Frequency Domain Magnitude Banded LMS Algorithm for Equalization of Rapidly Time Variant Channels

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Abstract: - **Frequency domain adaptive filtering has become increasingly popular due to its computational efficiency and excellent adaptation behavior. This paper proposes a new frequency domain based magnitude banded least mean-square(FDMBLMS) algorithm for the purpose of equalization of a rapidly time-variant channel. FDMBLMS implements a non-linear adaptation procedure based on the magnitude level of the Fourier transform of the channel output. Computer simulation results obtained by using a second order Markov communication channel model shows that the proposed FDMBLMS algorithm provides a significant performance improvement compared to the standard frequency domain LMS as well as the existing time domain uniform amplitude banded LMS algorithms.**

Keywords: - Time variant channel, LMS algorithm, Uniform ABLMS algorithm, FDLMS algorithm, FDMBLMS algorithm.

1. Introduction

Rapid time variation and multipath fading are the most serious problems affecting the reliability of communication systems such as mobile radio channels and high frequency(HF) channels. Due to the phenomenon of time variation these channels suffer from intersymbol interference(ISI). To compensate for channel distortions which cause ISI in communication systems, adaptive equalization techniques can be used [1].

The most common adaptive equalization method is based on the least-mean-square(LMS) algorithm[2] which updates the filter coefficients by a gradient based method in the time domain.

For the purpose of time variant channel estimation, Clark et.al[3] and Mclaughlin et.al[4] have shown that the LMS algorithm is suitable in a transversal filter structure due to its cost effectiveness.

Recently, Shimamura et.al[5], [6] derived a new LMS based nonlinear adaptive algorithm, called amplitude banded LMS(ABLMS) algorithm, which takes into account the amplitude information of a time-variant channel output in the coefficient adaptation process of the equalizer. It was shown that if the channel output is corrupted by white Gaussian additive noise uncorrelated with the input signal and when a second order Markov communication channel model is used, the ABLMS algorithm exhibits better performance than the conventional LMS algorithm for the purpose of equalization of time-variant multipath channels with only a small increase in computational complexity.

On the other hand, it is known that by implementing adaptive filtering in the frequency domain, great improvements in the convergence rate over the conventional time domain approach are achieved[7]. Moreover in [8] various types of the transform-domain LMS algorithm were used in tracking a class of time-varying plants.

In this paper, an ABLMS-like algorithm called frequency domain magnitude banded LMS(FDMBLMS) algorithm is proposed. Our motivation for considering such an adaptive algorithm is to retain the relative simplicity and better bit error rate(BER) performance the ABLMS algorithm has provided over the conventional LMS algorithm and yet to incorporate some of the benefits of the computational efficiency and faster tracking performance gained by frequency domain adaptation of equalizer coefficients. The resulting FDMBLMS algorithm is verified to be more robust in tracking rapidly time-varying channels than both the original ABLMS and the standard frequency domain LMS algorithms.

In the next section, the basic concepts of frequency domain adaptive algorithm are introduced. Section 3 discusses in detail the configuration and the formulation of the proposed FDMBLMS algorithm. Section 4 presents the experimental results obtained by computer simulations using a second order Markov communication channel model. Section 5 serves as the conclusion of the paper by summarizing the results of the proposed method.

2. Frequency Domain Adaptive Algorithm

Frequency domain adaptive filtering can be performed by Fourier transforming the input-signal vector and weighting the contents of each frequency bin[7],[9]. Figure 1 is a symbolic representation of the frequency domain adaptive filter.

The input signal is filtered by a bank of bandpass filters, implemented digitally by the discrete Fourier transform(DFT).

The vector Z_n

$$Z_n = [z_{n0}, z_{n1}, ..., z_{n(N-1)}]^T \tag{1}$$

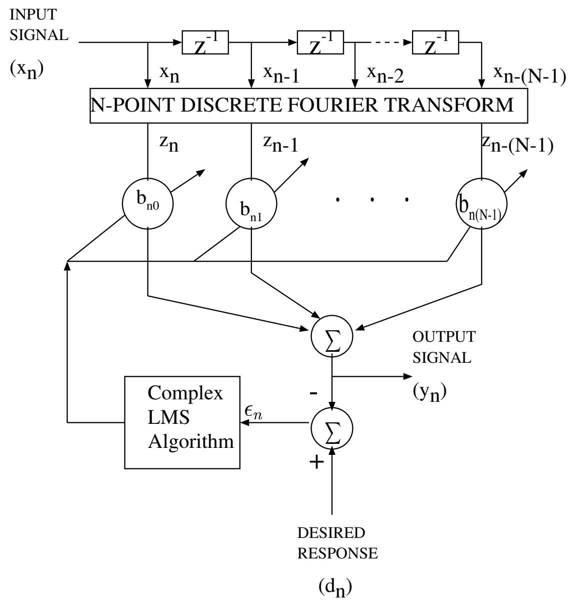

Figure 1: Symbolic representation of the frequency domain adaptive filter.

