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Signal Dependent Biorthogonal Wavelet Based Representation

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Abstract: - This work presents a framework for wavelet-based representation of a given deterministic signal. We propose a method for matched wavelet based representation of the signal. Our method requires a signal-conditioning step at each stage of the wavelet decomposition. The wavelet system designed in this work is biorthogonal. Both the signal-conditioning step and the wavelet system depend on the given signal. The proposed method is valid for signals having arbitrary spectra. The proposed method when applied to compression of different signals shows substantial improvement over.

Key-Words: - Signal-conditioning, Wavelet-based, Perfect reconstruction, FIR biorthogonal filter bank

1 Introduction

Appropriate signal representation is a fundamental issue in any signal processing task. Representation of a signal is either dictated by certain embedded features of the signal or is driven by the application at hand. The time impulse functions and complex exponentials are two extreme bases. The wavelet basis for the signal representation offers infinite number of possibilities between these two extremes cases and uses the scaled and translated version of some basic function. The non-uniqueness in the choice of basis is one of the important reasons for it to find widespread applications in signal processing. Depending on the application, it has to be suitably chosen. But finding a suitable basis is a nontrivial task and many researchers in recent past have attempted to answer this. Since wavelet representation is particularly useful for representing non - stationary signals it is interesting to find a basis that represents the given signal in some optimal way. Many applications of signal representation, such as image and speech compression, adaptive coding and pattern recognition require wavelets are matched to signal of interest.

Daubechies' classic technique [1, 2] for finding orthonormal and biorthogonal wavelet bases respectively with compact support is often used as default in many wavelet applications. However, the wavelets produced are independent of the signal being analyzed. Tewfik *et. al*.[3] have developed a technique for finding an optimal orthonormal wavelet basis for representing a specified signal within a finite a number of scales. Gopinath *et. al.*[4] extended the result of Tewfik, *et al.*, by assuming band-limited signals and finding the optimal M-band wavelet basis for representing a desired signal, again within a finite number of scales.

The wavelet design techniques developed by Mallat and Zheng [5] builds non-orthonormal wavelet bases from a library of existing wavelets in such a way that some error cost function is minimized. This technique is constrained by the library of functions used and do not satisfy the need for optimal correlation in both scale and translation. Aldroubi and Unser [6] match a wavelet basis to a desired signal by either projecting the desired signal onto an existing wavelet basis, or transforming the wavelet basis under certain conditions such that the error norm between the desired signal and the new wavelet basis is minimum. Both of these techniques are constrained by their initial choice of MRA. Chapa and Rao [7] have proposed an algorithm for designing wavelets matched to the given signal. Recently Anubha *et al.*, [8] have proposed a method for finding a matched wavelet for the given deterministic signal. The approach uses the deterministic autocorrelation function of decimated input signal.

In this research work we are proposing a scheme for wavelet based signal representation. Our approach uses a novel signal-conditioning step at each stage of wavelet decomposition. Our approach is valid for a wide class of signals having arbitrary spectra. This paper consists of the five sections. In section 2 we give motivation of the present work and formulate the problem. In section 3 we give the theory of the proposed solution in detail. Section 4 contains the simulations results and comparisons with the previous work. Section 5 contains the conclusions and discussion.

2 Motivation of the Problem

Wavelet offers infinite possibilities for signal representation. Different types of wavelets with different features have been proposed in the literature [9]. But most of them have been designed without reference to the particular signal at hand. The given basis is efficient if there is strong correlation between the given signal and the chosen basis. Therefore it is motivating to design the wavelet basis from the given deterministic signal. Also for a wavelet representation to be efficient it requires more than one level of decomposition. And this will be most efficient if the signal spectra also follow *1/f* law.

Recently Anubha *et al.,* [8] have proposed a method for finding a matched wavelet for a given deterministic signal. The proposed method finds a wavelet system such that on decomposition it projects maximum signal energy on the successive scaling sub space. This method works as long as the signal is dominantly a low pass i.e., the majority of signal energy is in the low frequency band. But if the signal has very less signal energy in the low frequency domain then the present framework is unable to design a wavelet system. The method does not perform well when signal is not a dominantly low pass signal and is not efficient when signal spectra do not follow *1/f* law.

