4th WSEAS International Conference on ELECTRONICS, CONTROL and SIGNAL PROCESSING, Miami, Florida, USA, 17-19 November, 2005 (pp.6-11)

Analog System Time-Optimal Design by Generalized Formulation

ALEXANDER ZEMLIAK
Department of Physics and Mathematics
Puebla Autonomous University
Av. San Claudio y 18 Sur, Puebla, 72570
MEXICO

Abstract: - The new general methodology for the electronic system design was elaborated by means of the optimum control theory formulation in order to improve the characteristics of the system design process. This approach generalizes the design process and generates a set of the different design strategies that serves as the structural basis to the optimal strategy construction. The principal difference between this new methodology and before elaborated theory is the more general approach on the system parameters definition. The main equations for the system design process were elaborated. These equations include the special control functions that are introduced into consideration artificially to generalize the total design process. Numerical results demonstrate the efficiency and perspective of the proposed approach.

Key-Words: - Time-optimal design algorithm, control theory approach.

1 Introduction

One of the main problems of the total quality design improvement is the problem of the computer time reduction for a large system design. This problem has a special significance for the VLSI electronic circuit design. The traditional system design methodology includes two main parts: the model of the system that can be described as algebraic equations or differential-integral equations and a parametric optimization procedure that achieves the cost function optimal point. By this conception it is possible to change optimization strategy and use different models and different analysis methods. However, the time of the large-scale circuit analysis and the time of optimization procedure increase when the network scale increases.

There are some powerful methods that reduce the necessary time for the circuit analysis. Because a matrix of the large-scale circuit is a very sparse, the special sparse matrix techniques are used successfully for this purpose [1]-[2]. Other approach to reduce the amount of computational required for the linear and nonlinear equations is based on the decomposition techniques. The partitioning of a circuit matrix into bordered-block diagonal form can be done by branches tearing as in [3], or by nodes tearing as in [4] and jointly with direct solution algorithms gives the solution of the problem. The extension of the direct solution methods can be obtained by hierarchical decomposition and macromodel representation [5]. An alternative approach for achieving decomposition at the nonlinear level consists on a special iteration techniques and has been realized in [6] for the iterated timing analysis and circuit simulation. Optimization technique that is used for the circuit optimization and design, exert a very strong influence on the total necessary computer time too. The numerical methods are developed both for the unconstrained and for the constrained optimization [7] and will be improved later on. The practical aspects of these methods were developed for the electronic circuits design with the different optimization criterions [8]-[9].

The system design ideas described above can be named as the traditional approach or the traditional strategy because the analysis method is based on the Kirchhoff laws.

The other formulation of the circuit optimization problem was developed in heuristic level some decades ago [10]. This idea was based on the Kirchhoff laws ignoring for all the circuit or for the circuit part. The special cost function is minimized instead of the circuit equation solving. This idea was developed in practical aspect for the microwave circuit optimization [11] and for the synthesis of high-performance analog circuits [12] in extremely case, when the total system model was eliminated. The last idea that excludes the Kirchhoff laws can be named as the modified traditional design strategy.

Nevertheless all these ideas can be generalized to reduce the total computer design time for the system design. This generalization can be done on the basis of the control theory approach and includes the special control function to control the design process. This approach consists of the reformulation of the total design problem and generalization of it to obtain a set of different design strategies inside the same optimization procedure [13]. The number of the different design strategies, which appear in the generalized theory, is equal to 2^M for the constant value of all the control functions, where M is the number of dependent parameters. These strategies serve as the structural basis for more strategies construction with the variable control functions. The main problem of this new formulation is the unknown optimal dependency of the control function vector that satisfies to the time-optimal design algorithm.

However, the developed theory [13] is not the most general. In the limits of this approach only initially dependent system parameters can be transformed to the independent but the inverse transformation is not supposed. The next more general approach for the system design supposes that initially independent and dependent system parameters are completely equal in rights, i.e. any system parameter can be defined as independent or dependent one. In this case we have more vast set of the design strategies that compose the structural basis and more possibility to the optimal design strategy construct.

