economics, reliability and flexibility emphasized as three important factors. Economics and reliability are indicated by the investment cost and the outage cost of customer respectively, and flexibility of the network is quantified by Available Transfer Capability (ATC) for the first time. By employing the FSPA method, these objectives are independently evaluated from viewpoints of identity-discrepancy-contrary (IDC), and then assembled into single index. An actual example is given to demonstrate the effectiveness of the proposed model.

## 2 Fuzzy Set Pair Analysis (FSPA)

### 2.1 Concept of Fuzzy Connection Degree

There are many approaches to represent and process uncertain knowledge, such as probability, grey, fuzzy set theory, etc. Set pair analysis (SPA) theory is another attempt to this problem proposed by Keqin Zhao [5]. The underlying idea of SPA is to consider the relation of certainties and uncertainties of two interrelated sets as a system of certainties and uncertainties, then set up an identity-discrepancy- contrary (IDC) connection degree formula of the two sets under certain circumstances. The theory has attracted attention of many researchers in various fields all over China, who contribute to its development and applications. Fuzzy Set Pair Analysis (FSPA) method is advanced with the integration of fuzzy set theory and SPA theory [6].

Fuzzy connection degree is a reification of the

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concept of SPA connection degree in the classification of fuzzy information [6], which is a microcosmic expression of fuzzy membership degree in research region. A piece of complete fuzzy information is depicted oppositely twice, forming the fuzzy set pair in different cognition layers. One depiction is to take object  $X$  as the research region, in which the fuzzy subsets  $A$  and  $A^c$  are certainty and uncertainty information respectively. The other is to take certainty information  $A$  as the research region, in which the fuzzy subsets  $B$  and  $B^c$  are the identity and contrary information respectively.

**Definition 1** [6] For any information  $x \in A$ , the membership degree of  $B$  and  $B^c$ , defined in symbols as  $\mu_B(x) = a$  and  $\mu_{B^c}(x) = c$ , are called the identity membership degree (for short, identity degree) and the contrary membership degree (contrary degree) respectively. Due to the objectivity of uncertainty and the subjectivity of cognition, relative uncertainties exist impersonally, so in general,  $\mu_B(x) + \mu_{B^c}(x) < 1$ , viz.  $B$  and  $B^c$  are not a couple of fuzzy complementary set.

**Definition 2** [6] For any information  $x \in X$ , the membership degree of  $A$  and  $A^c$ , defined in symbols as  $\mu_A(x) = a + c$  and  $\mu_{A^c}(x) = b$ , are called the certainty membership degree (certainty degree) and the uncertainty membership degree (discrepancy degree) respectively. According to unitary condition [5],  $A$  and  $A^c$  make a couple of fuzzy complementary set, and  $\mu_A(x) + \mu_{A^c}(x) = 1$ , viz.  $a + b + c = 1$ .

Therefore, the formula for the IDC fuzzy connection degree of a set pair in region  $X$  is obtained as follows [6]:

$$\mu = \mu_B(x) + \mu_{A^c}(x)i + \mu_{B^c}(x)j = a + bi + cj \quad (1)$$

where the  $j$  is the coefficient of the contrary degree, and is specified as  $-1$ . The  $i$  is the coefficient of the discrepancy degree, and is an uncertain value between  $-1$  and  $1$ , viz.  $i \in [-1, 1]$ , in terms of various circumstances,

With the opposite depiction as one characteristic, FSPA method depicts two independent opposite pairs: certainty-uncertainty and identity-contrary. It can also thoroughly describe the object and provide more information for estimation and analysis.

## 2.2 Formulations of Multi-objective Fuzzy Connection Degree

The same parameter ranges are required to assess the schemes by FSPA method. The ideal best scheme  $U$  and the ideal worst one  $V$  need to be predefined, where  $U = Z(u_1, u_2, \dots, u_n)$ ,  $V = Z(v_1, v_2, \dots, v_n)$ , in which  $u_r$  and  $v_r$  show the best and the worst value of index  $g_r$