In general a signal can have arbitrary spectra and also at a stage of wavelet decomposition the resultant signal can have arbitrary spectra. Therefore there is a need of a framework that represents a signal belonging to a wide class. Motivated by the above discussion, the problem is formulated as follows: Find a wavelet based system that can efficiently represent a signal having arbitrary spectra.

3 The Proposed solution

3.1 Introduction

As mentioned earlier, in this section we present a scheme for wavelet based signal representation of a given deterministic signal having arbitrary spectra. Our method is based on the fact that wavelet representation of the signal is efficient if on consecutive decomposition, signal has more energy in the scaling space and less in the subsequent wavelet space. But natural occurring

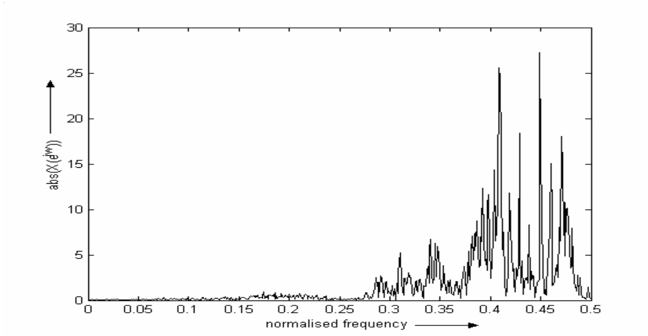

Fig. 1. A signal $X(e^{jw})$ having high pass spectra

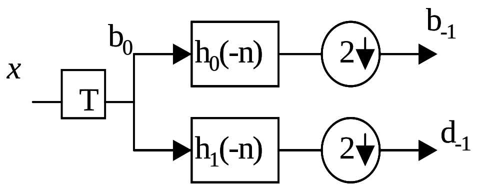

Fig. 2. A stage of the modified wavelet based decomposition

signal in general are not restricted to dominantly low frequency signal and also at any stage after the decomposition we can have an arbitrary signal spectra.

Consider a signal *x(n),* whose spectra is as shown in Fig. 1. If we want to represent the signal by wavelet, we will do first a stage of wavelet decomposition and then further decompose the low pass filter path a number of times. Since the signal does not have any energy in the subsequent low pass filter path the resultant representation is not efficient. But if we transform the signal to a low pass signal and do wavelet decomposition then the resultant wavelet representation will be efficient.

The scheme provides for a signal conditioning transformation *T*, such that it transforms the given signal to a dominantly low pass signal. After the signal conditioning, a wavelet system is designed from the transformed signal such that it projects maximum energy in the successive scaling space. In this way the transformed signal and hence the original signal will have an efficient representation by the modified wavelet based (**MWB**) system. Also the work presented in [8] becomes a special case of **MWB** system. The wavelet system is designed through a high pass analysis filter h_1 from the signal. A complete one level of signal decomposition step of **MWB** will have the form as shown in fig.1. From signal processing point of view perfect reconstruction property of the filter bank corresponding to wavelet is important. In this work the wavelet system designed is biorthogonal and it has compact support and allows perfect

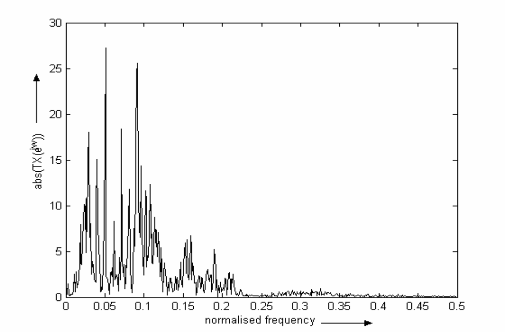

Fig. 3. The transformed signal TX(e^jw) 's spectra, showing the low pass nature.

reconstruction. The resultant representation will be efficient for any arbitrary signal. Following subsections gives the proposed theory in detail.

