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GA in Optimizing Ajichai Flood Levee's Encroachment

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Abstract: Floods are one of the major natural disasters that often threaten human lives and cause significant economic losses around the world. The history of mankind is filled with the stories of our struggles with floods to protect human races and to sustain the progression of our civilizations. Flood defense systems are designed and constructed to protect low-lying areas against flooding. Engineering design often is at the final stage for finding technical means to best accomplish the project goals. Conceptually, in the cost of the protection system, the most important factor in designing flood defense systems, should be in balance with the value of the protected area. Over the years, risk-based design and optimization methods have proven to be useful tools to obtain a balanced level of protection. The most common economic framework for floodplain management is minimization of expected annual damages and flood management expenses, structural and nonstructural flood control options. Levee systems have been built for flood protection in numerous rivers, lakes and coasts in the world over the long human history. Early flood levees usually were designed with scant quantitative analysis, relying primarily on occasional observations of flood stages and empirical judgments on required project scales. Economic design of a levee system for flood protection involves balancing costs of levee building (height), the losses of land value sacrificed for floodway expansion (Encroachment) and flood damages from inadequate channel capacity. The application of GAs to water resources problems has been increased in recent years. The study of genetic algorithms (GAs) originated, and has developed into a powerful optimization approach. GAs have so far had very little applications in flood defense systems optimization. In this paper several different approaches to GA formulation are considered, along with a range of sensitivity analysis. The object has been to present GAs as a practical tool in levee design optimization and to examine the potential of different GA formulations for solving the problem. It has been demonstrated that GAs provide robust and acceptable solutions to the levees encroachment optimization problem. The results obtained indicate that there is potential for application of GAs to levees optimization problems, where the objective function is nonlinear and other optimization techniques may be difficult to apply and find the global optimum.

Keywords: Optimization, Levee, Encroachment, Genetic Algorithm, Selection, Crossover, Mutation

1 Introduction

Floods are one of the major natural disasters that often threaten human lives and cause significant economic losses around the world. The history of mankind is filled with the stories of our struggles with floods to protect human races and to sustain the progression of our civilizations. Even with centuries of experiences on flood defense and tremendous amount of progresses have been achieved, flood still appears to enjoy being the main enemy of public in the category of natural disaster. Berz (2000) recently compares flood disasters with earthquakes, storms and other forms of nature disasters in the world [1]. His study indicates that floods contribute to 58% of total death and 33% of economic losses. Great majority of flood, related death and economic losses occurred in developing countries.

Flood defense systems are designed and constructed to protect low-lying areas against flooding. Decision for a flood defense system are multi-dimensional which involve a set of goals and constraints arising from political, economical and engineering aspects.

Over the years, risk-based design and optimization methods have proven to be useful tools to obtain a balanced level of protection. These methods have been explicitly accounted for in the design of various flood defense systems, such as storm sewer system (Yen and Ang, 1971, Yen et al, 1976, Yen and Jun, 1984), levees (Tung and Mays, 1981), dams and spillways (Tang and Yen, 1993), and storm surge protection work (Vrijling, 1993). Cheng et al. (1993) demonstrated how to apply the reliability analysis method to calculate the risk reduction associated with freeboard in dam design [2].

Levee systems have been built for flood protection in numerous rivers, lakes and coasts in the world over the long human history. Early flood levees usually were designed with scant quantitative analysis, relying primarily on occasional observations of flood stages and empirical judgments on required project scales. The achievements in experimental and theoretical hydraulics since the 18th century, rational estimation of storm discharge in the mid 19th century and the emerging of early economic-engineering analysis made possible the “modern sense” designs of flood levees. In recent decades, several studies have addressed the economic aspects of flood levee design, usually with benefit-cost analysis and optimization techniques (Tung and Mays, 1981, Wurbs, 1983, Lund, 2002).

GAs have so far had very little applications in flood defense systems optimization. Excellent introductions to GAs are given by Goldberg (1989) and by Michalewicz (1992) and several recent papers give summaries of the essentials (e.g. Oliveira and Loucks (1997)).The Primary objective of this paper is to explore the potential of alternative GA formulations in application to flood defense systems. The problem addressed here is about optimizing levee encroachment along the reaches of a river. In this paper the main object has been to present GAs as a practical tool in levee design optimization through a series of sensitivity analysis.