respectively. The values of  $U$  and  $V$  are brought from the inside or outside of the scheme set according to the objectives and the impersonal conditions of the system. The comparison space of the schemes is formed as  $[V, U]$ . Based on the multi-objective properties of transmission network planning, fuzzy connection degree of set pair  $\{d_{kr}, u_r\}$  for income-type index and cost-type index approaching optimum is given respectively as follows [6]:

$$\mu_{(d_{kr}-u_r)} = \frac{d_{kr}}{u_r + v_r} + \frac{(u_r - d_{kr})(d_{kr} - v_r)}{(u_r + v_r)d_{kr}}i + \frac{u_r v_r}{(u_r + v_r)d_{kr}}j \quad (2)$$

$$\mu_{(d_{kr}-u_r)} = \frac{u_r v_r}{(u_r + v_r)d_{kr}} + \frac{(v_r - d_{kr})(d_{kr} - u_r)}{(u_r + v_r)d_{kr}}i + \frac{d_{kr}}{u_r + v_r}j \quad (3)$$

Where  $d_{kr}$  represents the value of scheme  $k$  in index  $g_r$ .

The identity degree and contrary degree in both equation (2) and (3) show the closeness degree to the best value and to the worst value of the index  $g_r$ , and the discrepancy degree indicates the degree of uncertainty for the index  $g_r$ .

## 3 Application of Fuzzy Set Pair Analysis in Multi-objective Transmission Network Planning

### 3.1 Fuzzy connection degree of economics

In the calculation of economic property of transmission network planning, the total investment cost is acquired as follows:

$$F(Z_k) = \sum_{r=1}^{N_B} C_r z_{kr} \quad (4)$$

Where  $Z_k$  is the  $k$ th scheme in the scheme set  $Z$ ;  $C_r$  is the construction cost of each circuit of the branch  $r$ ;  $z_{kr} = 0, 1, \dots, z_r^{\max}$  is the number of circuits to be added to the branch  $r$  in scheme  $k$ ;  $z_r^{\max}$  is the limit number of candidate circuits in branch  $r$ ;  $N_B$  is the number of candidate branches. To describe in brief,  $F(Z_k)$  is simplified as  $F_k$ .

Suppose the best and the worst economics of the network are  $F_U$  and  $F_V$  respectively. As the economics scaled by investment is the cost-type, the value is smaller, the scheme is better. Thus the economics comparison space of the schemes is  $[F_U, F_V]$ . According to equation (3), the fuzzy connection degree of the set pair  $\{F_k, F_U\}$  formed by investment  $F_k$  and  $F_U$  is:

$$\mu_{k1} = a_{k1} + b_{k1}i + c_{k1}j \quad (5)$$

### 3.2 Fuzzy Connection Degree of Reliability

Many indexes can be adopted to evaluate the reliability of the network in transmission network planning. Outage cost in the demand side directly shows the economic expression of the reliability of the power supply, which is also an important factor in deciding the power price in the future. It is adaptable for the desire and the development of electricity market. The formula of outage cost [7] is:

$$R_k = \sum_{r \in S_{LD}} P_r T_r \sum_{i=1}^{N_D} I_{IEAR_i} \times E_{EENS_{i,r}} \quad (6)$$

Where  $R_k$  is the outage cost of scheme  $k$ ;  $N_D$  is the number of load buses in the system (the number of buses in the system is  $N$ );  $S_{LD}$  is the set of load levels;  $P_r$  and  $T_r$  are the probability and the duration of the  $r$ th load level respectively;  $I_{IEAR_i}$  is the interrupted energy assessment rate of the load bus  $i$  with unit of  $\text{¥/kWh}$ . It can be defined as the monetary loss of customers resulting from power supply interruption per kWh;  $E_{EENS_{i,r}}$  (kWh/period) is the expected energy not supplied at load bus  $i$  on the  $r$ th load level, which can be obtained by using equation (7):

$$E_{EENS_{i,r}} = \sum_{q \in S_F} L_{q,r} \prod_{j \in S_h} P_{qj} \prod_{k \in S_H} (1 - P_{qk}) \quad (7)$$

Where  $S_F$  is the contingency set of system;  $S_h$  and  $S_H$  are the fault equipment set and the normal equipment set respectively for contingency  $q$ ;  $P_{qj}$  and  $P_{qk}$  are the fault probability of equipment  $j$  and  $k$  respectively for contingency  $q$ ;  $L_{q,r}$  is the load curtailed at load bus  $i$  for contingency  $q$  on the  $r$ th load level. Calculation of the load curtailed is the key to estimate outage cost, whose process is shown in paper [7].

