

# Applying a Novel Method to the Analysis of Asymmetrical Transverse Slit in Microstrip Line

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*Abstract:* - In this paper a completely new method has been developed through employing a delicate combination of a reformed version of Rigorously Coupled Multi Strips (RCMS) and modal decoupling of Multi Transmission Lines (MTL) along with the available closed expressions for asymmetrical step, gap and open in microstrip line, to determine the distribution of currents separately versus its cross sectional positions and also to estimate the two-port parameters of a asymmetrical transverse slit under the examination. First, the currents' distributions are obtained by dividing the strip in a transversely non-uniform manner within three different longitudinal parts and using all the aforementioned closed expressions according to its corresponding parts. Then in order to calculate two-port parameters, unlike the previous approach, the strip is divided uniformly inside all parts and the closed expression of an asymmetrical step is simply exploited. A comparison between z-parameters of the novel and full wave analysis has been made at the end, via an example to validate the method discussed in this article.

*Key-Words:* - Asymmetrical transverse slit, gap, modal decoupling, MTL, open, RCMS, step.

## 1 Introduction

Analysis of discontinuities in microstrip line has been on top of the agenda in microwave literature due to its wide application in high-frequency circuits, for many years. The general solution to this sort of problem especially at very high frequencies is full wave analysis which is complicated and time-consuming.

An improved version of RCMS in conjunction with modal decoupling method of MTL is used to get a great perception of the form of currents' distribution on the strip readily particularly around the region of discontinuity in microstrip line [1], [2]. Also the closed expressions of discontinuities such as asymmetrical step, gap, open are taken into account [3]. Since one has already found himself got involved in MTL, a thorough examination and clarification of modal decoupling approach has already been discussed which would be very useful in the current procedures [4]. Using the simple quasi-static method for including dispersive effect in coupled MTL suggested by Tripathi, one may expand the frequency range of application of the novel technique even further to calculate scattering parameters of the structure in question [5]. Via the new method, not only is it possible to simulate and analyze an asymmetrical transverse slit, but also it may be employed to obtain the characteristic of any

transverse discontinuity with any shape, considering the fact that the only mathematical problem one is dealing with is a linear algebra on matrices realizable by any computer programming software on hand. Simplicity and flexibility of the method distinguish it from the others reported in literature so far. The following section delineates the analysis.

## 2 Method of Analysis

A uniformly divided asymmetrical transverse slit is illustrated in Fig.1. There are three longitudinal parts holding the lengths of  $d_1$ ,  $d_2$  and  $d_3$  in the z direction from left to right. Transversally the part 1, 2 and 3 are divided into N, M and K parallel transmission lines respectively. Determinations of MTL's parameters involved in the analysis such as mode propagation constant  $\gamma$ , characteristic impedance  $Z_C$ , current decoupling factor  $T_I$  and voltage decoupling factor  $T_V$  versus inductance and capacitance matrices have been explained in detail in [1]. Fig.1 is used to obtain distribution of currents; however for evaluation of two-port parameters one should take the same structure with a uniform division for all three parts. For that reason, two targets are going to be discussed in two separate subsections.

The actual voltages and currents in parallel narrow strip lines which are all longitudinally

located, are given in terms of decoupled forward and backward currents as follows.

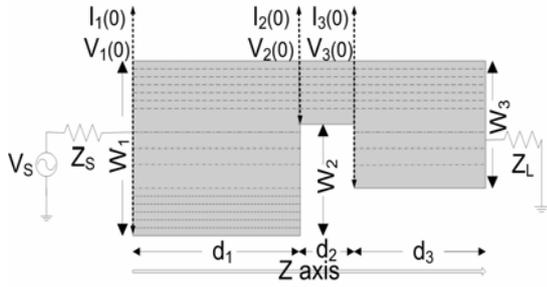


Fig.1 A non-uniformly divided asymmetrical slit

$$[I(z)] = [T_I] \cdot \left( [e^{-\gamma \cdot z}] \cdot [I^+] - [e^{+\gamma \cdot z}] \cdot [I^-] \right) \quad (1)$$

$$[V(z)] = [Z_C] \cdot [T_I] \cdot \left( [e^{-\gamma \cdot z}] \cdot [I^+] + [e^{+\gamma \cdot z}] \cdot [I^-] \right) \quad (2)$$

