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Optimal Short-Term Contract Allocation Using Particle Swarm Optimization

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Abstract: - In a liberalized electricity market, participants have several types of contracts to sell or buy electrical energy. Increasing electricity markets liquidity and, simultaneously, providing to market participants tools for hedging against spot electricity price were the two main reasons for the appearance of those types of contracts. However, due to the payoff nonlinearity characteristic of those contracts, deciding the optimal portfolio that best adjusts to their necessities becomes a hard task. This paper presents an optimization model applied to optimal contract allocation using Particle Swarm Optimization (PSO). This optimization model consists on finding the portfolio that maximizes the electricity producer results and simultaneously allows the practice of the hedge against the volatility of the System Marginal Price (SMP). Risk management is considered through the consideration of a mean-variance optimization function. An example for a programming period is presented using spot, forward and options contracts. PSO performance in such type of problems is evaluated by comparing it with the Genetic Algorithms (GA).

Key-Words: - Particle Swarm Optimization (PSO), Electricity Markets, Risk Management, Contracts

1 Introduction

Power systems have suffered on the last decades profound regulatory changes on the way that they were operated. Traditional vertically integrated power systems saw their organizational and operational structure completely changed. In countries where those changes occurred, we assisted to a complete separation between several activities (generation, transmission, distribution). However, transmission and distribution activities maintain the natural monopoly status.

In competitive electricity markets, charge characteristics (like seasonality, mean-reversion, stochastic growth), producers characteristics (like the technology used in generation, generators availability, fuel prices [1]) and technical constraints introduce big challenges but also big risk, like, for example, high price volatility.

Before the restructuration process, the model traditionally used was based on monopoly and regulated public utilities. Prices were stable and predictable over a relatively long time horizon and, therefore, the risk involved in the energy business was low. However, this has dramatically changed. The liberalization of the electric sector, besides the price volatility introduction, leads to a clear competition in several sectors of activity and in particular in the generation sector. Power producers have to change the way they do their business and evolve from monopoly to unbundled companies structure on direct competition. Power producers have to adapt themselves to the new reality, having to reduce the overcapacity by closing power plants or abandoning plans for the construction of new ones. They also have to rethink the entire productive process and study the possibility of constructing new power stations using new technologies.

In the new competitive environment, energy can be negotiated in a spot market, managed by the Market Operator (MO), where Producers and the Load Serving Entities (LSE) sell or buy, respectively, the energy on a half-hour or hour basis. The supply and demand bids are then aggregated creating the supply and demand curve, which sets the quantity of electric power to be traded at the price given by the intersection point of these curves, usually known as System Marginal Price (SMP). However, due to the demand and producer’s characteristics [2], the SMP is very volatile, being very difficult to predict.

This leads the agents of those markets to search for hedging tools that allow them to turn their results more predictable [3,4]. Responding to that necessity, derivatives markets allow negotiating contracts for which the underlying active is the electric energy. Those markets allow electric energy market participants to practice the hedge against the volatility of the SMP and simultaneously allow them to reduce the risk of credit and to turn the market more liquid.

Derivatives markets negotiate forward, futures and options contracts. The main difference between forward and futures contracts is that forward comprise the physical delivery of the negotiated electric energy and futures contracts are exclusively of financial type. Options contracts are similar to forward and futures having as main difference the fact that options give the buyer the power to decide or not the exercise of the option. For that he has to pay previously a certain amount of money designated by premium. Options exercise can be physical or financial. They are normally associated to forward contracts when they comprise the physical delivery of the underlying asset.

However, to make use of these types of contracts, agents of electricity markets (and producers in particular) need models to evaluate the correct price of contracts. Decision-support systems are required to define the type of contracts to establish and their characteristics.

Characteristics of electricity prices, such as: mean-reversion, high degree of skewness and non-constant volatility, exclude price modelling using commodity cost-of-carry models. So, Black & Scholes formula is not applied to electricity option pricing. A procedure to evaluate the option price in electricity markets, known as risk-neutral valuation, is presented in [2] and [5]. Binomial model could also be applied to evaluate options price on electricity markets but it requires some adjustments.

Finding the optimal portfolio that maximizes results and simultaneously practices the hedge against the volatility of the System Marginal Price (SMP) seems to be a hard task in electricity markets. An approach based on the efficient frontier concept is proposed in [6] and an approach based on maximization of a mean-variance function using genetic algorithms to solve the stochastic optimization problem is proposed in [5].