2 Problem Formulation

In accordance with the new design methodology [13] the design process is defined as the problem of the cost function $C(X)$ minimization for $X \in R^N$ by the optimization procedure, which can be determined in continuous form as:

$$\frac{dx_i}{dt} = f_i(X,U), \qquad (1)$$
$$i = 1,2,...,N$$

and by the analysis of the electronic system model in the next form:

$$(1-u_j)g_j(X) = 0, \qquad (2)$$
$$j = 1,2,\dots,M$$

where $N=K+M$, K is the number of independent system parameters, M is the number of dependent system parameters, X is the vector of all variables $X = (x_1, x_2, \dots, x_K, x_{K+1}, x_{K+2}, \dots, x_N)$; U is the vector of control variables $U = (u_1, u_2, \dots, u_M)$; $u_j \in \Omega$; $\Omega = \{0;1\}$.

The functions of the right part of system (1) are depended from the concrete optimization algorithm and, for instance, for the gradient method are determined as:

$$f_i(X,U) = -b\frac{d}{dx_i}\left\{C(X) + \frac{1}{e}\sum_{j=1}^{M} u_j g_j^2(X)\right\} \qquad (3)$$

for $i = 1,2,\dots,K$,

$$f_i(X,U) = -b\cdot u_{i-K}\frac{d}{dx_i}\left\{C(X) + \frac{1}{e}\sum_{j=1}^{M} u_j g_j^2(X)\right\} + \frac{(1-u_{i-K})}{dt}\left\{-x_i' + h_i(X)\right\} \qquad (3')$$

for $i = K+1, K+2, \dots, N$,

where b is the iteration parameter; the operator $\frac{d}{dx_i}$ hear and below means $\frac{d}{dx_i} j\,(X) = \frac{\P j\,(X)}{\P x_i} + \sum_{p=K+1}^{K+M} \frac{\P j\,(X)}{\P x_p}\frac{\P x_p}{\P x_i}$, x_i' is equal to $x_i(t-dt)$; $h_i(X)$ is the implicit function ($x_i = h_i(X)$) that is determined by the system (2), *C(X)* is the cost function of the design process.

The problem of the optimal design algorithm searching is determined now as the typical problem of the functional minimization of the control theory. The total computer design time serves as the necessary functional in this case. The optimal or quasi-optimal problem solution can be obtained on the basis of analytical [14] or numerical [15]-[16] methods. By this formulation the initially dependent parameters for $i = K+1, K+2, ..., N$ can be transformed to the independent ones when u_j=1 and it is independent when u_j=0. On the other hand the initially independent parameters for $i = 1,2,\dots,K$, are independent ones always.

We have developed in the present paper the new approach that permits to generalize more the above described design methodology. We suppose now that all of the system parameters can be independent or dependent ones. In this case we need to change the equation (2) for the system model definition and the equation (3) for the right parts description.

Equation (2) defines the system model and is transformed now to the next one:

$$(1-u_i)g_j(X) = 0 \qquad (4)$$

$i = 1,2,\dots,N$ and $j \;\hat{I}\; J$

where J is the index set for all those functions $g_j(X)$ for which $u_i = 0$, $J = \{j_1, j_2, \ldots, j_z\}$, $j_s \in P$ with $s = 1, 2, \ldots, Z$, P is the set of the indexes from 1 to M, $P = \{1, 2, \ldots, M\}$, Z is the number of the equations that will be left in the system (4), $Z \in \{0, 1, \ldots, M\}$. The right hand side of system (1) is defined now as:

$$f_i(X,U) = -b \cdot u_i \frac{d}{dx_i} F(X,U) + \frac{(1-u_i)}{dt}\{-x_i(t-dt) + h(X)\} \tag{5}$$

for $i = 1,2,\ldots, N$,

where $F(X,U)$ is the generalized cost function and it is defined as:

$$F(X,U) = C(X) + \frac{1}{e} \sum_{j \in \Pi \backslash J} g_j^2(X) \tag{6}$$

This definition of the design process is more general than in [13]. It generalizes the methodology for the system design and produces more representative structural basis of different design strategies. The total number of the different strategies, which compose the structural basis, is equal to $\sum_{i=0}^{M} C_{K+M}^{i}$. We expect new possibilities to accelerate the design process in this case.