is related to the input vector X_n by the orthogonal transformation:

$$Z_n = WX_n. \tag{2}$$

where W is an $N \times N$ DFT matrix whose (p,q)th element is $\exp(-j2\pi pq/N)$. The output and the corresponding error signal are

$$y_n = Z_n^T B_n \tag{3}$$

and

$$\epsilon_n = d_n - y_n \tag{4}$$

respectively, where

$$B_n = [b_{n0}, b_{n1}, ..., b_{n(N-1)}]^T \tag{5}$$

is the frequency domain weight vector. The complex LMS algorithm[10] is used to recursively update the weight vector B_n. The weight vector update equation is

$$B_{n+1} = B_n + 2\mu\Lambda^{-2}\epsilon_n \bar{Z}_n \tag{6}$$

where μ is the adaptive step size and Λ^2 is $N \times N$ diagonal matrix whose(i,i)th element is equal to the power estimate of the ith DFT output z_{ni}. Equations (3), (4) and (6) provide the fundamental adaptation concept of FDLMS algorithm.

3. The Proposed Method

3.1 Channel Model

The channel model assumed in this paper is given by

$$x_n = \sum_{i=0}^{L} h_i(n)u_{n-i} + v_n \tag{7}$$

where $h_o(n), h_1(n), ..., h_L(n)$ are the channel coefficients, u_n is the transmitted sequence, and v_n is a white Gaussian noise uncorrelated with u_n. The channel output x_n becomes the input for the equalizer.

3.2 FDMBLMS Algorithm

In implementing the FDLMS algorithm, the vector recursion(6) can be decomposed into the following N scalar recursions[11].

$$B_i(n+1) = B_i(n) + 2\frac{\mu}{\hat{\sigma}^2_{z,i}(n)} e(n)z_i(n), i = 0, 1, ..., N-1 \tag{8}$$

where

$$Z(n) = DFT[X(n)] \tag{9}$$

and $\hat{\sigma}^2_{z,i}(n)$ is an estimate of $E[z_i^2(n)]$. The $\hat{\sigma}^2_{z,i}(n)$s are the estimates of the signal powers at various taps of the filter and can be obtained using the recursion:

$$\hat{\sigma}^2_{z,i}(n) = \beta\hat{\sigma}^2_{z,i}(n-1) + (\beta-1)z_i^2(n), i = 0, 1, ..., N-1 \tag{10}$$

where β is a positive constant close to but less than one.

When $\mathbf{b}(n)$ and $\mathbf{z}(n)$ are given by $\mathbf{b}(n) = (b_0(n), b_1(n), ..., b_{M-1}(n))^T$ and $\mathbf{z}(n) = (z_n, z_{n-1}, , , , z_{n-M+1})^T$, respectively, Equation(8) provides the FDLMS adaptation procedure for an M length LTE.

For the frequency domain magnitude banded algorithm to be proposed here, in the case of an LTE, a Q by M coefficient matrix $\mathbf{B_a}(n)$ is prepared, elements of which are given by $b_{ij}(n), i = 1, 2, ..., Q, j = 1, 2, ..., M$. The $\mathbf{B_a}(n)$ is initialized at $n = 0$ where all the elements are set to zero. For the adaptation of the algorithm, the elements of $\mathbf{B_a}(n)$ are updated based on the operation of switching the elements to be updated; in such a fashion that among the Q by M elements of $\mathbf{B_a}(n)$, only M elements, $b_{q(j)j}(n), j = 1, 2, ..., M$, are selected for each iteration and a coefficient vector is formed as $\mathbf{b_a}(n) = (b_{q(1)1}(n), b_{q(2)2}(n), ..., b_{q(M)M}(n))^T$ where $q(j)$ is an integer and determined based on the magnitude level of each element z_{n-j+1} of the vector $\mathbf{z}(n)$ for $j = 1, 2, ..., M$ as follows:

- if $|z_{n-j+1}| \le Z_{max}/Q$, then $q(j) = 1$.
- if $Z_{max}/Q < |z_{n-j+1}| \le 2Z_{max}/Q$, then $q(j) = 2$.
- if $2Z_{max}/Q < |z_{n-j+1}| \le 3Z_{max}/Q$, then $q(j) = 3$.
- .
- .
- if $(Q-1)Z_{max}/Q < |z_{n-j+1}|$, then $q(j) = Q$.