In this paper, we introduce a new approach to the problem, making use of Particle Swarm Optimization [7] to find the optimal solution. A mean-variance optimization function is used to maximize the results and simultaneously to practice the hedge against the volatility of the SMP. Scenario prediction is not addressed in this paper; however, in [8], we present a technique to find the maximum and the minimum SMP for a programming period with a certain confidence level α.

At least, to demonstrate that PSO is a very successful meta-heuristic, namely, with problems of this complexity, a comparison is made with a Evolutionary Programming (EP) technique called Genetic Algorithms (GA).

2 Particle Swarm Optimization

Particle swarm optimization has roots on artificial life in general and on bird flocking, fish schooling and swarming theory in particular [7].

On a given iteration, a set of solutions called "particles" move around the search space from one iteration to another according to rules that depend on three factors [9]: inertia (the particles tend to move in the direction they have previously moved), memory (the particles tend to move in the direction of the best solution found so far on their trajectory) and cooperation (the particles tend to move in the direction of the global best solution, that is, in the direction found by all particles).

The movement rule followed by each particle can be expressed has:

$$X_i^{new}=X_i+V_i^{new} \tag{1}$$

Where:

X_i^{new}	represents the new position of particle i
X_i	represents the previous position of particle i
V_i^{new}	represents the velocity of particle i and is given by equation (2)

$$V_i^{new} = dec(t) \cdot V_i + rand_{i,k} \cdot \alpha_{i,k} \cdot (pbest_i - X_i) + rand_{i,j} \cdot \alpha_{i,j} \cdot \left[pbest(gbest) - X_i\right] \tag{2}$$

Where:

$\alpha_{i,k}$	represents a weight fixed at the beginning of the process designated by cognitive acceleration parameter
$\alpha_{i,j}$	represents a weight fixed at the beginning of the process designated by social acceleration parameter
$rand_{i.k}$ $rand_{i,j}$	represent random numbers from a uniform distribution on [0,1]
dec(t)	represents a function that will decrease with the iteration number reducing the importance of inertia term
$pbest_i$	represents the best position found so far by particle i
pbest(gbest)	represents the best global position of all particles

3 Problem Formulation

The problem related with the optimal contracts allocation for producers in a liberalized market is a very complex problem and has a very high importance. For a certain programming period, the producer has to decide which amount of energy he should sell and on what contractual forms to use.

Although the options used in electricity markets are of financial type, we aim to demonstrate that options with physical exercise could be perfectly used in electricity markets, as they appear to be a powerful tool for the producer to practice the hedge against the volatility of the SMP.

Options positions that have to be considered in the decision-support system are: short call[1] and long put[2]. Our decision-support system considers that producers can use forward contracts to sell the energy.

The developed decision-support system has as main objective finding the optimal portfolio of contracts that a certain producer should establish for a programming period i with a duration h. This decision-support system is based on the maximization of a Mean-Variance function of the profit (π) for a set of scenarios forecasted for the considered programming period.

3.1 Spot Market

Producers can make use of the spot market to sell energy. So, the producer spot position revenue for the scenario j and programming period i is given by:

$$r_{i,j}^{ss} = SMP_{i,j} \times e_i^{ss} \qquad (3)$$

Where,

$r_{i,j}^{ss}$	represents the revenue, in €, of the short position obtained by the producer in the spot market, for the programming period i and scenario j
$SMP_{i,j}$	represents the System Marginal Price for scenario j and for programming period i, in €/MWh
e_i^{ss}	represents the energy amount, in MWh, that the producer decides to sell in the spot market for the programming period i

3.2 Forward Contracts

As it was previously stated, the producer can make use of forward contracts to sell energy. The revenue for short forward contracts positions assumed by the producer is given by:

$$r_{i,j}^{sf} = k_i^{sf} \times e_i^{sf} \qquad (4)$$

Where,

$r_{i,j}^{sf}$	represents the revenue, in €, of the short position obtained by the producer on forward contracts, for the programming period i and scenario j
k_i^{sf}	represents the delivery price, in €/MWh, of the forward contract for the programming period i
e_i^{sf}	represents the energy amount, in MWh, that the producer decides to sell in forward contracts for the programming period i

3.3 Options Contracts

As previously referred, in electricity markets options are usually of financial type. However, we have assumed that their exercise is physical and that they are European-style options[3].

