Expantion of reliability models based on Markov Chain with consideration of Fuzzy Failure rates: System with two parallel and identical elements with

increasable failure rate.

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Abstract:

Reliability Models based on Markov Chains (except Queuing models) have extensive application in reliability of electrical and electronically equipments. In this article, a system with two parallel and identical elements with increasable failure rate is analyzed and the results are in consideration of failure rates in of triangular fuzzy numbers. Also, to reach a triangular fuzzy number related to failure rate the $\%(1-\beta)$ confidence interval of increasable failure rate is used.(λ of raily distribution).

1) Introduction:

Nowadays, Reliability models are considered one of the most important applications of Markov Chains, and most electronic systems come across these models.

This is an extensive system and for every electrical system a specific model is designed and implemented and in these models Failure Rate is definite.

Since these failure rates are driven from gathering data and usage of probability distribution functions, or the opinions of the experts on the matter, uncertainty is also, an obvious parameter.

Hence, in this article, one particular system with two parallel elements is reviewed and the results are in consideration of increasable failure rate (λt).

To demonstrate the uncertainty in calculation of failure rate, these parameters are estimated through a triangular fuzzy number and to calculate the parameters of this fuzzy number, the $\%(1-\beta)$ confidence interval [1] of this parameter witch is driven in respect to χ^2 p.d.f is used. Therefore, by solving one numeric sample while all the system elements are identical, comparing the driven results with the crisp model results indicates a more realistic and practical result than crisp condition

2) Notifications:

The notification that used in this article is as followed:

 λ_i : Failure rate of *i* element.

 L_i : Lower limit of triangular fuzzy number related to failure rate of *i* element.

 M_i : Medium of triangular fuzzy number related to failure rate of *i* element.

 U_i : Upper limit of triangular fuzzy number related to failure rate of *i* element.

 $P_i(t)$: Probability of the system at the *t* moment to be in condition *i*.

 $R_{P}(t)$: Probability of the functionality of the system.

MTTF : Mean time to failure.

3) Introduction to the discussed sample:

In this article, a system working with two parallel elements is considered. Assuming that the system will stop working when both elements have malfunctioned, we can consider the following four conditions for the system:

State	Condition of first	Condition of	
	part	second part	
1	Working	Working	
2	Not working	Working	
3	Working	Not working	
4	Not working	Not working	

Table 1 : States of the system

and the flow diagram for this system will be as follow:



Fig 1: States of the system

in respect to the above descriptions, the probability of the system functioning is calculated as followed

$$R_P(t) = P_1(t) + P_2(t) + P_3(t)$$
(01)

We also know that:

$$P_1(t) + P_2(t) + P_3(t) + P_4(t) = 1 \tag{02}$$

In this system, the purpose is to find $P_i(t)$, i = 1,2,3. For the nods 1 through 3 in figure 1 we have:

$$P_{1}(t + \Delta t) = P_{1}(t) - \lambda_{1} t \Delta t P_{1}(t) - \lambda_{2} t \Delta t P_{1}(t)$$
(03)

$$P_2(t + \Delta t) = P_2(t) + \lambda_1 t \Delta t P_1(t) - \lambda_2 t \Delta t P_2(t)$$

$$(04)$$

$$P_2(t + \Delta t) = P_2(t) + \lambda_1 t \Delta t P_1(t) - \lambda_2 t \Delta t P_2(t)$$

$$(05)$$

$$P_3(t + \Delta t) = P_3(t) + \lambda_2 t \Delta t P_1(t) - \lambda_1 t \Delta t P_3(t)$$
(05)

and by solving the equations (03) through (05) we can calculate the values $P_i(t)$, i = 1,2,3 as followed:

$$P_1(t) = e^{-\frac{1}{2}(\lambda_1 + \lambda_2)t^2}$$
(05)

$$P_2(t) = e^{-\frac{1}{2}\lambda_2 t^2} - e^{-\frac{1}{2}(\lambda_1 + \lambda_2)t^2}$$
(06)

$$P_3(t) = e^{-\frac{1}{2}\lambda_1 t^2} - e^{-\frac{1}{2}(\lambda_1 + \lambda_2)t^2}$$
(07)

$$R_P(t) = e^{-\frac{1}{2}\lambda_1 t^2} + e^{-\frac{1}{2}\lambda_2 t^2} - e^{-\frac{1}{2}(\lambda_1 + \lambda_2)t^2}$$
(08)

and the system MTTF is also calculated as followed:

$$MTTF = \int_{0}^{+\infty} R_P(t) dt = \sqrt{\frac{2\pi}{\lambda_1}} + \sqrt{\frac{2\pi}{\lambda_2}} + \sqrt{\frac{2\pi}{(\lambda_1 + \lambda_2)}}$$
(09)