The Z_{max} denotes the maximum magnitude of the frequency domain equivalent of the received sequence and Q corresponds to a division number to classify the magnitude level of the frequency domain equivalent of the received sequence. The output of the filter whose coefficient vector is $\mathbf{b_a}(n)$ is obtained by convolution between $\mathbf{b_a}(n)$ and $\mathbf{z}(n)$. Thus, the coefficient vector is also updated by the FDLMS algorithm (8), but with $\mathbf{b}(n)$ replaced by $\mathbf{b_a}(n)$. This algorithm is what we regard as the FDMBLMS algorithm throughout this paper.

To summarize, in the proposed method, the whole magnitude band of the frequency response of the channel output is first uniformly divided in to small classes in such a manner that

$$A_2 - A_1 = A_3 - A_2 = ... = A_Q - A_{Q-1} = Z_{max}/Q \tag{11}$$

where A_1, A_2, ..., A_Q stand for each division class. Then, only the coefficients which correspond to the frequency bins in the chosen class of the divided band are selectively updated instead of the whole coefficient matrix.

The FDMBLMS algorithm would update all the elements of the coefficient matrix $\mathbf{B}(n)$, because the input sequence is statistically distributed on a time variant channel. For each iteration, the FDMBLMS algorithm has M coefficients to be updated in the same manner as the standard FDLMS algorithm has. Therefore, the computational complexity of the FDMBLMS algorithm is quite comparable with that of the standard FDLMS algorithm.

The coefficient selection for each iteration in the FDMBLMS algorithm is based on the magnitude information of the DFT of the received sequence. Since we can determine the pattern of the channel impulse response from the magnitude response as well as the phase response of the Fourier transform of channel output, we can easily see that the magnitude of the frequency response of the received sequence is directly associated with the channel coefficients. Whenever the frequency transform of the received sequence, z_n, is allocated to one particular range among the Q ranges based on its magnitude level, only the coefficient corresponding to that range is always selected and updated in the FDMBLMS algorithm. Due to this fact, the time variation influence from the channel for the coefficient corresponding to that range may be decreased approximately by a factor of Q. This results in an adaptation where the influence of time variation of the channel is proportionally alleviated by a factor of Q. Therefore, even though all the elements of the coefficient matrix in the FDMBLMS algorithm are not updated for each iteration, the coefficients being selected and updated for each iteration are strongly related with the previously updated coefficients for each banded range. As a result, the number of updates for the coefficient corresponding to each banded range is decreased, and the convergence speed of the adaptive algorithm does not deteriorate[6].

3.3 Parallel Adaptation

Figure 2 shows a block diagram of the FDMBLMS equalizer in which the training mode is assumed (a delayed transmitted sequence is given at the receiver side). The FDMBLMS equalizer consists of two linear transversal equalizers whose coefficients are updated by the FDMBLMS and FDLMS algorithms, respectively. LTE(I) and LTE(II) are two linear transversal equalizers with the same filter length.

The superior tracking performance of the FDMBLMS algorithm might not be always guaranteed for all the adaptation process, due to

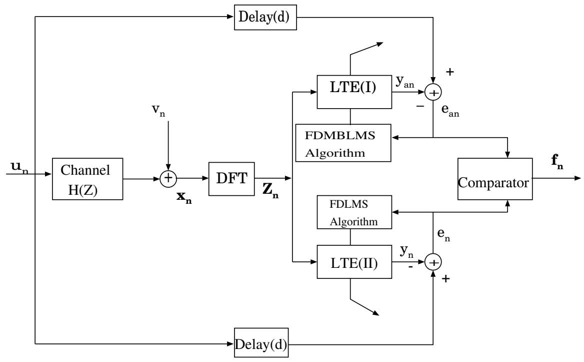

Figure 2: Configuration of the proposed FDMBLMS LTE.