However, options have non-linear characteristics that difficult their manipulation. The exercise depends on the SMP for the delivery date. So, the revenue for the assumed positions is dependent from the considered scenario.

As we assume that options exercise is physical, the options positions that a producer could establish to sell the produced energy are: short call and long put.

For the short call position, the buyer will exercise the option if the SMP if greater than the exercise price for the delivery date because, in this situation, the buyer spends less money to buy the same quantity of energy.

The revenue for the short call position is, as we have seen, dependent on the scenario considered for the delivery date and is given by:

$$r_{i,j}^{sc} = \begin{cases} p_i^{sc} & if \quad SMP_{i,j} \le k_i^{sc} \\ k_i^{sc} + p_i^{sc} & if \quad SMP_{i,j} > k_i^{sc} \end{cases} \qquad (5)$$

Where,

$r_{i,j}^{sc}$	represents the revenue, in €, of the short call position, for the programming

[1] Sell a call option.
[2] Buy a put option.

[3] Options are classified according to the exercise date in European and American options. European options can only be exercised at the exercise date whereas American options can be exercised at any time up to the exercise date.

period i and scenario j

p_i^{sc} represents the premium, in €/MWh, of the call option with delivery date coincident with the programming period i

k_i^{sc} represents the delivery price, in €/MWh, of the call option with delivery date coincident with the programming period i

$SMP_{i,j}$ represents the System Marginal Price, in €/MWh, for scenario j and programming period i

For the long put position, the buyer of the option will exercise it if the SMP is lower than the exercise price for the exercise date because, in this situation, the producer (buyer) will sell the same quantity of energy more expensive than if he sells it in the spot market.

The revenue of the long put position is given by:

$$r_{i,j}^{lp} = \begin{cases} k_i^{lp} - p_i^{lp} & if \quad SMP_{i,j} \le k_i^{lp} \\ -p_i^{lp} & if \quad SMP_{i,j} > k_i^{lp} \end{cases} \tag{6}$$

Where,

$r_{i,j}^{lp}$ represents the revenue, in €, of the long put position, for the programming period i and scenario j

k_i^{lp} represents the delivery price, in €/MWh, of the put option with delivery date coincident with the programming period i

p_i^{lp} represents the premium, in €/MWh, of the put option with delivery date coincident with the programming period i

$SMP_{i,j}$ represents the System Marginal Price, in €/MWh, for scenario j and programming period i

3.4 Optimization Problem

The optimization problem formulation aims to maximize a Mean Variance function and is given by:

$$Maximize \quad E_i(\pi) - \frac{\delta_i}{2} \times Var_i(\pi) \tag{7}$$

$$Subjectto:$$

$$e_{i,\min} \le e_i^{ss} + e_i^{sf} + e_i^{sc} + e_i^{lp} \le e_{i,\max} \tag{8}$$

$$e_i^{ss}, e_i^{sf}, e_i^{sc}, e_i^{lp} \ge 0 \tag{9}$$

Where,

$E_i(\pi)$ represents the expected value of the profit, in €, for the programming period i

δ_i represents the producer risk aversion factor for the programming period i

$Var_i(\pi)$ represents the variance of the profit, in €, for the programming period i

$e_{i,\min}$ represents the minimum energy, in MWh, that the producer can produce during the programming period i

$e_{i,\max}$ represents the maximum energy, in MWh, that the producer can produce during the programming period i

e_i^{ss} represents the energy, in MWh, sold by the producer in the spot market in the programming period i

e_i^{sf} represents the energy, in MWh, negotiated in forward contracts assumed by the producer for the programming period i

e_i^{sc} represents the energy, in MWh, associated to the short call position assumed by the producer for the programming period i

e_i^{lp} represents the energy, in MWh, associated to the long put position assumed by the producer for the programming period i

Equation (7) is the objective function of this problem and represents a mean-variance function of the profit. This type of function is useful in problems of this nature because it allows finding the portfolio that maximizes the profit and simultaneously allows the practice of the hedge against the volatility of the SMP. The constraint (8) represents production limits.