3.2 Signal Conditioning

Suppose signal $x(n)$ can have discrete Fourier transform $X(e^{j\omega})$. We now define Low pass signal energy (**LPSE**) and High pass signal energy (**HPSE**) as

$$LPSE = \int_{0}^{\pi/2} \left|X(e^{j\omega})\right|^2 d\omega \quad (1)$$

$$HPSE = \int_{\pi/2}^{\pi} \left|X(e^{j\omega})\right|^2 d\omega \quad (2)$$

We propose and define the following signal conditioning transformation before the wavelet decomposition:

$$(T\,x)[n] \equiv \begin{pmatrix} x(n), & LPSE \geq HPSE \\ (-1)^n x(n), & LPSE < HPSE \end{pmatrix} \quad (3)$$

The effect of this transformation is that, if the signal is dominantly a low pass then the signal is not transformed and if the signal is not dominantly low pass then spectra changes to $X(e^{j(\omega+\pi)})$ and becomes low pass signal. When signal-conditioning T is applied to the signal in Fig. 1, the transformed signal becomes a dominantly low pass signal from as shown in Fig. 3. This transformation ensures that we always have more energy in the low frequency band before the decomposition. Note that signal conditioning transformation T is invertible. Now application of wavelet decomposition on the transformed signal will provide better representation.

3.2 Design of Wavelet Analysis Filter

Let $b_0(n)$ be the signal we get after the signal conditioning transformation T applied. Consider a 2-band wavelet system to which the transformed

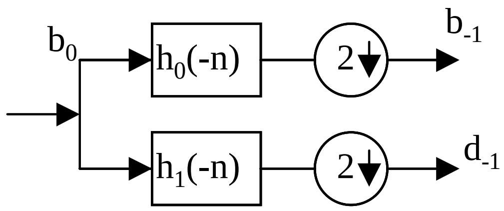

Fig. 4. A 2-band analysis filter bank

signal $b_0(n)$ is applied as shown in fig. 2. We want to derive a wavelet system such that it puts minimum energy in the successive wavelet space [8]. The aim is to find a closed form solution from the signal itself. The signal $b_0(n)$ can be decomposed into $b_{-1}(n)$ and $d_{-1}(n)$. The output $b_{-1}(n)$ and $d_{-1}(n)$ from the analysis filter bank are the coefficients of expansion in the next lower scaling space V_{-1} and the wavelet space W_{-1}, i.e.

$$b_{-1}(n) = \sum_k h_0(-k) b_0(2n-k) \quad (4)$$

and

$$d_{-1}(n) = \sum_k h_1(-k) b_0(2n-k) \quad (5)$$

Since our aim is to design high pass filter h_1, the d_{-1} should carry the details only. Let the length of the filter be N. Let the continuous time signal reconstructed from $b_0(n)$ and $d_{-1}(n)$ be $b(t)$ and $d(t)$ respectively. Therefore, if $\phi(t)$ and $\psi(t)$ be the scaling function and the wavelet function for the reconstruction, then

$$b(t) = \sum_k b_0(k)\phi\ (t-k), \quad (6)$$

$$d(t) = \sum_k \frac{1}{\sqrt{2}} d_{-1}(k)\psi\ \left(\frac{t}{2} - k\right) \quad (7)$$

The difference of the above signal is

$$s(t) = b(t) - d(t), \quad (8)$$

and the corresponding difference signal energy is

$$S = \int s^2(t)dt. \quad (9)$$

Now

$$s(t) = \sum_k b_0(k)\phi(t-k) - \sum_k \frac{1}{\sqrt{2}} d_{-1}(k)\psi\left(\frac{t}{2} - k\right) \quad (10)$$

Therefore,

$$S = \int s^2(t)dt$$

$$= \int \left(\sum_k b_0(k)\phi'(t-k) - \sum_k \frac{1}{\sqrt{2}} d_{-1}(k)\psi\left(\frac{t}{2} - k\right) \right)^2 dt \quad (11)$$