3 Numerical Results

Some non-linear passive and active electronic circuits have been analyzed to demonstrate developed general system design approach. The circuits have various nodal numbers from 3 to 5. The numerical results correspond to the optimized integration step for system (1) integration.

3.1 Example 1

The passive four-node nonlinear circuit is analyzed below (Fig. 1) on basis of the proposed general design methodology. This problem includes five independent parameters $(x_1, x_2, x_3, x_4, x_5)$, where $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5^2 = y_5$, and four originally dependent parameters (x_6, x_7, x_8, x_9), where $x_6 = V_1$, $x_7 = V_2$, $x_8 = V_3$, $x_9 = V_4$. The control vector U includes nine components $(u_1, u_2, \ldots, u_9)$.

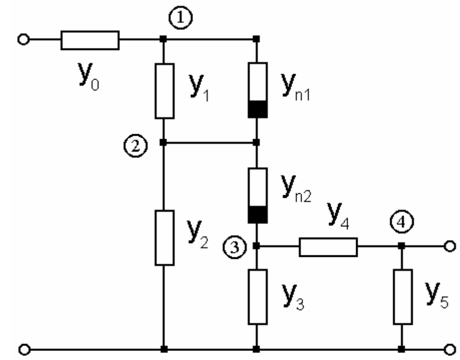

Fig. 1. Four-node circuit topology.

The mathematical model of the circuit can be writing as the next system:

$$g_1(X) \equiv y_0(V_0 - x_6) - \left[x_1^2 + a_{n1} + b_{n1}(x_6 - x_7)^2\right](x_6 - x_7) = 0$$

$$g_2(X) \equiv \left[x_1^2 + a_{n1} + b_{n1}(x_6 - x_7)^2\right](x_6 - x_7) - x_2^2 x_7 - \left[a_{n2} + b_{n2}(x_7 - x_8)^2\right](x_7 - x_8) = 0$$

$$g_3(X) \equiv \left[a_{n2} + b_{n2}(x_7 - x_8)^2\right](x_7 - x_8) - \left(x_3^2 + x_4^2\right)x_8 - x_4^2 x_9 = 0 \tag{7}$$

$$g_4(X) \equiv x_4^2 x_8 - \left(x_4^2 + x_5^2\right)x_9 = 0$$

where $y_{n1} = a_{n1} + b_{n1} \cdot (V_1 - V_2)^2$, $y_{n2} = a_{n2} + b_{n2} \cdot (V_2 - V_3)^2$. The system model (4) includes four equations where each function $g_j(X)$ is defined by (7). The optimization procedure (1) includes nine equations. System (7) is solved by the Newton-Raphson method. The cost function $C(X)$ of the design process is defined by the following form:

$$C(X) = (x_9 - k_0)^2 + (x_6 - x_7 - k_1)^2 + (x_7 - x_8 - k_2)^2. \tag{8}$$

The total number of the different design strategies that compose the structural basis of the generalized theory is equal to $\sum_{i=0}^{4} C_9^i = 256$. At the same time the structural basis of the previous developed theory includes 16 strategies only. It is clear that not all the new strategies lead to the design problem solution. Some strategies have a bad stability. Nevertheless there many new strategies that have very high design properties. The results of the structural basis strategies that include all the “old” strategies (the last 16 strategies) and some new strategies are shown in Table 1.

Table 1. Some strategies of the structural basis for four-node circuit.