## 2.1 Determination of Currents' Distribution

According to Fig.1, the first longitudinal part comprises four different groups of parallel lines, i.e. from bottom to top,  $(N - K)$  open lines,  $(K - M)$  gap lines between part 1 and 3, 1 step line between part 1 and 2, and at last  $(M - 1)$  ordinary transmission lines. In the second part, there are 1 step line and  $(M - 1)$  ordinary lines at the both sides. The third part consists of  $(K - M)$  gap lines between part 1 and 3, 1 step line between part 3 and 2, and  $(M - 1)$  ordinary lines between part 3 and 2. Ordinary lines are  $(W_1 - W_2)/M$  wide. Step lines have width ratio of  $W_1/(W_1 - W_2)$ . Open lines hold the width of  $(W_1 - W_3)/(N - K)$ . Depending on the number of gap lines and the width of the other sections, the gap width can be easily achieved. Bear in mind that the number of divisions are limited by the range of closed expressions specifically for the gap section.

Realizing the whole geometry, one is able to write the cross sectional conditions for longitudinal currents and voltages given below.

$$V_S [1]_{N \times 1} - Z_S [1]_{N \times N} \cdot [I_1(0)]_{N \times N} = [V_1(0)]_{N \times N} \quad (3)$$

$$[V_3(d_3)]_{K \times 1} = Z_L [1]_{K \times K} \cdot [I_3(d_3)]_{K \times 1} \quad (4)$$

$$[I_1(d_1 + d_{OC})]_{(N-K) \times 1} = [0]_{(N-K) \times 1} \quad (5)$$

$$[I_1(d_1 + d_P)]_{(K-M) \times 1} = [I_3(-d_P)]_{(K-M) \times 1} \quad (6)$$

$$(j\omega \cdot C_g) \cdot ([V_1(d_1 + d_P)] - [V_3(-d_P)])_{(K-M) \times 1} = [I_1(d_1 + d_P)]_{(K-M) \times 1} \quad (7)$$

$$[V_1(d_1 + d_S)]_{1 \times 1} = [V_2(-d_S)]_{1 \times 1} \quad (8)$$

$$[I_1(d_1 + d_S)]_{1 \times 1} - [I_2(-d_S)]_{1 \times 1} = (j\omega \cdot C_S) \cdot [V_1(d_1 + d_S)]_{1 \times 1} \quad (9)$$

$$[V_2(d_2 + d_S)]_{1 \times 1} = [V_3(-d_S)]_{1 \times 1} \quad (10)$$

$$[I_2(d_2 + d_S)]_{1 \times 1} - [I_3(-d_S)]_{1 \times 1} = (j\omega \cdot C_S) \cdot [V_2(d_2 + d_S)]_{1 \times 1} \quad (11)$$

$$[V_1(d_1)]_{(M-1) \times 1} = [V_2(0)]_{(M-1) \times 1} \quad (12)$$

$$[I_1(d_1)]_{(M-1) \times 1} = [I_2(0)]_{(M-1) \times 1} \quad (13)$$

$$[I_2(d_2)]_{(M-1) \times 1} = [I_3(0)]_{(M-1) \times 1} \quad (14)$$

$$[V_2(d_2)]_{(M-1) \times 1} = [V_3(0)]_{(M-1) \times 1} \quad (15)$$

In above equations,  $d_S$ ,  $d_P$  and  $d_{OC}$  account for equivalent length of the inductive effect in asymmetrical step, fringing capacitive effect in gap and fringing capacitive effect in open, respectively. Also  $C_g$  is the gap capacitance and  $C_S$  is the capacitance in equivalent circuit of an asymmetrical step. In equation (5) the first  $(N - K)$  rows of  $I_1$  are taken into account. In (6) and (7) the rows from 1 to  $(K - M)$  for  $I_3$  and  $V_3$ , and from  $(N - K + 1)$  to  $(N - M)$  for  $I_1$  and  $V_1$  are considered. The first row of  $I_2$  and  $V_2$ , and the  $(N - M + 1)^{th}$  row of  $I_1$  and  $V_1$  are involved in equations (8) and (9). For (10) and (11) one has the same row for the second part and the  $(K - M + 1)^{th}$  row for the third part. In the four last equations, the last  $(M - 1)$  rows are employed in all voltages and currents.

As one notices there exist  $2(N + M + K)$  variables, i.e.  $[I_1^+]_{N \times 1}$ ,  $[I_1^-]_{N \times 1}$ ,  $[I_2^+]_{M \times 1}$ ,  $[I_2^-]_{M \times 1}$ ,  $[I_3^+]_{K \times 1}$ ,  $[I_3^-]_{K \times 1}$ , which will be determined through the above equations. The distribution of the longitudinal current versus cross-sectional positions at each part on the  $z$  axis can be directly achieved. But the transverse distribution is determined qualitatively and implicitly by differentiating successive longitudinal voltages across the strip for each cross section and then divided by its corresponding characteristic impedance.