The profit π for each programming period i and scenario j is the sum of all revenues minus the costs of production. However, the costs of production are function of the scenario j considered for that period. The profit and the cost of production can be expressed as follows:

$$\pi_{i,j} = r_{i,j}^{ss} + r_{i,j}^{sf} + r_{i,j}^{sc} + r_{i,j}^{lp} - C_{i,j} \tag{10}$$

$$C_{i,j} = \begin{cases} C(e_i^{ss} + e_i^{sf}) \text{ if } k_i^{lp} < SMP_{i,j} < k_i^{sc} & (11) \\ C(e_i^{ss} + e_i^{sf} + e_i^{sc}) \text{ if } SMP_{i,j} > k_i^{lp} \text{ and } SMP_{i,j} \ge k_i^{sc} & (12) \\ C(e_i^{ss} + e_i^{sf} + e_i^{lp}) \text{ if } SMP_{i,j} \le k_i^{lp} \text{ and } SMP_{i,j} < k_i^{sc} & (13) \\ C(e_i^{ss} + e_i^{sf} + e_i^{sc} + e_i^{lp}) \text{ if } k_i^{sc} \le SMP_{i,j} \le k_i^{lp} & (14) \end{cases}$$

The profit is given by equation (10) and the production cost, expressed in €, by equations (11) to

(14). As we can see, production cost will depend on the considered scenarios due to the non-linear characteristics of options.

The satisfaction of constraints (8) and (9) can be achieved in practice by applying a penalty factor to the fitness function.

4 Study Case

The model presented in this paper is based on the maximization of the Mean Variance of the profit (π) for a certain programming period i. The expected value and the variance of the return are calculated for a set of scenarios S.

Let us consider that period i has the characteristics presented in Table 1.

Table 1- Characteristics of programming period i

Duration (h)	1
SMP scenario 1 (€/MWh; probability)	(26; 0.6)
SMP scenario 2 (€/MWh; probability)	(23; 0.4)

4.1 Contracts characteristics

The characteristics of options contracts, with delivery date coincident with the programming period i, are presented in Table 2.

Table 2 - Characteristics of options contracts for period i

	Exercise Price (€/MWh)	**Premium (€/MWh)**
Short Call	24.21	0.80
Long Put	25.32	1.82

Forward contracts with delivery date coincident with programming period i are negotiated at 23.25 €/MWh.

The production cost function considered is equal to

$$C(P_g) = 20 + 2 \times P_g + 0{,}1 \times P_g^2$$

with P_g in MW, C in €, $P_g^{\min}$ =200 MW and $P_g^{\max}$ =5 MW.

The risk aversion factor (δ_i) is assumed to be equal to 0.5.

4.2 Penalty functions

The following penalty function (15) has been used to satisfy restriction (8):

$$P_f = \begin{cases} 0 & if \quad e_{i,\min} \le e_{total} \le e_{i,\max} \\ \exp(10 * erra^2) - 1 & if \quad e_{i,\min} > e_{total} > e_{i,\max} \end{cases} \qquad (15)$$

where,

$$erra = \min\left[\left|(e_{total} - e_{\min})\right|, \left|(e_{total} - e_{\max})\right|\right] \qquad (16)$$

and,

$$e_{total} = e_i^{ss} + e_i^{sf} + e_i^{sc} + e_i^{lp} \qquad (17)$$

The following penalty function (18) can be used to satisfy restriction (9):

$$P_f = \begin{cases} 0 & if \ 0 \le e_i^{ss,sf,sc,lp} \le e_{i,\max} \\ \exp(errb^2) - 1 & if \ 0 > e_i^{ss,sf,sc,lp} > e_{i,\max} \end{cases} \qquad (18)$$

where,

$$erra = \min\left[\left|e_i^{ss,sf,sc,lp}\right|, \left|(e_i^{ss,sf,sc,lp} - e_{\max})\right|\right] \qquad (19)$$

4.3 PSO Parameters

The Particle Swarm Optimization (PSO) parameters used to find the optimal solutions are listed in Table 3.

Table 3 - Parameters used in PSO

Parameter	
Nº. of Particles	20
Nº. of Iterations	6000
Nº. of Evaluations	120000
Cognitive Acceleration	2
Social Acceleration	2
Initial inertia weight	0.9
Final inertia weight	0.4

4.4 Genetic Algorithm Parameters

To evaluate PSO performance and that it is a very successful optimization method, we need to compare it with others and for that we choose an evolutionary technique called Genetic Algorithms (GA). The GA parameters used to find the optimal solutions are listed in Table 4.