N	Control functions vector U (u1,u2,u3,u4,u5,u6,u7,u8,u9)	Calculation results	
		Iterations number	Total design time (sec)
1	(1 1 1 0 1 0 0 0 1)	5	0.0031
2	(1 1 1 1 1 0 0 0 1)	397	0.4312
3	(1 1 1 0 1 1 0 0 1)	5	0.0029
4	(1 1 0 1 1 1 1 1 0)	119	0.0209
5	(1 1 1 1 0 0 1 0 1)	101	0.0232
6	(1 1 1 0 1 0 0 1 1)	15	0.0134
7	(1 1 1 0 1 1 1 0 1)	5	0.0009
8	(1 1 1 0 1 1 1 1 1)	101	0.0243
9	(1 1 1 1 0 0 1 1 1)	185	0.0324
10	(1 1 1 1 0 1 0 0 1)	74	0.0102
11	(1 1 1 1 0 1 0 1 1)	121	0.0254
12	(1 1 1 1 0 1 1 1 1)	159	0.0127
13	(1 1 1 1 1 0 0 0 0)	33	0.0263
14	(1 1 1 1 1 0 0 0 1)	397	0.4317
15	(1 1 1 1 1 0 0 1 0)	6548	7.1392
16	(1 1 1 1 1 0 0 1 1)	76	0.0122
17	(1 1 1 1 1 0 1 0 0)	456	0.5113
18	(1 1 1 1 1 0 1 0 1)	24	0.0052
19	(1 1 1 1 1 0 1 1 0)	3750	4.3661
20	(1 1 1 1 1 0 1 1 1)	90	0.0095
21	(1 1 1 1 1 1 0 0 0)	68	0.0354
22	(1 1 1 1 1 1 0 0 1)	596	0.6213
23	(1 1 1 1 1 1 0 1 0)	5408	6.2191
24	(1 1 1 1 1 1 0 1 1)	78	0.0255
25	(1 1 1 1 1 1 1 0 0)	238	0.2104
26	(1 1 1 1 1 1 1 0 1)	77	0.0227
27	(1 1 1 1 1 1 1 1 0)	139	0.0131
28	(1 1 1 1 1 1 1 1 1)	131	0.0103

The strategy 13 is the traditional one. There are seven different strategies among “old” group that have the design time less that the traditional strategy. These are the strategies 16, 18, 20, 24, 26, 27 and 28. The strategy 18 is the optimal one among all the “old” strategies and it has the time gain 5.06 with respect to the traditional design strategy. On the other hand the best strategy among all the strategies (number 7) of the Table 1 has the time gain 29.2. So, we have the additional acceleration 5.77 times. This effect was obtained on basis of more extensive structural basis and servers as the principal result of the new generalized methodology. The posterior analysis and the control vector U optimization can increase this time gain as shown in [17].

3.2 Example 2

In Fig. 2 there is a circuit that has 6 independent variables as admittance $y_1, y_2, y_3, y_4, y_5, y_6$ ($K=6$) and 5 dependent variables as nodal voltages V_1, V_2, V_3, V_4, V_5 ($M=5$) at the nodes 1, 2, 3, 4, 5.

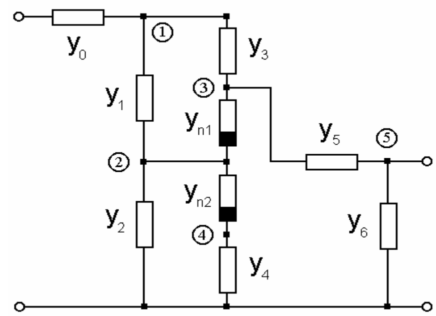

Fig. 2. Five-node circuit topology.

The nonlinear elements have next dependency: $y_{n1} = a_{n1} + b_{n1} \cdot (V_3 - V_2)^2$, $y_{n2} = a_{n2} + b_{n2} \cdot (V_4 - V_2)^2$. The vector X includes eleven components. The first six components are defined as: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5^2 = y_5$, $x_6^2 = y_6$. The others components are defined as: $x_7 = V_1$, $x_8 = V_2$, $x_9 = V_3$, $x_{10} = V_4$, $x_{11} = V_5$. The control vector U includes eleven components too. The total structural basis includes 1024 different strategies in the limits of the new approach. The previous structural basis includes 32 strategies only.

