A Novel Frequency-Domain Independent Component Analysis Approach for Wireless Communications

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Abstract: - In this paper, a novel Frequency-Domain Independent Component Analysis (ICA-F) approach is proposed to blindly separate and deconvolve the convolutive combinations of digitally modulated signals in wireless communications. This approach relies on the simple observation that if signals are independent in one domain, their corresponding components in a linearly transformed domain are also independent. The proposed ICA-F lends itself to computationally efficient Fast Fourier Transform (FFT) implementation, which converts the convolutive combination in the time domain into multiple instantaneous combinations in the frequency domain. Then, the natural-gradient Independent Component Analysis (ICA) algorithm is employed in each frequency bin to the separate frequency components of source signals. The permutation and gain ambiguities associated with the ICA algorithm are successfully solved. The ICA-F has lower computational complexity and faster convergence than the existing time-domain approach. Simulation results confirm the effectiveness of the proposed ICA-F.

Key-Words: - Blind Equalizer, Blind Source Separation, Constant Modulus Algorithm, Gain Ambiguity, Independent Component Analysis, Permutation Ambiguity, Short-Time Fourier Transform

1 Introduction

Blind Source Separation (BSS) is one of the most prominent research areas with numerous potential Independent Component Analysis applications. (ICA) is the most widely used methodology to In ICA, the source signals are perform BSS. extracted from the received signals, which are the unknown combinations of the source signals [1]. If the source signals are combined instantaneously, an instantaneous ICA algorithm can be directly employed to separate the received signals. In numerous practical situations, such as digitally modulated signals traveling through frequency-selective, slow fading channels in wireless communications, the received signals are the convolutive combinations of the source signals. Many ICA approaches have been proposed to separate the convolutive combination, and they are classified into two approaches: the time-domain approach and the frequency-domain approach.

The first approach is the time-domain ICA approach [2]. This approach is theoretically sound and achieves good separation performance once it converges. However, the time-domain ICA approach is computationally extensive since the adaptation includes convolution operations. In addition, statistical dependencies between filter taps reduce the convergence speed since updating a filter tap influences adaptation of the ones succeeding it.

The second approach is the frequency-domain ICA approach [3-8]. In this approach, the convolutive combination in the time domain is converted into multiple instantaneous combinations in the frequency domain. Then, these instantaneous combinations are individually separated by an instantaneous ICA algorithm. The advantage of the frequency-domain ICA approach lies in the fact that the convolutive combination with a large number of unknown parameters is decomposed into multiple, independent instantaneous combinations, each with fewer parameters to be estimated. The frequency-domain ICA approach is computationally efficient since the convolution in the time domain becomes computationally efficient multiplications in the frequency domain. In addition, adaptation of the ICA algorithm in one frequency bin does not interference with others, which results in fast convergence. In general, the frequency-domain ICA approach is more attractive than the time-domain ICA approach.

In existing literatures, many frequency-domain ICA approaches [3-6] are proposed to separate the convolutive combinations of speech signals. However, few references [7, 8] are known for digitally modulated signals in wireless communications. Moreover, the approaches in [3-6] do not fully solve the gain ambiguity, that is, the recovered signals are the filtered versions of the source signals. However, the technique presented here, the Frequency-Domain ICA approach (ICA-F), utilizes blind equalizers to successfully solve the gain ambiguity. Thus, the signals recovered by the ICA-F are the delayed versions of the source signals. In addition, the ICA-F uses the fourth-order cross-cumulant to solve the permutation ambiguity.

This contribution is organized as follows. Section 2 formulates the convolutive combination model in the time and frequency domains. In Section 3, the proposed ICA-F is developed. In addition, the permutation and gain ambiguities associated with the frequency-domain ICA approach are successfully resolved. Simulation results are presented in Section 4. Finally, conclusions are drawn in Section 5.

The following notations are used in this contribution. The superscripts, *, T, and H, denote conjugate, transpose, and conjugate transpose, respectively. The operators E() and \otimes denote expectation and convolution, respectively. \leftarrow means substitution, i.e., the variable of the right-hand side is computed and substituted in the left-hand side. diag(W) is to form a diagonal matrix from the diagonal elements of the matrix W.

2 Convolutive Combination Mode

In the general convolutive combination model, the number of the source signals, M, equals the number of the received signals. There are the source signal vector S(n) whose elements are the source signals $s_l(n)$'s, l = 0,1,...,M-1, and the received signal vector X(n) whose elements are the received signal vector X(n) whose elements are the received signals $x_m(n)$'s, m = 0,1,...,M-1. The convolutive combination model is expressed in the time domain as:

$$X(n) = H(n) \otimes S(n) \tag{1}$$

where H(n) is an unknown $M \times M$ matrix, whose $(m \ l)$ entry, $h_{m,l}(n)$, represents the impulse response from the transmitter l to the receiver m.