Table 4 - Parameters used in Genetic Algorithm

Parameter	
Population Size	50
Max. Generations	2400
Nº. of Evaluations	120000
Crossover Rate	0.8
Mutation Rate	0.2

4.5 PSO Results vs. GA Results

To demonstrate PSO superiority over GA in this particular problem, we use as stopping criteria the maximum number of evaluations and for each variable and was fixed in 120000. To achieve that, in PSO we use 20 particles and 6000 iterations and in GA were necessary 2400 generations to get the same number of evaluations because we use a population size of 50. Due to random initialization, the trajectory for each run is different. To compare PSO and GA were used 1000 runs to calculate averages and standard deviation of the results. The results for the considered case study using PSO are presented in Table 5.

Table 5 - Optimal contractual positions using PSO

Positions	Average Quantity (MWh)	Std. Deviation (MWh)	% of total production capacity
Short Spot	10.15	0.001419	5.1
Short Forward	59.13	0.006179	29.6
Short Call	31.56	0.003317	15.8
Long Put	28.35	0.003274	14.2
Total	**129.19**	**--**	**64.7**

GA results for the case study are presented in Table 6.

Table 6 - Optimal contractual positions using GA

Positions	Average Quantity (MWh)	Std. Deviation (MWh)	% of total production capacity
Short Spot	10.15	0.006567	5.1
Short Forward	59.13	0.020780	29.6
Short Call	31.55	0.009529	15.8
Long Put	28.35	0.011058	14.2
Total	**129.18**	**--**	**64.7**

As we can see from Table 5 and Table 6, PSO reveals superiority in terms of its robustness, evaluated by the standard deviation of the best solution obtained in 1000 runs. Also from Table 5 and Table 6 we can see that only 64.7 % of the total production capacity should be used and that the producer should sell 5.1% of his total production capacity in the spot market.

In Table 7 is made a comparison between PSO and GA for the fitness function and computational time.

Table 7 – Comparison of PSO with GA

Algorithm	Mean Fitness Value	Std. Fitness Value	Mean Time (sec.)
PSO	1132.38604	6.2707e-7	10.765
GA	1132.38604	5.8599e-6	67.914

From Table 7, beside the mean fitness values being equals for the two algorithms, PSO reveals again to be more robust (PSO standard deviation fitness value is 89.30 % lower than GA). PSO reveals also to be faster than GA to achieve better results. For the case study, the PSO mean computational time is 84.15 % lower than GA.

Fig. 1 presents the energy to be sold under each type of contract. From this figure we can see that the majority of the energy should be sold in forward contracts, corresponding to 46% of the total energy that producer should sell.

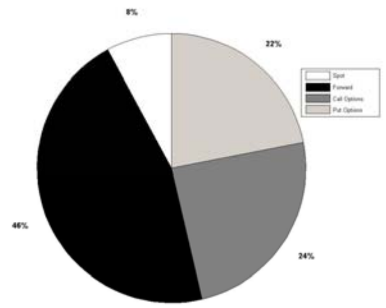

Fig.1 – Contractual positions

Fig.2 shows the graphical distribution for all contractual positions.

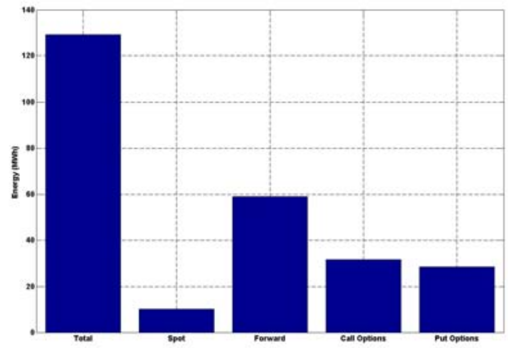

Fig.2 - Graphical distribution by contractual position

The analysis of producer results for the scenarios S is shown in Fig. 3.