The mathematical model (4) of this circuit is defined on the basis of nodal method and includes five equations in this case. The optimization procedure includes eleven equations and it is based on formulas (1) and (5). The cost function $C(X)$ is defined by the formula similar to (8) with the necessary index correction for all the components:

$$C(X) = (x_{11} - kk_0)^2 + \left[(x_8 - x_9)^2 - kk_1\right]^2 + \left[(x_9 - x_{10})^2 - kk_2\right]^2.$$

The results for old structural basis strategies are shown in Table 2a for those strategies that have the computer time less than the traditional one. The results for some new structural basis strategies are shown in Table 2b. The strategy 1 of Table 2a is the traditional one. The time gain of the best old strategy (23 from Table 2a) with respect to the traditional strategy is equal to 1158. This is a significant time gain, but we have more perspective strategies between the new structural basis. The design time for strategies 11,12,14,15 from Table 2b is less than the best strategy 23 from Table 2a.

Table 2a. Some strategies of old structural basis.

N	Control functions vector U (u1,u2,u3,u4,u5,u6,u7,u8,u9,u10,u11)	Calculation results	
		Iterations number	Total design time (sec)
1	(1 1 1 1 1 1 0 0 0 0 0)	15026	11.587
2	(1 1 1 1 1 1 0 0 0 1 1)	4387	1.522
3	(1 1 1 1 1 1 0 0 1 1 0)	1479	2.043
4	(1 1 1 1 1 1 0 0 1 1 1)	340	0.041
5	(1 1 1 1 1 1 0 1 0 1 0)	1480	1.743
6	(1 1 1 1 1 1 0 1 0 1 1)	563	0.072
7	(1 1 1 1 1 1 0 1 1 0 0)	154	0.021
8	(1 1 1 1 1 1 0 1 1 0 1)	174	0.023
9	(1 1 1 1 1 1 0 1 1 1 0)	368	0.043
10	(1 1 1 1 1 1 0 1 1 1 1)	688	0.051
11	(1 1 1 1 1 1 1 0 0 1 0)	65	0.011
12	(1 1 1 1 1 1 1 0 0 1 1)	4312	0.821
13	(1 1 1 1 1 1 1 0 1 0 0)	5601	7.112
14	(1 1 1 1 1 1 1 0 1 0 1)	854	0.081
15	(1 1 1 1 1 1 1 0 1 1 0)	483	0.052
16	(1 1 1 1 1 1 1 0 1 1 1)	367	0.031
17	(1 1 1 1 1 1 1 1 0 0 0)	354	0.352
18	(1 1 1 1 1 1 1 1 0 0 1)	548	0.063
19	(1 1 1 1 1 1 1 1 0 1 0)	98	0.012
20	(1 1 1 1 1 1 1 1 0 1 1)	1144	0.102
21	(1 1 1 1 1 1 1 1 1 0 0)	80	0.009
22	(1 1 1 1 1 1 1 1 1 0 1)	535	0.044
23	(1 1 1 1 1 1 1 1 1 1 0)	194	0.01
24	(1 1 1 1 1 1 1 1 1 1 1)	254	0.011

Table 2b. Some strategies of new structural basis.

N	Control functions vector U (u1,u2,u3,u4,u5,u6,u7,u8,u9,u10,u11)	Calculation results	
		Iterations number	Total design time (sec)
1	(1 0 1 1 1 1 0 1 1 1 1)	95361	24.254
2	(1 0 1 1 1 1 1 1 0 1 1)	16457	14.521
3	(1 0 1 1 1 1 1 1 1 0 1)	2649	0.311
4	(1 0 1 1 1 1 1 1 1 1 0)	458	0.901
5	(1 1 1 0 0 1 1 1 1 1 1)	227	0.201
6	(1 1 1 0 1 0 1 1 1 1 1)	956	0.109
7	(1 1 1 0 1 1 0 1 1 1 1)	958	0.111
8	(1 1 1 0 1 1 1 0 1 1 1)	1369	0.162
9	(1 1 1 0 1 1 1 1 0 1 1)	1352	0.141
10	(1 1 1 0 1 1 1 1 1 1 0)	13556	1.733
11	(1 1 1 1 0 1 0 0 0 0 1)	5	0.001
12	(1 1 1 1 0 1 0 0 0 1 1)	20	0.002
13	(1 1 1 1 0 1 0 1 1 1 1)	134	0.011
14	(1 1 1 1 0 1 1 0 1 1 1)	51	0.0095
15	(1 1 1 1 0 1 1 1 0 1 1)	45	0.0022
16	(1 1 1 1 0 1 1 1 1 0 1)	82	0.012
17	(1 1 1 1 0 1 1 1 1 1 1)	142	0.013
18	(1 1 1 1 1 0 0 1 1 1 1)	221	0.032
19	(1 1 1 1 1 0 1 0 1 1 1)	742	0.091
20	(1 1 1 1 1 0 1 1 0 1 1)	77	0.011
21	(1 1 1 1 1 0 1 1 1 0 1)	266	0.033