To find a closed form solution for $h_1(n)$, we use the orthogonality condition of wavelet basis in the expression for S. We get,

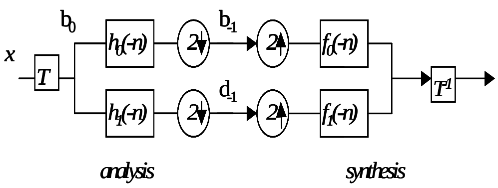

Fig. 5. The complete modified wavelet based analysis and synthesis filter bank

$$S=\sum_m (b_0(m))^2 - \sum_q \sum_k \sum_p h_1(p)h_1(q)b_0(2k+q)b_0(2k+p) \quad (12)$$

By maximizing S, as a function of $h_1(n)$, the resulting equation is

$$\sum_p h_1(p)\left[\sum_q b_0(2k+p)b_0(2k+r)\right]=0$$

for $r=0,1,...,j-1,j+1,...,N-1$. (13)

Here, *jth* filter weight is kept constant to value 1.The proposed method leads to a closed form expression and it can this error is indeed maximized.

3.3 Complete Modified Wavelet Based System

Once we get the high pass analysis filter $h_1(n)$ from the signal statistics, we can impose condition for the resultant filter bank to be biorthogonal. The resultant complete modified wavelet based system will be as shown in Fig. 5. The four filters h_0, h_1, f_0, f_1 are related by following equations for the condition of perfect reconstruction [2].

$$h_1(n)=(-1)^n f_0(N_1-n) \quad (14)$$

$$f_1(n)=(-1)^n h_0(N_1-n) \quad (15)$$

where, N_1 is any odd delay.

From (14), the scaling filter f_0 is computed. Since the integer translates of $\phi(t)$ and $\psi(t)$ form the basis of V_0 and W_0 respectively, $f_0(2m-n)$ and $f_1(2m-n)$ form the basis for integer values of m. Similarly, $h_0(n-2m)$ and $h_1(n-2m)$ form the dual basis of $l^2(Z)$ for integer values of *m*. Therefore

$$\sum_n h_0(n-2m_1)f_0(n-2m_2)=\delta(m_1-m_2)$$

$$\forall\, m_1, m_2 \in Z \quad (16)$$

The analysis filter h_0 can be found by solving (16). After solving for h_0, the filter f_1 can be found using (15). Hence all the four filters of 2-band wavelet system can be computed to form a perfect reconstruction biorthogonal FIR filter bank.

4 Simulation Results

Three different deterministic signals have been taken for the simulation purposes. The simulations are done on **MATLAB** software. The first signal is a predominantly a high frequency signal as shown in Figs. 6(a, b). First, simulations are carried out using the signal matched wavelet (**SMW**) method as proposed in [8] recently. When we are designing the high pass analysis filter of the match wavelet, the algorithm gives instead a low pass filter as shown in Fig. 6(c). Which is in contradictory to the requirement of the wavelet system.

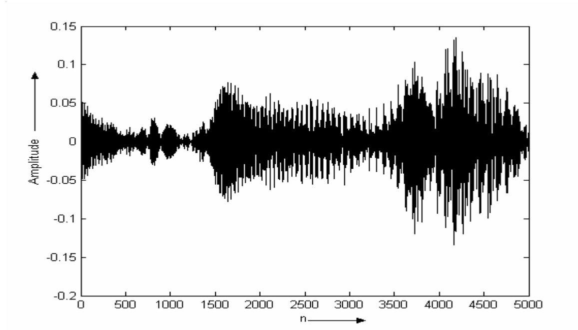

Fig. 6. (a) Test signal 1.