The convolutive combination model in (1) allows for two important propagation effects typically found in fading channels of wireless communications. First, the source signal $s_l(n)$ does not arrive at receivers simultaneously; the $s_l(n)$ arrives at receivers at different instants. Second, this model describes that the $s_l(n)$ arrives at a receiver via more than one path. This is known as multipath propagation in wireless communications.

The convolutive combination model, shown in (1), is represented in the z-transform domain as: X(z) = H(z)S(z) (2)

where the vectors S(z) and X(z) denotes the z-transforms of S(n) and X(n), respectively, and H(z) is a matrix whose (m, l) entry, $H_{m,l}(z)$, is the z- transform of the $h_{m,l}(n)$.

(1) and (2) indicate that the convolutive combination in the time domain corresponds the instantaneous combinations in the frequency domain. This observation provides insight into the proposed Frequency-Domain ICA algorithm (ICA-F), which is developed in Section 3.

3 Proposed Frequency-Domain ICA Approach

The structure of the proposed ICA-F, which is comprised of five processing stages, is shown in Fig. 1. The operations in these stages are described in the following.

3.1 Discrete Short-Time Fourier Transform (STFT)

In this stage, the discrete STFT is applied to the received signals $x_m(n)$'s. The analysis window used in the discrete STFT, win(n), is a rectangular window of length L. Then, the K-point Fast Fourier Transform (FFT) is performed over the windowed section of the $x_m(n)$'s. The number K is larger than or equal to the window length L.

The discrete STFT of the $x_m(n)$, $X_m(r,k)$, is expressed as:

$$X_m(r,k) = \sum_{n=0}^{+\infty} x_m(n) win(rL-n) e^{-j2\pi k(rL-n)/K}$$
(3)

where r, (r = 1,2,...), is the frame number, and k, (k = 0,1,...,K-1), is the frequency bin index.

In the same way as (3), the $S_l(r,k)$ is denoted as the discrete STFT of the $s_l(n)$. The convolution combination specified in (1) is converted into *K* instantaneous combinations in the frequency domain as:



Fig. 1 The structure of the proposed frequency-domain Independent Component Analysis (ICA-F) approach with correcting the permutation and gain ambiguities

X(r,k) = H(k)S(r,k)(4)

where the vector X(r,k) whose elements are the $X_m(r,k)$'s, the vector S(r,k) whose elements are the $S_l(r,k)$'s, and H(k) is given by:

$$H(k) = H(z)\Big|_{z=e^{-j2\pi k/K}}$$
⁽⁵⁾

3.2 Natural-Gradient ICA Algorithm

The source signals $s_l(n)$'s are assumed to be complex-valued, zero-mean, stationary, nongaussian, and independent. In [9], one theorem states that functions of independent random variables are also statistically independent. In the proposed ICA-F, the $s_l(r,k)$'s are the linear functions of the $s_l(n)$'s, and hence are independent. Consequently, (4) is a valid instantaneous combination ICA model in each frequency bin.

The natural-gradient ICA algorithm [10] is used to adapt the separating matrix W(k) to obtain the Y(r,k) in each frequency bin. The Y(r,k) is the estimate of the S(r,k), and suffers from the permutation and gain ambiguities, given by:

$$Y(r,k) = [Y_0(r,k),...,Y_l(r,k),...,Y_{M-1}(r,k)]^T$$

= W(k)X(r,k) (6)

The natural-gradient ICA algorithm implicitly incorporates high-order statistics by using the nonlinear function f(x). Since the digitally modulated signal is generally complex-valued with a negative kurtosis, the nonlinear function f(x) is chosen as [2]:

$$f(x) = \left|x\right|^2 x \tag{7}$$

In each frequency bin, the update rule of the W(k) is given by:

 $W(k) \leftarrow W(k) + \mu \left(I - E(Y_{non}(r,k)Y^{H}(r,k)) \right) W(k)$ (8) where μ is the convergence factor, *I* is the identity matrix, and the vector $Y_{non}(r,k)$ is given by:

$$Y_{non}(r,k) = \left[f(Y_0(r,k)), \dots, f(Y_l(r,k)), \dots, f(Y_{M-1}(r,k))\right]^{l}$$
(9)

The update rule in (8) runs iteratively until the W(k) converges. Following the Minimal Distortion Principle (MDP) in [6], the final value of the W(k) is given by:

$$W(k) \leftarrow diag(W^{-1}(k))W(k) \tag{9}$$

3.3 Solving Permutation Ambiguity

One potential problem with the update rule in (8) is that it is insensitive to the row permutations of the W(k). Since (8) is individually adapted in each frequency bin, the $Y_l(r,k)$'s are extracted with arbitrary orders. If the $Y_l(r,k)$'s are extracted from the different source signals, they are independent. Otherwise, they are statistically dependent. This independent property is measured by the fourth-order cross-cumulant. The fourth-order cross-cumulant between $Y_l(r,0)$ and $Y_m(r,k), k \neq 0$, $CUM_{l,m}(k)$, is defined as:

$$CUM_{l,m}(k) = E(|Y_{l}(r,0)|^{2}|Y_{m}(r,k)|^{2}) - E(|Y_{l}(r,0)|^{2})E(|Y_{m}(r,k)|^{2}) - |E(Y_{l}(r,0)Y_{m}^{*}(r,k))|^{2} - |E(Y_{l}(r,0)Y_{m}(r,k))|^{2}$$
(10)

In principle, $CUM_{l,m}(k)$ is zero when $Y_l(r,0)$ and $Y_m(r,k)$ are from the different source signals. Otherwise, $CUM_{l,m}(k)$ is non-zero.