2 Economic Design of Flood Levees

Economic design of a hydraulic structure, by nature is an optimization problem consisting of an analysis of the hydraulic performance of the structure to convey flow across or through the structure and a determination of the most economical design alternative [11]. The objective is to minimize the sum of capital investment cost, the expected flood damage costs and operation and maintenance costs.

Economic design of a levee system for flood protection involves balancing construction costs of levee, the losses of land value sacrificed for floodway expansion (Encroachment) and flood damages from inadequate channel capacity. The most common economic objective for floodplain management is minimization of expected annual damages and flood management expenses. Under static conditions, the flood frequency distribution is stationary and economic factors, such as the value of damage to properties, construction cost, and floodplain land values, are constant.

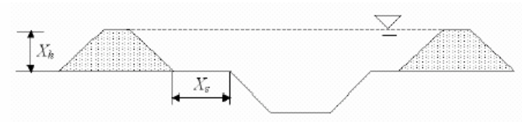


Fig. 1: Levee Height and Encroachment

2.1 Optimal Tradeoff of Levee Encroachment and Levee Height

A static model is formulated to minimize the sum of expected flood damage, considering levee construction cost and resultant land value loss due to floodway occupancy. This simple model allows preliminary quantitative examination of the tradeoff between optimal encroachment and optimal height in designing a new levee. The objective function is:

$$Max \frac{B}{C}(Xs, Xh) = \frac{B(Xs, Xh)}{C(Xs)} \tag{1}$$

Where

B/C( ) =ratio between benefit and cost

X_s =designed levee encroachment

X_h =designed levee height

C(0) =Construction cost of a levee of height X_h

B(0) =benefit of land value

The land value benefit function B( ) depends not only on levee encroachment but also on levee height because the bottom width of levee cross-section may change with levee height (e.g., a trapezoid cross-section).The first order condition for maximizing the expected total benefit of flood control is that the first partial derivatives of $B/C(X_s,X_h)$, with respect to X_s and X_h equal zero.

$$\frac{\partial(B/c)}{Xs} = 0 \tag{2}$$

$$\frac{\partial(B/c)}{Xh} = 0 \tag{3}$$

Given a levee overtopping flow Q(Xs,Xh), we have

$$\frac{\partial Q}{\partial Xh} / \frac{\partial Q}{\partial Xs} = \frac{\partial(C+B)}{\partial Xh} / \frac{\partial B}{\partial Xs} \tag{4}$$

Equation (4) holds for the optimal levee height and encroachment. The optimal levee height and encroachment can be found by numerically solving combined equations (2) and (3) and verifying that a minimum has been found, even though the expected total cost function in Equation (1) is not convex [11].

3 Genetic Algorithms

Evolutionary Algorithms can be divided into three main areas of research: Genetic Algorithms (GA), Evolution Strategies (ES) and Evolutionary Programming (EP). Genetic Programming began as a general model for adaptive process but has since become effective at optimization while Evolution Strategies was designed from the beginning for variable optimization. The schematic diagram of these algorithms which are made of the several iterations of basic Evolution Cycle is shown below:

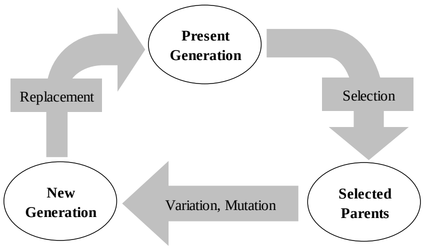


Fig. 1: Schematic Diagram of Evolution Cycle

A GA is a search algorithm based upon the mechanics of natural selection, derived from the theory of natural evolution. GAs simulate mechanisms of population genetics and natural rules of survival in pursuit of the ideas of adaptation, indeed this has led to a vocabulary borrowed from natural genetics [4].