Suppose the best and the worst outage costs are  $R_U$  and  $R_V$  respectively. Then the comparison space of outage cost of planning schemes is  $[R_U, R_V]$  since the outage cost is the cost-type index. Using equation (3), the fuzzy connection degree of the set pair  $\{R_k, R_U\}$  formed by outage cost  $R_k$  and  $R_U$  is:

$$\mu_{k2} = a_{k2} + b_{k2}i + c_{k2}j \quad (8)$$

### 3.3 Fuzzy Connection Degree of Flexibility

With electricity market gradually open to the demand side, the customers can freely choose power suppliers. The supply and demand relationship between the customers and power plants will change more frequently, and the large-scale and long distance power trade may come into being. These require adequate flexibility of the network to adapt the uncertainties in the future environment.

Available Transfer Capacity (ATC) [8] is the residual capacity in the actual network which is available for

commercial usage on the basis of the existing power contract. It is the maximal power transfer likely to be added among districts or point-to-point under the environment of electricity market, which is the scale of holding transfer capacity in the actual network. ATC is greater, the freedom to choose trade is larger for the market participators, and the relation of supply and demand is more flexible. Therefore ATC can be used to measure the flexibility of the network.

For different demands, various methods can be used to calculate ATC, such as the methods of linear distribution factors, continuous power flow, optimal power flow, etc. The essence of the linear distribution factors method [9] is to analyze the actual network sensitivity factors based on the DC power flow. It can provide a reasonably accurate approximation of ATC to meet the requirements of transmission network planning with fast calculation speed. The overload restriction of line flow and  $N-1$  static security restriction is also considered effectively in this method. The steps of calculating ATC by the method are as follows:

#### (a) Power Transfer Distribution Factors (PTDF's)

Power transfer is to transfer power from source buses to sink buses. The power transfer distribution factor (PTDF),  $S_{ij}$ , indicates the sensitivity of power flow in branch  $ij$  due to unit power transfer increase, which is computed as:

$$S_{ij} = b_{ij} e_{ij} X Q \quad (9)$$

Where  $b_{ij}$  is the susceptance of the branch  $ij$ ;  $e_{ij}$  is a row vector with  $N$  dimensions, which is null except for 1 in column  $i$  and -1 in column  $j$ ;  $X$  is the branch reactance matrix of the system of  $N \times N$ ;  $Q$  is a column vector with  $N$  dimensions, which is null except for proportional injections for unit power transfer defined by the participation factors of the source/sink buses. For source bus,  $Q_i > 0$  and  $\sum_i Q_i = 1.0$ ; for sink bus,  $Q_j < 0$ , and  $\sum_j Q_j = -1.0$ ; and for other bus,  $Q_k = 0$ .

#### (b) Branch power flow margin

Let  $p_{ij}$  be the power flow in branch  $ij$ , and the positive direction is from  $i$  to  $j$ ;  $\bar{p}_{ij}$  is the power flow limit in the positive direction, and  $\underline{p}_{ij}$  is the power flow limit in the negative direction, then the margin of the branch power flow from  $i$  to  $j$  is:

$$M_{ij} = \bar{p}_{ij} - p_{ij} \quad (10)$$

and the margin from  $j$  to  $i$  is:

$$M_{ji} = \underline{p}_{ij} - p_{ij} \quad (11)$$

#### (c) Available Transfer Capacity (ATC)

As a result of overload restriction of power flow rating, each branch  $ij$  has a maximum power transfer value  $T_{ij}$ . The minimum  $T_{ij}$  of all branches in the

network is defined as the Available Transfer Capacity (ATC) in the current system conditions, which can be expressed as follows:

$$T = \min\{T_{ij}\} \quad (12)$$

$$\text{in which, } T_{ij} = \begin{cases} M_{ij}/S_{ij} & S_{ij} > 0 \\ M_{ji}/S_{ij} & S_{ij} < 0 \end{cases}$$