## 2.2 Calculation of Two-port Parameters

Unlike the previous subsection, all parts are divided in a uniform manner and the only closed expression for an asymmetrical step is incorporated in the analysis. Notice that here one has two different step width ratio which are  $W_1/(W_1 - W_2)$  and  $W_3/(W_3 - W_2)$ . Their related parameters as described earlier are  $d_{S1}$ ,  $C_{S1}$  and  $d_{S3}$ ,  $C_{S3}$  respectively. As a matter of convenience two series of new parameters are defined for the currents of second part. These parameters are obtained in terms of the old parameters as follows.

$$\begin{bmatrix} I_{n2}^+ \\ I_{n2}^- \end{bmatrix}_{N \times 1} = \begin{bmatrix} [0]_{(N-M) \times 1} \\ [I_2^+]_{M \times 1} \end{bmatrix}, \quad \begin{bmatrix} I_{n2}^- \\ I_{n2}^+ \end{bmatrix}_{N \times 1} = \begin{bmatrix} [0]_{(N-M) \times 1} \\ [I_2^-]_{M \times 1} \end{bmatrix} \quad (16)$$

$$\begin{bmatrix} \gamma_{n2} \\ \gamma_{n2} \end{bmatrix}_{N \times N} = \begin{bmatrix} [0]_{(N-M) \times (N-M)} & [0]_{(N-M) \times M} \\ [0]_{M \times (N-M)} & [\gamma_2]_{M \times M} \end{bmatrix} \quad (17)$$

$$\begin{bmatrix} T_{In2} \\ T_{In2} \end{bmatrix}_{N \times N} = \begin{bmatrix} [0]_{(N-M) \times (N-M)} & [0]_{(N-M) \times M} \\ [0]_{M \times (N-M)} & [T_{I2}]_{M \times M} \end{bmatrix} \quad (18)$$

$I_{k2}$ ,  $\gamma_{k2}$  and  $T_{Ik2}$  are akin to the three above equations. The conditions at the beginning and end of the line are exactly the same as defined in (3) and (4). The rest of equations are

$$[V_1(d_1 + d_{S1})]_{M \times 1} = [V_2(-d_{S1})]_{M \times 1} \quad (19)$$

$$[V_2(d_2 + d_{S3})]_{M \times 1} = [V_3(-d_{S3})]_{M \times 1} \quad (20)$$

$$(j\omega \cdot C_S)^{-1} \cdot [1]_{N \times N} \cdot ([I_1(d_1 + d_{S1})] - [I_{n2}(-d_{S1})]_{N \times 1}) = [V_1(d_1 + d_{S1})]_{N \times 1} \quad (21)$$

$$(j\omega \cdot C_S)^{-1} \cdot [1]_{K \times K} \cdot ([I_{K2}(d_2 + d_{S3})] - [I_3(-d_{S3})]_{K \times 1}) = [V_3(-d_{S3})]_{K \times 1} \quad (22)$$

If one replaces  $Z_L$  with a large number such as  $10^8$  and goes through equations (16)-(22) in order to find out the forward and backward currents belonging to the first and third part, it will result in the three following equations.

$$[I_1(0)]_{N \times 1} = [T_{I1}] \cdot ([I_1^+] - [I_1^-])_{N \times 1} \quad (23)$$

$$[V_1(0)]_{N \times 1} = [Z_{C1}] \cdot [T_{I1}] \cdot ([I_1^+] + [I_1^-])_{N \times 1} \quad (24)$$

$$[V_3(d_3)]_{K \times 1} = [Z_{C3}] \cdot [T_{I3}] \cdot ([e^{-\gamma_3 d_3}] \cdot [I_3^+] + [e^{+\gamma_3 d_3}] \cdot [I_3^-])_{K \times 1} \quad (25)$$

Then  $Z_{11}$  and  $Z_{21}$  of the two-port network are defined.

$$Z_{11} = \frac{1}{N} \frac{\left( \sum_{i=1}^N V_1^i(0) \right)}{\left( \sum_{i=1}^N I_1^i(0) \right)} \quad (26)$$

$$Z_{21} = \frac{1}{K} \frac{\left( \sum_{i=1}^K V_3^i(d_3) \right)}{\left( \sum_{i=1}^N I_1^i(0) \right)} \quad (27)$$

In (26) and (27), 'i' stands for the row's number in matrices. Calculations of  $Z_{12}$  and  $Z_{22}$  have the similar procedure by changing simply the role of port 1 and 2. Exploiting the conversion between two-port network parameters, one deduces any two-port parameters.