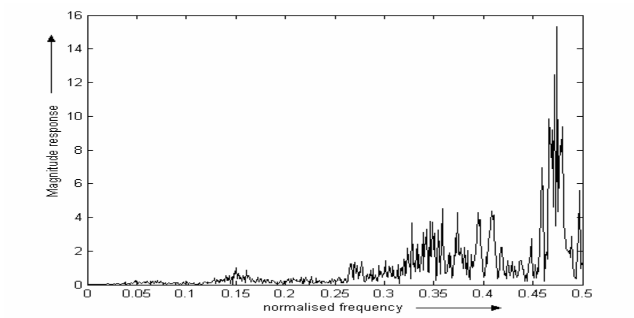

Fig. 6. (b) Frequency Magnitude response of the test signal.

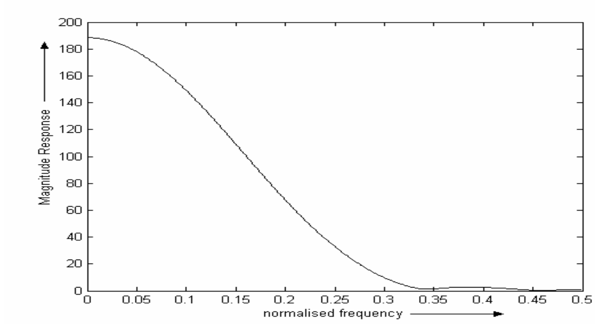

Fig. 6. (c) Frequency Magnitude response of the analysis filter h_1.

