## A Method for Designing CDO Conformed to Investment Parameters

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*Abstract:* - We have proposed the method for designing CDO (Collateralized Debt Obligation) that meets investor needs of attributes of CDO and minimizes capital loss risks that the agent takes. Then we assumed fictitious obligation pool and investor needs and constructed portfolio using the method. We confirmed that opportunity losses and dead stock losses did not occur and the capital loss risks stayed within the practical range.

Key-Words: - market-based financing, collateralized debt obligations, investment parameter, risk minimization

### **1** Introduction

Recently market-oriented indirect financing have gotten a lot of attention, and the market of CDO (Collateralized Debt Obligation), which is one of a form of market-oriented indirect financing, is expanding more and more. CDOs are issued in various schemes, and they are categorized variously according to underlying asset, the way of management of the CDO cash flow, the motive of trade etc. [1]. In the scheme of issuing traditional CDOs like those, the result of tranching is limited by the natures of underlying asset and the target rating of each tranche. So we cannot design CDOs importing investor needs of their merchantability that are credit capability, purchase amount, etc. Therefore dead stocks and opportunity losses may occur.

In this paper we propose the method for designing CDOs adjusting investment parameters and minimizing capital loss risks of the issuer.

### 2 Traditional Method for Issuing CDO

### 2.1 Traditional Method for Issuing CDO

We illustrate a typical procedure for issuing and repaying CDO with Fig. 1.

- (1) A bank lends small and medium-sized enterprises (SME) some money. The bank is called "originator" by occasion of holding obligations that are origins of CDOs.
- (2) Originator sells SPC (Special Purpose Company) the pool of loan obligations.



Fig. 1. Traditional scheme for issuing CDOs

- (3) The SPC gets information of credit risk of the obligations from the originator or something like rating agency, and sets target rating to each tranche, and decides the amount of issuance of each tranche tailored to the target rating [2][3]. The number of tranches is usually 2 to 5.
- (4) The SPC sells (institutional or retail) investors CDOs. However the most subordinated tranche, equity, is sold to prescribed institution.
- (5) The originator gets repayment of the loan obligations from SMEs at the expiration date.
- (6) The originator transfers the repayment money to the SPC.
- (7) The SPC pays off investors according to the total amount of repayment money.

### 2.2 Traditional Portfolio of CDO

Each tranche that are created by SPC has a contract condition. A contract condition is a rule of redemption, which establishes the relationship between the total repayment and redemption rate, e.g. "this tranche is paid off \_\_ yen per unit of face value when total



Fig. 2. Traditional CDO portfolio

repayment is \_\_\_\_\_yen." Contract conditions and amounts of issuance of all tranches are necessary items that have to be decided at CDO issuance, and we define them "CDO portfolio" in this paper.

Fig. 2 is an example of traditional CDO portfolio. It has three tranches, A, B, and Equity. Their amounts of issuance are 10 B yen, 6 B yen and 9 B yen respectively, and therefore CDOs of 25 B yen in total are issued. Contract conditions are represented as relationships among repayments and redemptions as shown in Fig. 2. The contract condition of tranche A, for example, is "none of the principal is redeemed when the repayment is 0, a part of the principal is redeemed when the repayment is 0 to 10, and all of the principal is redeemed when the repayment is more than 10." Redemption amount per unit of face value or redemption rate at partial redemption.

In the example of Fig. 2 the border of contract condition between A and B (that is 10 B yen) and the one between B and Equity (that is 16 B yen) are correspond to the border of issue amount between A and B and the one between B and Equity respectively. At the traditional CDO portfolio contract condition and issue amount are integrated like this, so we cannot construct a portfolio adjusted to investor needs even if the needs are known. Thereby the opportunity of liquidation of obligations may be lost as a result of mismatch between tranches based on the obligation pool and investor needs, accrual of dead stock of CDOs, and appearance of investors who could not find CDOs that have the merchantability they expect.

# 3 Needs-based CDO Design Method

In order to solve the problem of the traditional method for issuing CDO we propose "a method for designing





CDO conformed to investment parameters." The following is the details of the method.

### 3.1 Approach

Investors input these two parameters as the investment parameters for the purpose of reflecting investor needs to CDO portfolio, and SPC derives a portfolio after collecting a certain number of investment parameters.