4) Calculations with respect to Fuzzy failure Rates:

Most important consideration is that the Values of λ_i , i = 1,2 are not fixed. Since they are driven from collected data or the opinions of the experts, uncertainty of the value is an undeniable fact.

Most of the times, these failure rates considered as a known value or have a known distributions function.

In this article we assume that the failure rates have raily p.d.f. Of course in this condition, failure rates, $\lambda_i i = 1,2$ is considered in the form of a Triangular Fuzzy Number as follows:

$$\overline{\lambda_i} = \left(L_i / M_i / U_i\right) \qquad i = 1,2 \tag{10}$$

In which L_i are the Lower Limits of $\%(1-\beta)$ confidence interval, M_i are the Point estimations, and U_i are the upper limits of $\%(1-\beta)$ confidence interval of λ_i .

The α -cut of these failure rates can be calculated as follow:

$$\overline{\lambda_i}[\alpha] = [L_i + \alpha (M_i - L_i), U_i - \alpha (U_i - M_i)] \qquad i = 1, 2 \qquad (11)$$

Now we can calculate the $P_i(t)$, i = 1,2,3 in Fuzzy condition by using extension principle [2].

Assume $\overline{P_i}[t,\alpha] = [P_{i1}(t,\alpha), P_{i2}(t,\alpha)], i = 1,2,3$ therefore we will have [3]:

$$\overline{P}_{1}[t,\alpha] = [P_{11}(t,\alpha), P_{12}(t,\alpha)]$$

$$P_{11}(t,\alpha) = e^{-\frac{1}{2}[(U_{1}+U_{2})-\alpha(U_{1}+U_{2}-M_{1}-M_{2})]t^{2}}$$

$$P_{12}(t,\alpha) = e^{-\frac{1}{2}[(L_{1}+L_{2})+\alpha(M_{1}+M_{2}-L_{1}-L_{2})]t^{2}}$$
(12)

$$\overline{P}_{2}[t,\alpha] = [P_{21}(t,\alpha), P_{22}(t,\alpha)]$$

$$P_{21}(t,\alpha) = e^{-\frac{1}{2}[U_{2}-\alpha(U_{2}-M_{2})]t^{2}} - e^{-\frac{1}{2}[(L_{1}+L_{2})+\alpha(M_{1}+M_{2}-L_{1}-L_{2})]t^{2}}$$

$$P_{22}(t,\alpha) = e^{-\frac{1}{2}[L_{2}+\alpha(M_{2}-L_{2})]t^{2}} - e^{-\frac{1}{2}[(U_{1}+U_{2})-\alpha(U_{1}+U_{2}-M_{1}-M_{2})]t^{2}}$$
(13)

$$\overline{P}_{3}[t,\alpha] = [P_{31}(t,\alpha), P_{32}(t,\alpha)]$$

$$P_{31}(t,\alpha) = e^{-\frac{1}{2}[U_{1}-\alpha(U_{1}-M_{1})]t^{2}} - e^{-\frac{1}{2}[(L_{1}+L_{2})+\alpha(M_{1}+M_{2}-L_{1}-L_{2})]t^{2}}$$

$$P_{32}(t,\alpha) = e^{-\frac{1}{2}[L_{1}+\alpha(M_{1}-L_{1})]t^{2}} - e^{-\frac{1}{2}[(U_{1}+U_{2})-\alpha(U_{1}+U_{2}-M_{1}-M_{2})]t^{2}}$$
(14)