The best strategy 11 has the time gain 11587, i.e. ten times more. These examples show that the time gain of the new structural basis increases when the circuit size and complexity increase.

3.3 Example 3

It is interesting to analyze the active circuit with at least one transistor. This circuit is shown in Fig. 3.

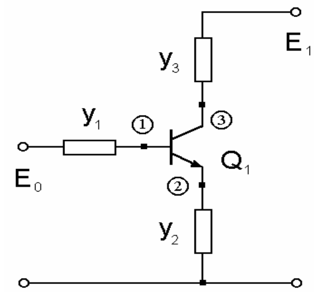

Fig. 3. One transistor amplifier.

In this case there are three independent variables y_1, y_2, y_3 as admittance ($K=3$) and three dependent variables V_1, V_2, V_3 as nodal voltages ($M=3$). The state parameter vector X includes six components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4 = V_1$, $x_5 = V_2$, $x_6 = V_3$. The design process has been realized on DC mode. The Ebers-Moll static model of the transistor has been used. The cost function $C(X)$ has been determined as the sum of the squared differences between beforehand-defined values and current values of the voltages for the transistor junctions. The old structural basis includes 8 strategies only, and the new basis includes 32 strategies. The results of this circuit design are shown in Tables 3a and 3b. Table 3a includes all strategies of old structural basis and Table 3b includes some strategies of new structural basis.

Table 3a. Old structural basis strategies.

N	Control functions vector U (u1, u2, u3, u4, u5, u6)	Calculation results	
		Iterations number	Total design time (sec)
1	(1 1 1 0 0 0)	826	3.108
2	(1 1 1 0 0 1)	707	1.813
3	(1 1 1 0 1 0)	1791	4.594
4	(1 1 1 0 1 1)	1224	2.709
5	(1 1 1 1 0 0)	887	2.163
6	(1 1 1 1 0 1)	153	0.335
7	(1 1 1 1 1 0)	1045	2.222
8	(1 1 1 1 1 1)	309	0.217

The best strategy of old basis (8 from Table 3a) has time gain 14.3. The best strategy of new basis (1 from Table 3b) has time gain 58.6. So, we have an additional acceleration more than 4 times. This is the main result of new generalized system design methodology.

Table 3b. Some strategies of new structural basis.

N	Control functions vector U (u1, u2, u3, u4, u5, u6)	Calculation results	
		Iterations number	Total design time (sec)
1	(1 0 1 1 1 1)	30	0.053
2	(1 1 0 1 1 1)	778	1.391
3	(1 0 1 1 1 0)	5599	25.094
4	(0 1 1 1 0 0)	1285	10.902
5	(0 1 1 1 1 0)	3015	10.998
6	(0 1 1 1 0 1)	47	0.089
7	(1 1 0 0 1 1)	174	0.465
8	(1 1 0 1 0 1)	606	1.223

4 Conclusion

The traditional method for the analog circuit design is not time-optimal. The problem of the optimal algorithm construction can be solved more adequately on basis of the optimal control theory application. The time-optimal design algorithm is formulated as the problem of the functional optimization of the optimal control theory. In this case it is necessary to select one optimal trajectory from quasi-infinite number of different design strategies that are produced. The new and more complete approach to the electronic system design methodology has been developed now by means of broadened structural basis definition. The total number of the different design strategies, which compose the structural basis by this approach, is equal to $\sum_{i=0}^{M} C_{K+M}^{i}$. This new structural basis serves as the necessary set for the optimal design strategy search. This basis includes new and very perspective strategies that can be used for the time-optimal design algorithm construction. This approach can reduce considerably the total computer time for the system design. Analysis of the different electronic systems gives the possibility to conclude that the potential computer time gain that can be obtain by means of the broadened structural basis is significantly larger than for previous developed methodology.