From the explanation above, a method to solve the permutation ambiguity is given as following. First, the order of the $Y_l(r,0)$'s is chosen as the reference order. Then, the order of the $Y_l(r,k)$'s, $k \neq 0$, is adjusted such that it is the same as the reference order. To do so, the recovered source components without the permutation ambiguity, $U_l(r,k)$'s, are given by:

$$U_l(r,0) = Y_l(r,0)$$
 (11)
and

 $U_l(r,k) = Y_m(r,k), \quad k \neq 0$ (12)

where the $Y_m(r,k)$ has the maximum absolute value of $CUM_{l,m}(k)$ for m = 0,1,..,M-1.

3.4 Discrete Inverse Short-Time Fourier Transform (ISTFT)

In this stage, the overlap-add method [11] is used to implement the discrete ISTFT. The overlapping occurs when the points of the FFT, K, is larger than the window length L.

3.5 Solving Gain Ambiguity

Due to the gain ambiguity, the $U_l(r,k)$'s are subjected to arbitrary complex gains in frequency bins. Thus, the $\tilde{s}_l(n)$'s are the filtered versions of the source signals, and encounter both magnitude and phase distortions. In this paper, blind equalizers employing Constant Modulus Algorithm (CMA) [12] are used to compensate for these distortions. CMA is a blind equalization technique that restores modulus of source signals. Consequently, the $\hat{s}_l(n)$'s are the delayed versions of the $s_l(n)$'s, and are not subjected to the phase and amplitude distortion as the $\tilde{s}_l(n)$'s.

4 Simulation Results

Computer simulations are performed to confirm the effectiveness of the proposed ICA-F. The simulation setting, shown in Table 1, is used throughout this paper. The computer simulations employ the third-order convolutive combination system with the following coefficients as:

$$H(0) = \begin{pmatrix} 0.87 + 0.18j & 0.09 + 0.04j \\ 0.02 + 0.1j & 0.06 + 0.91j \end{pmatrix}$$
(13)

$$H(1) = \begin{pmatrix} -0.39 - 0.21j & 0.05 + 0.01j \\ 0.03 + 0.06j & -0.28 + 0.24j \end{pmatrix}$$
(14)

and

$$H(2) = \begin{pmatrix} 0.05 + 0.08j & 0.02 + 0.002j \\ 0.01 + 0.001j & 0.15 + 0.1j \end{pmatrix}$$
(15)

The space diagrams of the received signals without additive noises are shown in Fig.2a and Fig.2b, respectively. These space diagrams do not resemble characteristics of DQPSK due to the convolutive combination. The space diagrams of the recovered source signals employing the ICA-F are shown in Fig. 3a and 3b, which resemble the characteristic DQPSK constellation up to phase These space diagrams show that the rotations. ICA-F successfully separates proposed and deconvolves the convolutive combination. Figure 4a and Figure 4b present the space diagrams of the recovered signals without solving the permutation and gain ambiguities. These results demonstrate that the frequency domain ICA approach cannot achieve a good performance improvement without solving the permutation and gain ambiguities.

5 Conclusion

In wireless communications, the received signals are the convolutive combinations of the source signals in case of slow, frequency-selective fading channels. In

this contribution, a novel Frequency-Domain Independent Component Analysis approach (ICA-F) is proposed to blindly separate and deconvolve the source signals. In the ICA-F, the convolutive combinations in the time domain are converted to multiple instantaneous combinations in the frequency domain. The proposed ICA-F is more computationally efficient and converges faster than the existing time domain approach. The ICA-F successfully solves the permutation and gain ambiguities, which are the major obstacles to implement the frequency-domain ICA approach. Computer Simulations illustrate the performance of the proposed ICA-F.

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Table 1. Simulation Parameters	
Source signals	Uniformly-distributed, independent Differential Quadrature Phase Shift Keying (DQPSK)
The number of source and received signals	2
Samples of the received signals	90,000
The length of the rectangular window	3
The points of the FFT	8



Fig. 2. Space diagrams of the received signals $x_0(n)$ and $x_1(n)$



Fig. 3. Space diagrams of the recovered signals $\hat{s}_0(n)$ and $\hat{s}_1(n)$, where the permutation and gain ambiguities have been corrected using ICA-F



Fig. 4. Space diagrams of the recovered signal 1 and recovered signal 2, where the permutation and gain ambiguities have not been corrected

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