Suppose the best and the worst ATC of the network are  $T_V$  and  $T_U$  respectively. As ATC index is benefit-type, the larger value is the ATC index, the scheme is better. Thus the comparison space of ATC of the schemes is  $[T_V, T_U]$ . According to equation (2), the fuzzy connection degree of the set pair  $\{T_k, T_U\}$  formed by the Available Transfer Capacity  $T_k$  and  $T_U$  of scheme  $k$  is:

$$\mu_{k3} = a_{k3} + b_{k3}i + c_{k3}j \quad (13)$$

### 3.4 Multi-objective Transmission Network Planning Model Based on Fuzzy Set Pair Analysis

Economics, reliability and flexibility indexes make up of multi-objective transmission network optimization planning (MTNOP). The optimization degree depends on the balance of these three parts. In the actual MTNOP, the absolute optimal scheme does not exist. For the convenience of the following text, an ideal optimal scheme  $U=Z(F_U, R_U, T_U)$  is assumed, in which economics, reliability and flexibility reach the best at the same time, then the fuzzy connection degree of set pair  $\{Z_k, U\}$  formed by the scheme  $Z_k$  and  $U$  is:

$$\mu_k = a_k + b_k i + c_k j \quad (14)$$

$$\text{Where } a_k = \sum_{r=1}^3 \omega_r a_{kr}, b_k = \sum_{r=1}^3 \omega_r b_{kr}, c_k = \sum_{r=1}^3 \omega_r c_{kr},$$

$\sum_{r=1}^3 \omega_r = 1$ ; weight  $\omega_1, \omega_2$  and  $\omega_3$  are used to express the relative importance of economics, reliability and flexibility respectively, which are valued according to the actual network and the decisions of the operators.

The values of  $a_k$  and  $c_k$  are relatively certain, which represent positive and negative attitude to  $Z_k$ 's closeness to the ideal optimal scheme  $U$  respectively. In the relative certainty condition, the relative closeness degree of  $Z_k$  to  $U$  is defined as:

$$\gamma_k = \frac{a_k}{a_k + c_k} \quad (15)$$

For the same scheme,  $\gamma_k$  monotonously increases with decreases of the investment cost and the outage cost; while  $\gamma_k$  increases with increment of the ATC. The candidate schemes may be evaluated according to  $\gamma_k$ , in which the one with maximum  $\gamma_k$  has the best

total benefit.

Based on the above analysis, the transmission network planning model with Fuzzy Set Pair Analysis method considering economics, reliability and flexibility is given:

$$\text{Obj. } \max_{k \in Z}(\gamma_k) \quad (16)$$

$$\text{s.t. } B\theta = P_G - P_D \quad (17)$$

$$\underline{p}_{ij} \leq p_{ij} \leq \bar{p}_{ij} \quad i, j \in N \quad (18)$$

Where  $B$  is the bus susceptance matrix of system;  $\theta$  is the bus voltage angle vector;  $P_G$  and  $P_D$  are the bus generation vector and load demand vector respectively.

## 4 Simulation Analysis

The simulation network is a simplified actual system in the western part of China with 18 buses and 27 branches. The network path schematic diagram and the system parameters can be found in [10]. To meet  $N-1$  reliability rule, all candidate branches in the network are supposed to have three candidate circuits. Table 1 lists the economic and reliable parameters. The interrupted energy assessment rates ( $I_{IEAR}$ ) are chosen as ten times of load power price of each bus, which are  $I_{IEAR}=(5, 5, 6.5, 5.5, 6.2, 6.8, 5.2, 5.6, 5.4, 6, 7, 5.8, 6, 6.8, 5, 5.6, 6.2, 6.4)$ ; let the comparison spaces of investment, outage cost and ATC be  $[80000, 120000]$ ,  $[20, 150]$  and  $[100, 2000]$  respectively; suppose all the weights of economics, reliability and flexibility of the scheme are 1/3.

Table 1 Economic and reliable parameters

Parameters	Setting
Line investment cost (1000,000 ¥/km)	0.40
Line fault possibility(occ/yr-km)	0.05
Line fault recovery(yr/occ)	$9.13 \times 10^{-4}$
Load duration(h)	3500

The results of the proposed method in this paper are shown in Table 2, where the column of added circuits represents the number of new-added candidate circuits, for example 13(2) means that 2 candidate circuits need to be added in the 13th branch.