Acknowledgment

This work was supported by the Mexican National Council of Science and Technology CONACYT, under project SEP-2004-C01-46510.

References:

[1] J.R. Bunch and D.J. Rose, (Eds.), *Sparse Matrix Computations*, Acad. Press, N.Y., 1976.

[2] O. Osterby and Z. Zlatev, *Direct Methods for Sparse Matrices*, Springer-Verlag, N.Y., 1983.

[3] F.F. Wu, Solution of Large-Scale Networks by Tearing, *IEEE Trans. Circuits Syst.*, Vol. CAS-23, No. 12, 1976, pp. 706-713.

[4] A. Sangiovanni-Vincentelli, L.K. Chen and L.O. Chua, An Efficient Cluster Algorithm for Tearing Large-Scale Networks, *IEEE Trans. Circuits Syst.*, Vol. CAS-24, No. 12, 1977, pp. 709-717.

[5] N. Rabat, A.E. Ruehli, G.W. Mahoney and J.J. Coleman, A Survey of Macromodeling, *Proc. of the IEEE Int. Symp. Circuits Systems*, April, 1985, pp. 139-143.

[6] A.E. Ruehli and G. Ditlow, Circuit Analysis, Logic Simulation and Design Verification for VLSI, *Proc. IEEE*, Vol. 71, No. 1, 1983, pp. 36-68.

[7] R. Fletcher, *Practical Methods of Optimization*, John Wiley and Sons, N.Y., Vol. 1, 1980, Vol. 2, 1981.

[8] R.K. Brayton, G.D. Hachtel and A.L. Sangiovanni-Vincentelli, A survey of optimization techniques for integrated-circuit design, *Proc. IEEE*, Vol.69,1981,pp.1334-1362.

[9] R.E. Massara, *Optimization Methods in Electronic Circuit Design*, Longman Scientific & Technical, Harlow, 1991.

[10] I.S. Kashirsky and Ia.K. Trokhimenko, *The Generalized Optimization of Electronic Circuits*, Tekhnika, Kiev, 1979.

[11] V. Rizzoli, A. Costanzo and C. Cecchetti, Numerical optimization of broadband nonlinear microwave circuits, *IEEE MTT-S Int. Symp.*, Vol. 1, 1990, pp. 335-338.

[12] E.S. Ochotta, R.A.Rutenbar and L.R. Carley, Synthesis of High-Performance Analog Circuits in ASTRX/OBLX, *IEEE Trans. on CAD*, Vol.15, No. 3, 1996, pp. 273-294.

[13] A.M. Zemliak, Analog System Design Problem Formulation by Optimum Control Theory, *IEICE Trans. on Fundament.*, Vol. E84-A, No. 8, 2001, pp. 2029-2041.

[14] L.S. Pontryagin, V.G. Boltyanskii, R.V. Gamkrelidze and E.F. Mishchenko, *The Mathematical Theory of Optimal Processes*, Interscience Publishers, Inc., New York, 1962.

[15] J.B. Rosen, Iterative Solution of Nonlinear Optimal Control Problems, *J. SIAM, Control Series A*, 1966, pp. 223-244.

[16] R. Pytlak, *Numerical Methods for Optimal Control Problems with State Constraints*, Springer-Verlag, Berlin, 1999.

[17] A. Zemliak, Novel Approach to the Time-Optimal System Design Methodology, *WSEAS Trans. on Systems*, Vol. 1, No. 2, 2002, pp. 177-184.