## 3 Example and Results

The properties of the structure under investigation are listed in table 1. In that table, h is the thickness of the substrate and  $\epsilon_r$  is the relative dielectric constant in microstrip line. Voltage source ( $V_s$ ) is equal to 1 volt.

For evaluation of currents' distributions, N, M and K are chosen 10, 5 and 7 respectively. In the second approach those are taken 21, 7 and 17 instead.

Fig.2 demonstrates the both longitudinal and transverse current distribution ( $I_z$  and  $I_x$ ) versus transverse positions in the close proximity to the discontinuity. The currents' distributions in the middle of the region of discontinuity are shown in Fig.3, and eventually the distribution of longitudinal and transverse current in the third part in the vicinity of discontinuity are illustrated in Fig.4. A comparison has been made by estimating z-parameters via the both novel and full wave method. The results of that have been reported in table 2 for a few frequencies.

Table 1  
Structure Properties

PARAMETERS	VALUES
h	0.5 mm
$\epsilon_r$	8.8
$W_1$	1.5 mm
$W_2$	0.5 mm
$W_3$	1.2 mm
$d_1$	6 mm
$d_2$	0.25 mm
$d_3$	6 mm

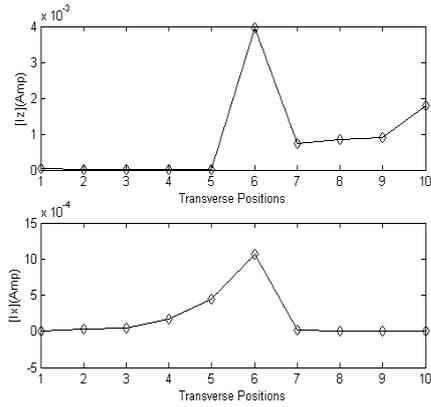


Fig.2 Distribution of longitudinal and transverse currents ( $I_z$ ,  $I_x$ ) V.S. transverse positions at  $z = 6mm$  and  $f = 1GHz$

As it is clear, a symmetrical structure has been exemplified here. The real parts of the impedances are zeros which have been ignored in table 2.

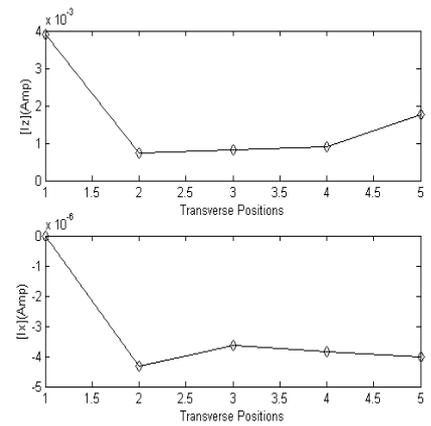


Fig.3 Distribution of longitudinal and transverse currents ( $I_z$ ,  $I_x$ ) V.S. transverse positions at  $z = 6.15mm$  and  $f = 1GHz$

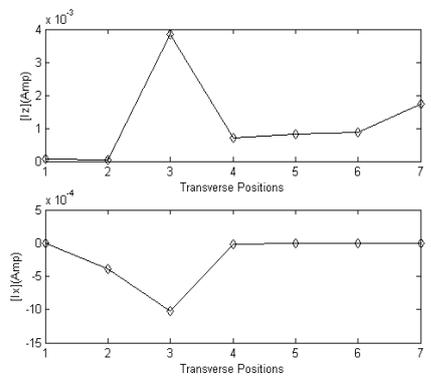


Fig.4 Distribution of longitudinal and transverse currents ( $I_z$ ,  $I_x$ ) V.S. transverse positions at  $z = 6.3mm$  and  $f = 1GHz$

Table 2  
Z-Parameters of the Two-Port Network

Novel Technique		
f(GHz)	$Z_{11}$	$Z_{21}$
1	-j38.121	-j47.780
2	-j8.552	-j30.611
3	+j10.981	-j33.308
Full Wave Analysis		
1	-j37.000	-j46.720
2	-j7.956	-j30.706
3	+j13.016	-j35.888

### 4 Conclusion

Two novel methods which are a little bit different from each other have been developed to specify the distribution of longitudinal and transverse currents qualitatively and also two-port parameters in an easy way. Indeed those give one a great perception of what's happening to the currents on the strip especially around discontinuities. An amalgam of RCMS and modal decoupling technique in conjunction with closed expressions of the open, gap and step are applied to create these totally novel techniques. One may ameliorate the results even at higher frequencies by incorporating dispersive effect readily.

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