$$\overline{R}_{p}[t,\alpha] = [R_{p_{1}}(t,\alpha), R_{p_{2}}(t,\alpha)]$$

$$R_{p_{1}}(t,\alpha) = P_{1_{1}}(t,\alpha) + P_{2_{1}}(t,\alpha) + P_{3_{1}}(t,\alpha) =$$

$$e^{-\frac{1}{2}[(U_{1}+U_{2})-\alpha(U_{1}+U_{2}-M_{1}-M_{2})]t} - 2e^{-\frac{1}{2}[(L_{1}+L_{2})+\alpha(M_{1}+M_{2}-L_{1}-L_{2})]t^{2}}$$

$$+ e^{-\frac{1}{2}[U_{1}-\alpha(U_{1}-M_{1})]t^{2}} + e^{-\frac{1}{2}[U_{2}-\alpha(U_{2}-M_{2})]t^{2}}$$

$$R_{p_{2}}(t,\alpha) = P_{1_{2}}(t,\alpha) + P_{2_{2}}(t,\alpha) + P_{3_{2}}(t,\alpha) =$$

$$e^{-\frac{1}{2}[(L_{1}+L_{2})+\alpha(M_{1}+M_{2}-L_{1}-L_{2})]t^{2}} - 2e^{-\frac{1}{2}[(U_{1}+U_{2})-\alpha(U_{1}+U_{2}-M_{1}-M_{2})]t^{2}}$$

$$+ e^{-\frac{1}{2}[L_{1}+\alpha(M_{1}-L_{1})]t^{2}} + e^{-\frac{1}{2}[L_{2}+\alpha(M_{2}-L_{2})]t^{2}}$$
(15)

Assume $\overline{MTTF}_1[\alpha] = [MTTF_1(\alpha), MTTF_2(\alpha)]$ therefore we will have:

$$MTTF_{1}(\alpha) = \int_{0}^{+\infty} R_{P1}(t,\alpha)dt = \sqrt{\frac{2\pi}{(U_{1}+U_{2})-\alpha(U_{1}+U_{2}-M_{1}-M_{2})}} + \sqrt{\frac{2\pi}{U_{1}-\alpha(U_{1}-M_{1})}} + \sqrt{\frac{2\pi}{U_{2}-\alpha(U_{2}-M_{2})}} - 2\sqrt{\frac{2\pi}{(L_{1}+L_{2})+\alpha(M_{1}+M_{2}-L_{1}-L_{2})}} + (16)$$

$$MTTF_{2}(\alpha) = \int_{0}^{+\infty} R_{P2}(t, \alpha) dt =$$

$$\sqrt{\frac{2\pi}{(L_{1} + L_{2}) - \alpha(M_{1} + M_{2} - L_{1} - L_{2})}}$$

$$+ \sqrt{\frac{2\pi}{L_{1} + \alpha(M_{1} - L_{1})}} + \sqrt{\frac{2\pi}{L_{2} + \alpha(M_{2} - L_{2})}}$$

$$- 2 \times \sqrt{\frac{2\pi}{(U_{1} + U_{2}) + \alpha(U_{1} + U_{2} - M_{1} - M_{2})}}$$
(17)

and to calculate the Fuzzy Number related to the mean time to failure of system (\overline{MTTF}) we have:

 $\overline{MTTF} = (A/B/C) \tag{18}$

$$A = MTTF_{1}(0) = \left(2\sqrt{2} + 1\right)\sqrt{\frac{\pi}{U}} - 2\sqrt{\frac{\pi}{L}}$$
(19)

$$B = MTTF_{1}(1) = MTTF_{2}(2) = \left(2\sqrt{2} - 1\right)\sqrt{\frac{\pi}{M}}$$
(20)

$$C = MTTF_2(0) = \left(2\sqrt{2} + 1\right)\sqrt{\frac{\pi}{L}} - 2\sqrt{\frac{\pi}{U}}$$

$$\tag{21}$$

Now assuming that both elements are identical and because Fuzzy rates of these Elements have exponential distribution with unknown parameter (λ), the point estimation and %(1- β) confidence intervals (two ways) of this parameter are:

$$M = \frac{1}{X^2} \tag{22}$$

$$L = \frac{\chi^{2}_{(1-\beta/2),(2n_{i})}}{n \times X^{2}}$$
(23)

$$U = \frac{\chi^{2}_{(\beta/2),(2n_{i})}}{n \times \overline{X}^{2}}$$
(24)