A GA is a robust method for searching the optimum, solution to a complex problem, although it may not necessarily lead to the best possible solution, A GA generally represents a solution using strings (also referred to as chromosomes) of variables that represent he problem. In early GAs (Goldberg and Kuo 1987, Wang 1991) these strings were comprised of binary bits. In binary representation the bits may encode integers, real numbers, sets or whatever else is appropriate to the problem. Real-value coding is now proving more effective in many problems than binary coding (e.g. Oliveira and Loucks 1997).

Coding components of possible solutions into a chromosome is the first part of a GA formulation. Each chromosome is a potential solution and is comprised of a series of substrings or genes, representing components or variables that either form or can be used to evaluate the objective function of the problem. The fitness of a chromosome as a candidate solution to a problem is an expression of the value of the objective function represented by it. It is also a function of the problem constraints and may be modified through the introduction of penalties when constraints are not satisfied.

A GA starts with a population of chromosomes, which are combined through genetic operators to produce successively better chromosomes, The genetic operators used in the reproductive process are selection, crossover and mutation. Chromosomes in the population with high fitness values have a high probability of being selected for combination of other chromosomes of high fitness. Combination is achieved through the crossover of pieces of genetic material between selected chromosomes. Mutation allows for the random mutations of bits of information in individual genes. Through successive generations fitness should progressively improve. Various schemes for selection, crossover, and mutation exist and will be discussed.

3.1 Genetic Algorithm Process

Canonical form of GA approach can be summarized as:

1. Define the objective function (environment) which is appropriate to conditions of problem.
2. Present the possible solutions (phenotype) as real value (genotype or chromosome). All the optimization parameters should be placed somewhere inside chromosome. The chromosome is defined by individual genes.
3. Generate a random population of specific size. The population size affects the efficiency and performance of GA. GA does poorly for very small size of populations and very large population size impacts performance of the algorithm. For typical applications, the suggested range is between 10-160 chromosomes.
4. Evaluate the fitness of every solution over the objective function. There are many methods to evaluate fitness and assign a real number to each chromosome, the most popular one is called proportional selection method which will be discussed.
5. Select a population of chromosomes of the same size of initial population for mating, by a random selection method. There are some selection algorithms like tournament selection and roulette-wheel selection which are discussed later.
6. Apply crossover operation on selected pairs if they have been chosen for crossover (based on probability of crossover).

7. Replace the parent population with new generation.
8. Applying mutation operator based on the probability of mutation. At this point the process of producing a pair of offspring from two selected parents is finished.
9. Go through steps 4 to 8 until the termination criteria met.

3.2 Representation Schemes

Traditionally GAs have used binary coding, in which a chromosome is represented by a string of binary bits that can, encode integers, real numbers, or anything else appropriate to a problem. Real-value chromosomes have been also used with success by various authors (e.g., Oliveira and Loucks 1997 [8]). In a real-value representation, individual genes of a chromosome are initially allocated values randomly within feasible limits of the variable represented, with a sufficiently large population of chromosomes adequate representation will be achieved. There is a significant advantage in not wasting computer time on decoding for objective function evaluation, although a more careful approach to mutation is required. In real-value coding there is no discretization of the decision variable space. This is another advantage of this approach.

3.2.1 Selection Approaches

Selection is the procedure by which chromosomes are chosen for participation in the reproduction process. A popular approach has been fitness proportionate selection (Goldberg 1989 [4]), in which the probability P of an individual k being selected is given by:

$$p_k = \frac{f_k}{\sum f_j} \qquad (5)$$

where f is fitness of individuals along the population.

Various rank selection schemes are in use (Michalewict 1992) that tend to ensure that good chromosomes have higher chances of being selected for the next generation. Ranking schemes operate by sorting the population on the basis of fitness values and then assigning a probability of selection based upon the rank. The roulette wheel approach is one of ranking schemes of selection.

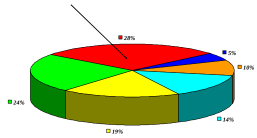


Fig. 2: Roulette Wheel Selection

There are also other selection techniques such as constant selection differential, drawback and tournament. Goldberg and Deb (1990) have compared various selection schemes, and indicated a preference for the tournament selection scheme.