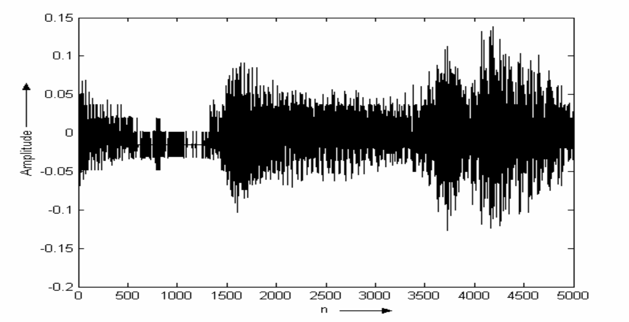

Fig. 7. (a) Test signal 1 reconstructed from SBW method

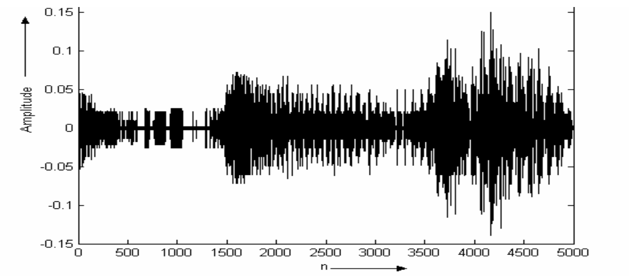

Fig. 7. (b) Test signal 1 reconstructed from MWB method

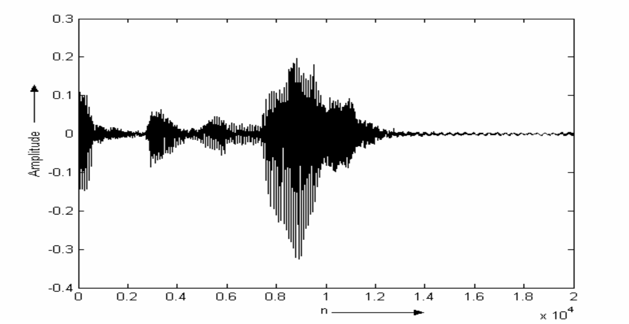

Fig. 8. (a) Test signal 2

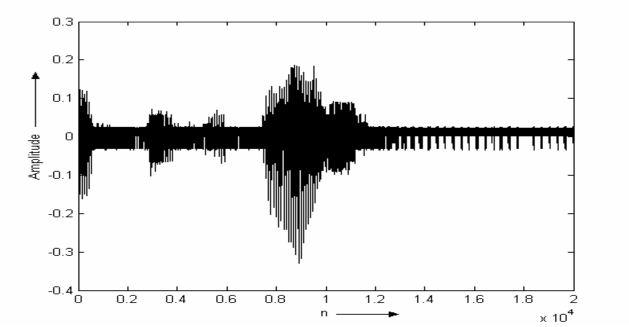

Fig. 8. (b) Test signal 1 reconstructed from SBW method

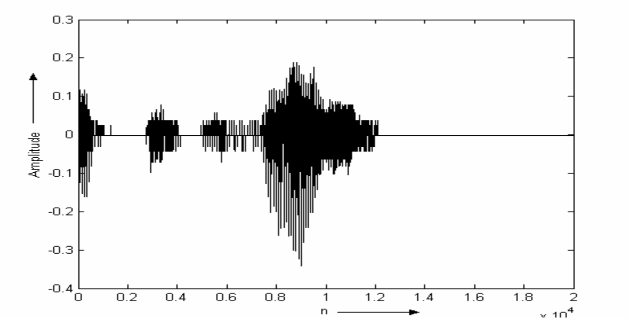

Fig. 8. (c) Test signal 1 reconstructed from MWB method

Further simulations are done in terms of the compression power of the two methods of wavelet-modified wavelet based (**MWB**) method as proposed by us. For the **SBW** we have used '**bior 4.4**' as listed in Wavelet Toolbox of the **MATLAB** software. Since the above two methods uses different wavelets and are designed with different criterion, we have adopted different mechanism for their bit allocation.

For the **SBW** method following is the method for bit allocation in different band. If an overall bit rate of *R* bits per sample is assigned, then the bit allocation for the sub-bands is:

$$R_k = R + \frac{1}{2}\log_2\left(\frac{\sigma_k^2}{\sigma_1\sigma_2}\right) \quad for \;\; k = 1,2 \qquad (17)$$

where, σ_1^2 is the variance of scaling sub-band and σ_2^2 is the variance of the signal in wavelet sub-band. And R_1 and R_2 are the bits required for the scaling and wavelet sub-bands respectively. For each sub-band number of quantization levels are chosen depending on the bits for the sub-band and uniform quantization is applied. The compression is in terms of energy they project in the scaling space of the wavelet system used for decomposition. The reconstructed signal energy is a good measure of compression power of the wavelet system chosen.

For **MWB** method, since the wavelet is designed such that maximum signal energy in the scaling space, a different strategy for bit allocation is assigned. For the given signal, we use *R1* as it is in **SBW** but for *R2* we kept the value to zero. The performance is measured in terms of peak signal to noise ratio (**PSNR**), which is defined as

$$PSNR \equiv 10\log_{10}\frac{\max_n |x(n)|^2}{\sum e^2(n)/N} \qquad (18)$$

Here, N is equal to total number of samples in the input signal *x(n)* and *e(n)* is the error between input signal sample and the reconstructed signal sample. Following are the simulation results for various test signal.

signal	Bit allocated				PSNR (dB)	
	Scaling space		Wavelet space		SBW	MWB
	SBW	MWB	SBW	MWB		
Test signal 1	4	4	1	0	19.69	26.05
Test signal 2	4	4	1	0	26.95	33.21

Table 1. Performance in terms of **PSNR** of **SBW** and **MWB** methods

5 Conclusions and Discussion

The simulation on Test signal 1 has shown the limitation of the present framework of signal-matched wavelet. This is intuitively also correct, for the signal, which is dominantly a high pass signal, we cannot have wavelet filter as high pass and still projecting minimum energy in the wavelet space. The simple signal-conditioning step allows us to project maximum signal energy in the successive scaling space. This signal-conditioning step is easier to implement in the algorithm. Since we project maximum energy in the scaling space, it is useful in terms of efficient signal representation. This is demonstrated in the Fig. 7 and Fig. 8. Table 1 also shows the compression performance in terms of **PSNR**. The simulation results shows that **MWB** method has better performance than the **SBW** method.

But biorthogonal wavelets are no longer energy conserving. Thus bit allocation algorithms as given by (17) are not optimal. Therefore coding for designed biorthogonal system is under investigation. Also in this work we have done only one level of wavelet decomposition. Therefore the work for a number of levels of decomposition is also under investigation.

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