Credit Capability:

The value concerned with the credit risk of the CDO the investor desires. Expectation of amount of principal redemption per unit of face value is applied in this paper.

Purchase Amount:

The amount of money the investor is to put in. We removed the constraint of alignment of contract condition borders and issue amount borders, so that each contract condition and issue amount is decided independently and investor needs about credit capability and purchase amount can be reflected to the CDO portfolio. As shown in Fig. 3 contract conditions of CDO are decided based on investor needs about the credit capability, whereas issue amounts are decided based on investor needs about the purchase amount. That makes it possible to design portfolios based on investor needs.

### **3.2** A New Risk Caused by Separation of Contract Condition and Issue Amount

If you permit the disagreement between contract condition borders and issue amount borders mentioned above, there is a possibility that the amount of redemption money differs from the one of repayment money at the expiration date. Fig. 4 shows an example of a portfolio, repayment-by-repayment surplus or deficit of the cash for redemption, and



repayment-by-repayment possibility (in other words probability density function). Taking note of the instance when the total repayment is 5 B yen, you can read the contract condition which indicates that only creditors of tranche A can get full redemption and creditors of the

# Fig. 4. Examples of portfolio and surplus or deficit of redemption money

others cannot get any redemption, whereas the issue amount of tranche A is 3 B yen, so the amount of repayment exceeds the one of redemption by 2 B yen (= 5 B yen 3 B yen). When the total repayment is 19 B yen, the amount of repayment is below the one of redemption by 3 B yen to the contrary. Thus the risk of surplus or deficit of redemption money occurs to the SPC as a result of separation between contract condition and issue amount.

However taking such risks and receiving premiums in return are not primary services of SPC. So we suggest transferring the risk into a certain guarantee institution. The guarantee institution receives risk premiums, makes up a shortfall when the redemption money runs short, and gets redundancy when the repayment money exceeds the redemption money. Now we define the amount of money guarantee institution receives or pays at the expiration date as *PL (Profit and Loss) of guarantee institution.* PL of guarantee institution receives the surplus and is negative when it covers shortfall.

Making the PL expectation of guarantee institution equal to the risk premium is equivalent to paying the guarantee institution the risk premium. E(P), the PL expectation of guarantee institution with a portfolio P is expressed following equation.



Fig. 5. Deriving probability density function from obligation data



Fig. 6. Incorporate investment prameters into CDO portfolio

*x* is the amount of repayment, W(P,x) is the PL of guarantee institution, f(D,x) is the probability density function of the amount of repayment with the obligation pool *D*, and *s* is the maximal value *x* can take. When the risk premium is *G*, E(P)=G is the constrained condition of determining the portfolio.

The smaller risk the guarantee institution takes the better. We use "the largest loss" as the parameter that represents the risk the guarantee takes. The largest loss is the value given by multiplying the minimum value of PL by minus 1, or 100% VaR, which is expressed as the length of "a" in Fig. 4. Minimizing the largest loss is the objective function.

### **3.3** Outline of the Method

First, derive the probability density function of the total repayment based on the obligation data represented in Fig. 5 using a certain approximate algorithm such as binomial expansion technique model, CreditRisk+ model, Monte Carlo simulation, etc.[4-7].



Fig. 7. Image of generating contract conditions

On the other hand investment parameters are accumulated as the table at the upper left in Fig. 6. Group them according to their credit capabilities, determine tranches of investor needs by allocating 1 tranche per the group (the table at the upper right in Fig. 6), and then assign the tranches of investor needs to the CDOs for investors in the portfolio (the table at the bottom in Fig. 6). You can adopt any ways for separating the credit capability at grouping investment parameters, i.e. separating it (i) at constant intervals or (ii) so that the total purchase amount per group becomes a certain value, and also adopt any values as the required credit capability of the tranche of investor needs, i.e. (i) an average of the credit capabilities in the group or (ii) 50 percentile of the credit capabilities in the group.

In addition to CDOs for investors, super senior debt (SS debt), which has the highest credit capability and lowest profitability, and equity, which has the lowest credit capability and highest profitability are made as CDOs for surplus securities negotiation agency. Total profitability of CDOs can be controlled by changing their credit capabilities and issue amounts.