3.2.2 Crossover Approaches

The general theory behind the crossover operation is that, by exchanging important building blocks between two strings that perform well, the GA attempts to create new strings that preserve the best material from two parent strings. The number of strings in which material is exchanged is controlled by the crossover probability forming part of the parametric data. Goldberg (1989) describes the following methods of crossover (I) one-point crossover; (2) two- point crossover and (3) uniform crossover [4].

Crossover occurs between two selected chromosomes with some specified probability. In one-point crossover, a crossover point is selected at random at some point C in the chromosome length L and two new individuals are created by swapping all genes between positions C and L. In two-point crossover, genetic material between two positions chosen at random along the length of the chromosomes, C1 and C2, is exchanged. Uniform crossover operates on individual genes of the selected chromosomes, rather than on blocks of genetic maternal, and each gene is considered in turn for crossover or exchange.

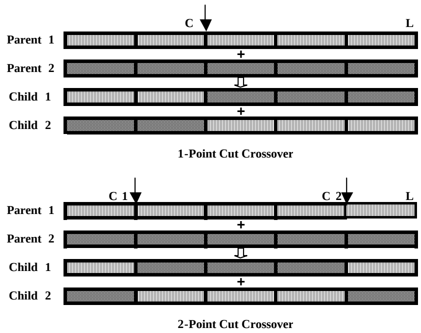


Fig. 3: Crossover Approaches

3.2.3 Mutation Approaches

Mutation is an important process that permits new genetic material to be introduced to a population. A mutation probability is specified that permits random mutations to be made to individual genes. The two basic approaches to mutation for teal-value representations are uniform ruination and non-uniform mutation (Michalewicz 1992). Uniform mutation permits the value of a gene to be mutated randomly within its feasible range of values; possibly resulting in significant modification of otherwise good solutions. Modified uniform mutation permits modification of a gene by a specified amount, which may be either positive or negative. In non-uniform mutation, the amount by which genes are mutated can be reduced as a run progresses, and can therefore help in the later generations to fine tune the solutions. This operator is particularly suited to problems where high precision is required.

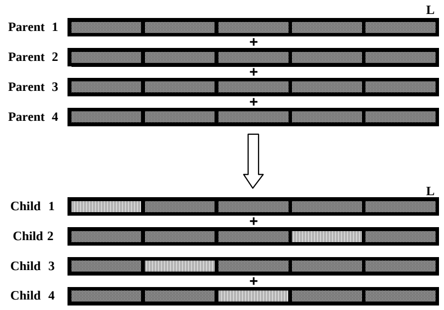


Fig. 4: Uniform Mutation

4 Methodology

In this paper, the HEC-HMS was applied to conduct hydrologic analysis. The Corps' HEC-HMS is a Windows-based program with significant improvements over its predecessor. The resulting peak flows of 25-year and 100-year hydrograph is to extract from the outputs of the model.

Afterwards, the digital topographic maps of study area were used to make the required DEM of study area and the cross-sections were defined and extracted to use in next steps.

For the hydraulic modeling of this study, the steady uniform flow is applied along the river. The design discharge for optimizing the encroachments of levees of study area was selected as the peak flow of hydrograph with return period of 100 year.

For accomplishing hydraulic analysis in this study, HEC-RAS model, (HEC-RAS version 3.0.1), as developed by the Hydrologic Engineering Center, U.S. Army Corps of Engineers, was used as a basis model to verify water surface profiles based on the flood discharges which are excluded from a visual basic code determining water level of each section. This part of analysis is done to enable the hydraulic simulator to participate in the iterative process of optimization.

Economic analysis and optimization is the next step. In this stage, we continue to use a visual basic code written to analyze the cost and benefit of flood defense system and determine the optimum levees encroachment with the maximum economic benefits. Genetic algorithm is applied to achieve best solution. Substituting the hydrologic and hydraulic study outputs in the optimization analysis and refining the construction cost estimate based on developing knowledge of cost-sensitive features such as water level, we make it possible to determine the optimum design.

4.1 Genetic Algorithm Formulation

Since the objective function is based on maximizing ratio between benefit and cost of flood levees, levee height in each section should be the decision variable on which the GA is based. Construction cost is calculated using levee fill volume and its unit cost. Benefit is calculated from the value of protected land due to constructed hydraulic structure (Levee). With 10 sections and one levee height, there are thus 10 discrete variables to be represented in the GA. Each of these may be considered to be a gene. Elevation values are to be considered as non-integer quantities with precision of 0.1 meter. This is toward defining the problem and is not a limitation for GAs.