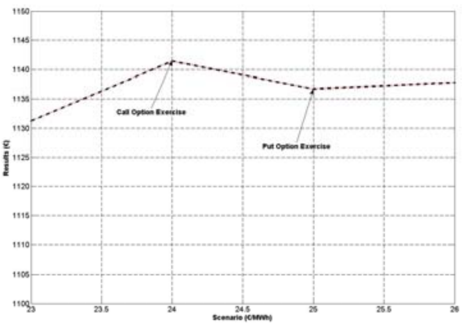

Fig.3 – Producer results for the set of scenarios S

From Fig.3 we can see when options are exercised and that producer results for the set of scenarios S are very stable.

PSO revealed to be an important tool for finding the optimal portfolio that allows the practice of the hedge against the volatility of the SMP and simultaneously for increasing the producer results. This meta-heuristic revealed also to be faster and more robust when compared with GA.

In Fig.4 is presented PSO and GA fitness functions evolution along the number of evaluations.

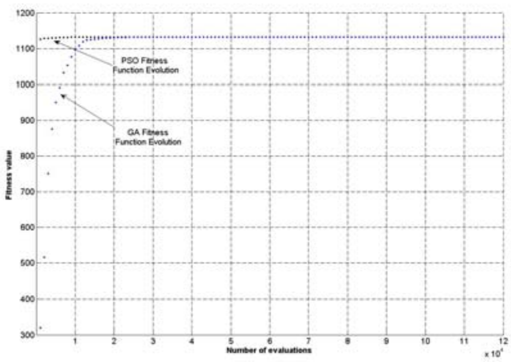

Fig.4 – PSO and GA fitness functions evolution

5 Conclusions

All over the world, the electric sector is undergoing restructuration and liberalization processes resulting in competitive electricity markets.

However, electricity markets are not like traditional markets due to the specific characteristics of the negotiated "product"– the electric energy. One of the characteristics of the electricity markets that take more concerns to their participants is the volatility of the System Marginal Price (SMP). Derivatives markets introduce a set of tools (contracts), making these markets more liquid and allowing their participants the practice of the hedge against the spot market price.

In this paper we presented a Mean Variance optimization method that allows the participants of electricity markets (and in particular the producers) to practice the hedge against the volatility of the System Marginal Price, using forward and options contracts.

This method is based on the use of Particle Swarm Optimization (PSO), which revealed to be a successful meta-heuristic tool for problems with this complexity. For the study case presented in this paper, Particle Swarm Optimization (PSO) proved its superiority over Genetic Algorithms (GA) in robustness and in computational time necessary to find the optimal solutions.

References:

[1] G. Gutiérrez and G. B. Sheblé, "Spot Fuel Markets' Influence on the Spot Electricity Market Using Leontief Model", *IEEE Bologna Power Tech Conference*, June 2003.

[2] M. V. Pereira et al., "Methods and Tools for Contracts on a Competitive Framework," *Task Force 38.05.09*, Cigré, February 2001.

[3] M. Wagner et al., "Hedging Optimization Algorithms for Deregulated Electricity Markets", *ISAP Intelligent Systems Application to Power Systems Conference 2003*, Lemnos-Greece, September.

[4] E. Tanlapco, J. Lawarrée, "Hedging with Futures Contracts on a Deregulated Electricity Industry", *IEEE Transactiins on Power Systems*, Vol. 17, N.º 3, August 2002.

[5] F. Azevedo et al., "Decision-Support Tool for the Establishment of Contracts on the Electricity Market", *IEEE Bologna Power Tech Conference*, June 2003.

[6] R. Bjorgan, et al., "Financial Risk Management on a Competitive Electricity Market", *IEEE Transactions on Power Systems*, Vol. 14, N.º 4, November 1999.

[7] J. Kennedy, R. Eberhart, "Particle Swarm Optimization", *Proceedings of the 1995 IEEE International Conference on Neural Networks*, pp 1942-1948, Perth, Australia, 1995.

[8] F. Azevedo and Zita A. Vale, "Short-term Price Forecast from Risk Management Point of View", *Submitted to the 13th International Conference on Intelligent Systems Application to Power Systems ISAP 2005*, EUA, November 2005.

[9] Y. Shi, R. C. Eberhart, "Parameter Selection in Particle Swarm Optimization", *Proceedings of the Seventh Annual Conference on Evolutionary Programming*, 1998.

[10] M. Shahidehpour et al., "Restructured Electrical Power Systems", *Marcel Dekker*, 2001.