Determining a portfolio is nothing less than assigning values to the variables a to l in Fig. 6. *Left* of contract condition in Fig. 6 means the largest value in the amount of repayment at which investors can receive no redemption from the tranche, and is 5 B yen on tranche B in Fig. 4. On the other hand *right* in Fig. 6 means the smallest value in the amount of repayment at which investors can receive full redemption from the tranche, and is 14 B yen on tranche B in Fig. 4. *Left* and *right* of a security  $S_k$  are  $L_k$  and  $R_k$  respectively in the sentence below.

The constraint conditions and the objective function for determining a portfolio are following.

Constraint Conditions:

- (1) Credit capabilities of CDOs for investors that are calculated from probability density function of repayment and their contract conditions are equal to required credit capabilities based on the investment parameters.
- (2) Expected value of profit and loss of guarantee institution is equivalent of the guarantee charge.
- *Objective Function:* Minimizing the largest loss. Portfolio decision problem is minimizing the largest loss under the constraint conditions mentioned above and the inputs of obligation data and investment parameters. A portfolio derived as a solution to the problem is defined "best portfolio."

We decide the best portfolio by following two steps.

- (a) Derive any portfolios that fulfill the constraint conditions. The credit capabilities of SS and equity, which are necessary to calculation of *expected value of PL of guarantee institution* in the constraint condition (2), is derived by fixing the contract conditions of them. So the procedure for deriving the portfolios is, first work out any contract conditions that meet the constraint condition (1), and secondly calculate issue amount for each contract condition so that each portfolio satisfy the constraint condition (2).
- (b) Derive the largest loss of above each portfolio, and identify the portfolio that has the smallest largest loss. Assuming that the redemption rate of the part redemption is calculated by linear interpolation method, the minimum value of PL of guarantee institution is observed only when repayment value x is any one of  $R_0, R_1, ..., R_{n+1}$ .

Next we go into details about each step.

### **3.4** Generation of Portfolios

The  $C_k$  that satisfies following Eq. (2) is in the range of  $L_k$   $C_k$   $R_k$  as long as constraint condition (1) is fulfilled.

$$\int_{C_k}^{s} f(x) dx = U_k \tag{2}$$



Fig. 8. Profit and loss of guarantee institution

 $U_k$  is a required credit capability of CDOs for investors,  $S_k$ . We describe the  $U_k$  as *center of contract condition* of the securities  $S_k$ .

Deriving  $C_k$  of CDOs for investors, you can limit the range that  $L_k$  and  $R_k$  can exist. Generate contract condition patterns by means of changing *left* and *right* of each tranche  $\delta$  by  $\delta$ (which is a minute value) in the limited range. As a result you can derive any contract conditions that meet constraint condition (1). Fig. 7 is an image of generating contract conditions.

Determine issue amount for each contract condition so that constraint condition (2) can be met. E(P), the expected value of PL, which is represented in Eq. (1), includes f(D,x), the probability density function of the amount of repayment. f(D,x) is an internally-calculated function at SPC based on the obligation data. So it is difficult for f(D,x) to bear an endorsement from an impartial party like a rating agency. Therefore we use E(P) at Eq. (3), which is expressed as the difference between the profitability of whole obligation pool and the one of whole CDOs. Determine issue amounts of SS and equity so that E(P) is equal to the risk premium *G*.

$$E(P) = \sum_{j} v_{j} t_{j} y_{j} - \sum_{k} V_{k} T_{k} Y_{k} \qquad .....(3)$$

 $v_j$ ,  $t_j$ ,  $y_j$  are face value, credit capability (=1 default ratio), and yield of underlying obligation  $D_j$  respectively, and  $V_k$ ,  $T_k$ ,  $Y_k$  are face value, credit capability, and yield of tranche  $S_k$  respectively.  $T_k$  is calculated based on contract conditions of CDOs and f(D,x), but it can be endorsed by rating agencies indirectly by acquiring credit ratings to the CDOs from the rating agencies before or sometime after the

Table 1. Investor needs

tranches	credit capability (yield)	purchase amount
Α	99.90% (1.0103)	600 M yen
В	99.00% (1.0222)	800 M yen
С	97.50% (1.0426)	900 M yen

CDO issuance. In the result E(P) also can be endorsed.

#### 3.5 Identification of the Best Portfolio

You do not need to calculate w(x), the PL of the guarantee institution, corresponding to all x, the amount of repayment, but corresponding to only n+2 of x which are  $R_0$ ,  $R_1$ , ..., and  $R_{n+1}$  (when the number of CDOs for investors are n). We explain this as follows.