the non-linearity which the magnitude banded technique inherently has. Magnitude ambiguities sometimes cause an unstable phenomenon which appear as "spikes" in the mean square error convergence. This is possibly because the magnitude response of the channel output can not be perfectly and uniquely associated with the channel coefficients. For example, it is possible that different channel coefficient pairs might provide with same magnitude response channel outputs. Therefore, a parallel adaptation scheme is here utilized to at least retain the performance of the conventional FDLMS in case of sudden instability. The two LTEs are individually updated based on the error sequences e_{an} and e_n, respectively. The comparator provides $f_n = e_{an}$ if $(e_{an})^2 \leq (e_n)^2$ and $f_n = e_n$ otherwise. Based on the comparator output, the FDMBLMS-FDLMS LTE outputs y_{an} when $f_n = e_{an}$, and y_n when $f_n = e_n$. As the FDMBLMS-FDLMS LTE requires a parallel adaptation of the FDMBLMS algorithm with the FDLMS algorithm, the result is that the whole computational complexity for implementing the FDMBLMS-FDLMS LTE is approximately twice of that required for implementing the FDLMS LTE. However, with VLSI digital processors having increased computational resourses becoming cheaper and readily available, the benefit of the proposed parallel structure is far more advantageous.

4. Performance Evaluation on a Time Variant Channel

The performance of the proposed FDMBLMS LTE equalizer shown in Figure 2 was investigated using a second order Markov communication channel model. The channel is given by:

$$H(z) = h_0(n) + h_1(n)z^{-1} + h_2(n)z^{-2} \qquad (12)$$

where the time variant coefficients, $h_0(n)$, $h_1(n)$ and $h_2(n)$ are generated by passing Gaussian white noise at 2400 sample/s through second order Butterworth filters with 3 dB bandwidths on the order of the fade rate. The input sequence of this channel is an uncorrelated, pseudo-random sequence with values of $+1$ or -1. This channel is an HF channel model $H_3(z)$ used in [12].

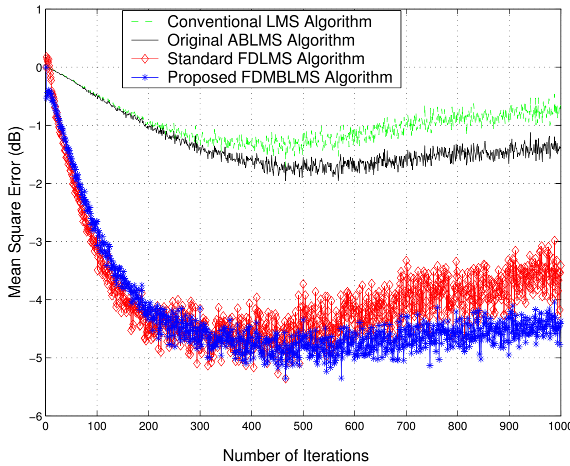

Figure 3: Comparison of MSE convergence at $Q = 2$ and channel fade rate $fd = 5Hz$.

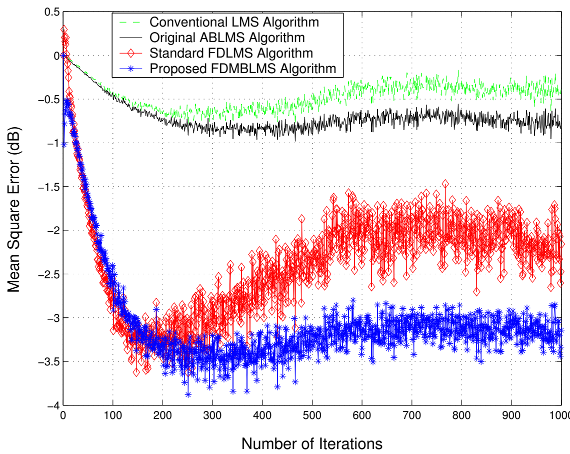

Figure 4: Comparison of MSE convergence at $Q = 4$ and channel fade rate $fd = 10Hz$.

Figures 3 shows mean square error(MSE) convergence plots for the conventional LMS(dash line), original ABLMS(solid line), standard

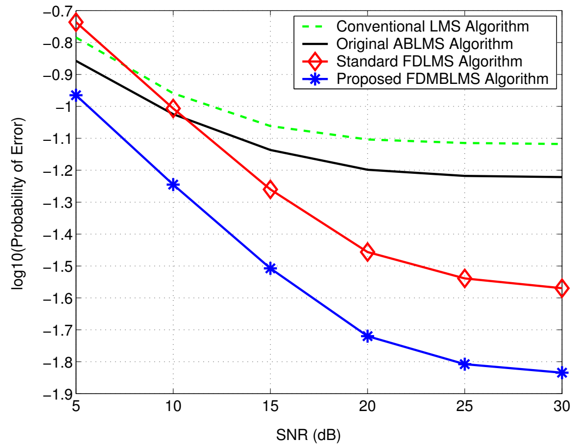

Figure 5: Comparison of BER performance at a channel fade rate of $5Hz$ for $Q = 4$.