In here

$$\overline{X^{2}} = \sum_{i=1}^{n} X_{i}^{2}$$
(25)

Now, by plugging the driven values from equations (22) to (24) in equations (19) to (21) we will have:

$$A = MTTF_{1}(0) = \left(2\sqrt{2} + 1\right) \sqrt{\frac{\pi \times n \times \overline{X^{2}}}{\chi^{2}_{(\beta/2),(2n)}}} - 2\sqrt{\frac{\pi \times n \times \overline{X^{2}}}{\chi^{2}_{(1-\beta/2),(2n)}}}$$
(26)

$$B = MTTF_{1}(1) = MTTF_{2}(2) = \left(2\sqrt{2} - 1\right)\sqrt{\pi \times \overline{X^{2}}}$$
(27)

$$C = MTTF_2(0) = \left(2\sqrt{2} + 1\right) \sqrt{\frac{\pi \times n \times \overline{X^2}}{\chi^2_{(1-\beta/2),(2n)}}} - 2\sqrt{\frac{\pi \times n \times \overline{X^2}}{\chi^2_{(\beta/2),(2n)}}}$$
(28)

At the end, we defuzzy \overline{MTTF} and then we will have:

$$MTTF = \left(A/B/C\right) = \left(\frac{2\sqrt{2}-1}{3}\right) \sqrt{\frac{\pi \times n \times \overline{X^2}}{\chi^2_{(\beta/2),(2n)}}} + \left(\frac{2\sqrt{2}-1}{3}\right) \sqrt{\frac{\pi \times n \times \overline{X^2}}{\chi^2_{(1-\beta/2),(2n)}}} + \left(\frac{2\sqrt{2}-1}{3}\right) \sqrt{\pi \times \overline{X^2}}$$
(29)

5) Numeric Sample

Assumptions based on an independent example with volume n = 25, failure rate is estimated at $\overline{X^2} = 920$,

therefore $\overline{\lambda} = 0.001087$. In the example in the crisp condition, the results are clocked at *MTTF* = 98.2970, in table 2.

α	Α	В	С	MTTF
0.0010	0.0000	98.2970	155.6540	84.6503
0.0020	2.3166	98.2970	149.4235	83.3457
0.0025	3.8291	98.2970	147.3830	83.1697
0.0050	8.6812	98.2970	140.9160	82.6314
0.0075	11.6434	98.2970	137.0276	82.3227
0.0100	13.8098	98.2970	134.2132	82.1066
0.0200	19.2893	98.2970	127.2053	81.5972
0.0250	21.1444	98.2970	124.8691	81.4368
0.0500	27.2610	98.2970	117.2977	80.9519
0.0750	31.1446	98.2970	112.5955	80.6790
0.1000	34.0730	98.2970	109.1038	80.4913
0.1500	38.5025	98.2970	103.9105	80.2367
0.2000	41.9110	98.2970	99.9863	80.0648
0.2500	44.7445	98.2970	98.2970	80.4461
0.3000	47.2081	98.2970	98.2970	81.2673
0.3500	49.4143	98.2970	98.2970	82.0028
0.4000	51.4321	98.2970	98.2970	82.6754
0.4500	53.3070	98.2970	98.2970	83.3003
0.5000	55.0709	98.2970	98.2970	83.8883

Table 2: The value of A, B, C and the MTTF

Values of \overline{MTTF} are calculated and presented based on the values of β . Also in this table, where the values of the lower limits of the Fuzzy Triangular Number were negative (column A), they were replaced with 0, and where the values of the upper limit is lower than B, we replaced it with B.

6- Conclusion, and Further research

Since we can see in figure 2, the system did not works more stable than system function in the crisp condition, but, because of using fuzzy failure rates, the system condition is more realistically than crisp condition, and these are the advantages of this model.



Fig2: The value of A, B, C and the MTTF

The system discussed in this article is one of hundreds of the actual existing systems that have already been produced based on definite parameters.

By considering any of the existing systems and by changing the p.d.f of failure rate of each element, we can drive different values and compare the results with each other.

7-Refrences:

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