Table 2 shows the optimal solution is scheme 1 by optimizing economics, reliability and flexibility. In scheme 1, the investment cost, the outage cost and the ATC are 1029.2 million yuan, 0.6371 million yuan, and 966.03MW respectively, and the relative closeness degree is 0.549146. If only the investment cost is considered, both scheme 1 and 3 are optimal with the investment cost of 1029.2 million yuan. The difference of the outage cost between scheme 1 and 3 is little, whereas the ATC of scheme 1 have a 65% increase to that of scheme 3. It indicates that with the same

investment scheme 1 is much better adaptable to the uncertain future than scheme 3. When the power transfer increases 2.5 percent, to meet  $N-1$  static security rule, the compensation investment of scheme 2,3 and 4 are 16,28 and 16 million yuan respectively, while no additional investment need to be superadded to scheme 1. Fig. 1 shows the network structure of scheme 1, in which the solid and dashed lines represent the existing and added circuits respectively.

In the electricity market, not only economics and reliability need to be taken into account in the transmission network planning, but also flexibility needs to be involved into the objective functions. In this instance the superaddition of a little compensation

investment may meet the system security requirements when the demand and supply relation varies.

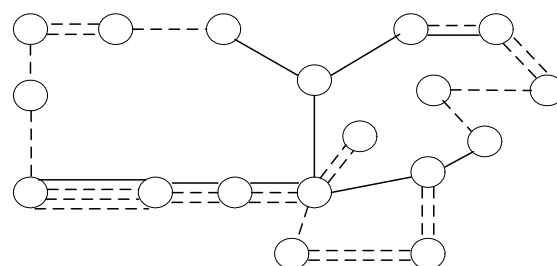


Fig. 1 The optimal planning network of 18-bus system

Table 2 Comparison of Planning Schemes

Scheme	Added circuits	Investment cost (1000,000 ¥)	Outage cost (1000,000 ¥)	ATC (MW)	Relative closeness degree $\gamma_k$
1	1,2(2),7,10,13(2),14(2),16(2),17,18(2),19(3),21,22,25(2),26(2),27	1029.2	0.6371	966.03	0.549146
2	1,2(2),7,10(2),12(2),13(2),14(2),17,18(2),19(3),21,22,25(2),26(2),27	1045.2	0.6609	741.78	0.509351
3	1,2(2),7,10,12,13(2),14(2),16,17,18(2),19(3),21,22,25(2),26(2),27	1029.2	0.6610	587.03	0.487287
4	1,2(2),7,10,12(2),13(3),14(2),18(2),19(3),21,22,25(2),26(2),27,	1046.0	0.5587	193.44	0.392080

## 5 Conclusions

Fuzzy Set Pair Analysis (FSPA) method is adopted to solve the multi-objective optimizing problems in transmission network planning, and a novel model is presented. It is verified by an actual example, and the conclusions are as follows:

(1) Using FSPA method, economics, reliability and flexibility of transmission network planning are integrated. The optimal scheme with best total benefit is achieved, which meets the actual demands of transmission network planning division.

(2) The planning scheme is assessed independently by two opposite pairs: certainty- uncertainty and identity-contrary, by FSPA method, which makes the evaluation more comprehensive and provides the planners more information for decision-making.

(3) The FSPA method has strong expansibility. New objectives can be easily added, and this method is adaptable to seek the optimal solution with meta-heuristic algorithms.

(4) Available Transfer Capability (ATC) is proposed to quantify the flexibility of the network for the first time, which improves adaptability of optimal scheme to the future uncertainties and avoids unnecessary compensation investment.

(5) Relative closeness degree is used to evaluate the scheme. The concept is clear and the optimal solution is easy to be achieved by programming in the computer.

(6) Multi-objective transmission network planning is

a mixed-integer nonlinear optimization problem, which causes the convex characteristic of objective function hard to reach. In general, the absolute optimal solution of a fixed multi-objective optimization model does not exist; however, a set of available solutions can be established. Therefore, it is a key decision to seek the best satisfaction solution from the available solution set for the multi-objective planning problem.

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