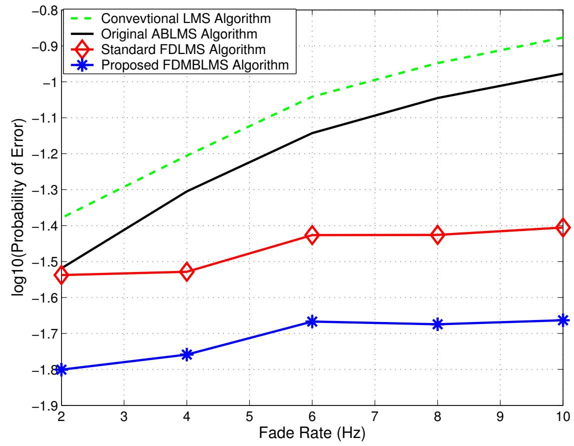

Figure 6: Comparison of BER performance at $SNR = 20dB$ for $Q = 4$.

FDLMS(solid line with diamond) and the proposed FDMBLMS(solid line with asterisk) algorithms at a division number $Q = 2$ and a channel fade rate $fd = 5Hz$. $Q = 2$ implies division of the whole magnitude band into 2 small classes. Figure 4 shows MSE convergence plots for the conventional LMS(dash line), original ABLMS(solid line), standard FDLMS(solid line with diamond) and the proposed FDMBLMS(solid line with asterisk) algorithms at a channel model with a fade rate $fd = 10Hz$ and $Q = 4$ when dividing the whole magnitude band into 4 small classes. A training sequence of 1000 data samples with an additive noise of $SNR = 20$ dB were used. The equalizers have a filter length of $M = 10$ and a delay $d = 5$ and the constant parameters have been commonly set to $\mu = 0.005$ and $\beta = 0.9$.

From Figures 3 and 4 we can observe that the proposed FDMBLMS algorithm is more convergent than the conventional LMS, original ABLMS and standard FDLMS algorithms. It also retains the lowest steady state MSE value. This implies that the proposed FDMBLMS provides the best tracking capacity of rapidly time varying channels among the four algorithms.

Figure 5 illustrates the BER performance at $Q = 4$ for the conventional LMS(dash line), original ABLMS(solid line), standard FDLMS(solid line with diamond) and the proposed FDMBLMS(solid line with asterisk) algorithms against additive noise on a channel model with a channel fade rate of $fd = 5Hz$.

Figure 6 illustrates the BER performance at $Q = 4$ for the conventional LMS(dash line), original ABLMS(solid line), standard FDLMS(solid line with diamond) and the proposed FDMBLMS(solid line with asterisk) algorithms against channel fade ratefd on a channel model corrupted with an additive noise of $SNR = 20dB$.

The equalizers used in Figures 5and 6 have a filter length of $M = 6$ and a delay $d = 3$ both of which provided the best performance for the filter structure used. The step size parameters have been optimized to $\mu = 0.5$ for the conventional LMS and the original ABLMS algorithms while it was optimized to $\mu = 0.08$ in case of the standard FDLMS and the proposed FDMBLMS algorithms. The power normalization factor $\beta = 0.9$ and 50,000 data samples have been commonly used in the training mode.

From Figures 5 and 6 it is clear that the BER performance of the proposed FDMBLMS algorithm is significantly better than those of the conventional LMS, original ABLMS algorithm and the standard FDLMS algorithms. Moreover, from Figure 6, we observe that the proposed algorithm is highly robust against increase in fade rate. This implies that the proposed FDMBLMS has the best capability of compensating for ISI under even severe fading channel conditions.

5. Conclusion

A novel magnitude division frequency domain LMS algorithm known as FDMBLMS algorithm has been proposed for the purpose of equalization of rapidly time-variant multipath channels. Observation of simulation results have demonstrated that the proposed FDMBLMS algorithm has highly outsmarted the conventional LMS-LTE, the original ABLMS as well as the standard FDLMS algorithms in convergence and tracking performances.

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