Fig. 8 shows w(x) of a certain portfolio. Assuming that the redemption rate of the part redemption is calculated by linear interpolation method, w(x) is an upward or downward straight line in the section  $[L_k, R_k]$ . In the section of the gap between two contract conditions such as  $[R_0, L_1]$  in Fig. 8, w(x)increases as much as repayment increases, so it is a straight line of which gradient is 1. Therefore it is obvious that the x at which w(x)shows the minimum value is any one of the value  $L_0$ ,  $L_1$ , ...,  $L_{n+1}$ ,  $R_0$ ,  $R_1$ , ..., and  $R_{n+1}$ . Additionally  $L_k$  which share borders with gaps such as  $L_1$  in Fig. 8 cannot be local minimum points because w(x) is less than  $w(L_k)$  in relation to x that is neighborhood of  $L_k$  and is less than  $L_k$ . Hence the largest loss of the portfolio or the minimum value of w(x) is any one of  $w(R_k)$ (k=0,1,...,n+1).

### **4 Prototype Experiment**

We assumed fictitious investor needs and an obligation pool and derived the best portfolio using a prototype application of proposed method compared to the traditional method on which contract conditions and issue amounts influence each other.

Table 1 shows the hypothetical investor needs. The credit capabilities of A, B and C are set in reference to 5-year-elapsed cumulative average broad-definition default ratios of the ratings "AA", "A" and "BBB" respectively which are defined by R&I (Rating and Investment Information, Inc.)[8]. Accordingly the credit capabilities of A, B and C are



considered to be equivalent to those of 5-year obligations that have above ratings.

First we designed CDOs using the proposed method. The guarantee charge was 150 M yen, and the minute value  $\delta$  that was taken into account at generating contract conditions was 25 M yen, which is the total face value of obligations divided by 1000. The yield of CDO was calculated on the basis of following equation.

yield = 
$$\frac{1.009 + 0.3 \times (1 - \text{credit capability})}{\text{credit capability}}$$

The proposed method derived the best portfolio of which the largest loss was 0.67 B yen and lead to no dead stock loss and no opportunity loss because the CDOs of A to C was structured in just proportion according to the investor needs. The largest loss, 0.67 B yen, is not much by any means compared to 1,020 B yen, which is the total amount of payment under guarantee for fiscal 2004 in Japan by National Federation of Credit Guarantee Corporations [9].

Fig. 9 shows f(x), line graph of PL, and the contract condition in the section [23 B, 25 B]. The f(x) in Fig. 9 was derived by Monte Carlo method of which the trial number was 1 million. The PL keeps positive until the total repayment reaches 24.1 B yen, and stands the minimum value, minus 0.67 B yen, at the *right* of equity, 24.525 B yen.

Then we structured CDOs using the traditional method, with the result that the issue amounts of tranche A, B, C and Equity was 24.770, 0.026, 0.153, and 0.353 B yen respectively. Comparing those values to the investor needs, a lot of dead stock loss and opportunity loss are occurred.

We also examined the sensitivity of the largest loss of the best portfolio to changes of the mean and the standard deviation of f(x), assuming that f(x) is represented by the normal distribution. Fig. 10 shows the result of the experiment in



Fig. 10 Sensitivity of the largest loss to the changes of the mean of f(x)



Fig. 11 Sensitivity of the largest loss to the changes of the standard deviation of f(x)

which we fixed the standard deviation of f(x) at the one in Fig. 9 and changed the mean of f(x). Fig. 11 shows the result of the experiment in which we fixed the mean of f(x) at the one in Fig. 9 and changed the standard deviations of f(x). The horizontal axis in Fig. 11 is represented by ratios to the standard deviation of f(x) in Fig. 9. These results suggest that the largest loss is sensitive to the collective credit capability of the underlying assets rather than the variance of them.

### **5** Conclusion

In our research we proposed the method for bridging the gap between debtor needs and investor needs adjusting CDO portfolio to investment parameters, and minimizing capital loss risks caused by such an adjustment. We designed CDOs using a prototype application of the proposed method and confirmed that opportunity losses and dead stock losses did not occur and the capital loss risks stayed within the practical range.

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# Fig. 9 Contract condition and PL of the best portfolio

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