An alternative approach to formulation of the GA is to use a representation appropriate to the components of the problem. Here, Real-value chromosomes have been used with success where individual genes of a chromosome are initiated by randomly within feasible zone. With a sufficiently large population good representation will be achieved.

The schematic diagram of steps accomplished during the optimization process using genetic algorithm is shown below:

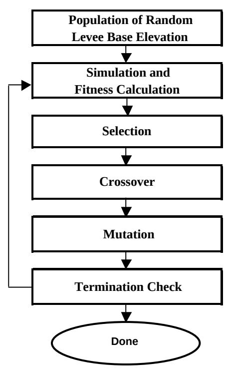


Fig. 5: Schematic Diagram of GA Formulation

For generating a random population, the population size 400 was selectedal selection method is used here to evaluate the fitness of every solution over the objective function. In selection step, roulette wheel approach is applied to select population of chromosomes of the same size of initial population. In the crossover step, 1-point cut crossover is applied on selected pairs based on probability of 0.2. After replacing the parent population with new generation, these changed chromosomes have not to be selected again for further crossover. Having finished crossover along the population, it is necessary to apply mutation operator based on the probability of mutation. The probabilities of 0.005 was used in this step. This process is repeated until the termination criteria met.

5 Study Area

The Ajichai watershed is located in Tabriz. It flows from the southern part of the Sabalan Mountain in a westerly direction across the urbanized Herris and Sarab County and through the city of Tabriz to its confluence with the Uroumieh lake.

The Ajichai River natural valley flood plain averages about 600 meters wide while the main channel averages about 50 meters wide through the study reach.

The Ajichai catchment elevation ranges from 1458 m to 3883 m above sea level and the annual average precipitation is approximately 300.

Fig. 6: Ajichai River and Basin

Bank-full discharge corresponding to an event with the 4-percent chance of exceedance (25-year) is 400 cubic meters per second and the 1-percent chance of exceedance (100-year) event is 700 cubic meters per second. The average bed slope through the project reach is on the order of 20 centimeters per 100 meters.

A subset area was selected along the river with a length of 2 kilometers and used as study area during the hydraulic and optimization process. Such a subset is just large enough to represent the river and the surrounding surfaces so that the computing time is reduced to minimum in the optimization algorithm.

6 Sensitivity analysis

A series of sensitivity analyses were carried out to establish appropriate parameter settings under real-value coding and alternative operators. In many practical problems, GA results are found to be sensitive to crossover and mutation probabilities. This is because genetic material lost at the stag of a run, through either crossover or mutation, may be needed to improve fitness. Sensitivity to crossover and mutation probability is discussed below for real-value coding scheme in GA formulation

6.1 Sensitivity to Crossover and Mutation Approaches

In this study, sensitivity to crossover and mutation probability was carried out using a population size of 80. The roulette wheel selection approach was adopted with 1-point cut crossover with probabilities from 0.2 to 0.6, and a uniform mutation operator with probability of 0.005 to 0.02 were considered through runs with a fixed length of 500 generation. Fig. 6 and 7 show the sensitivity of the achieved fitness to crossover probability for each of the schemes considered. Fitness is expressed as maximum and average values of objective function, as the ratio between benefit and cost of levee problem.

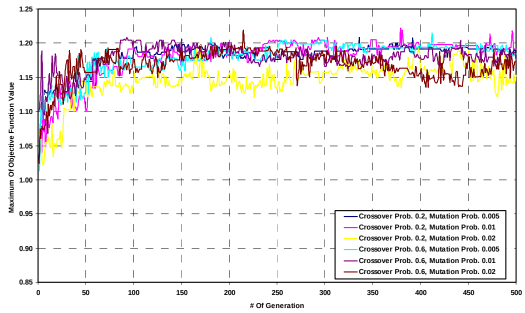


Fig. 7: sensitivity to crossover and mutation probability, maximum of objective function values (Standard GA)

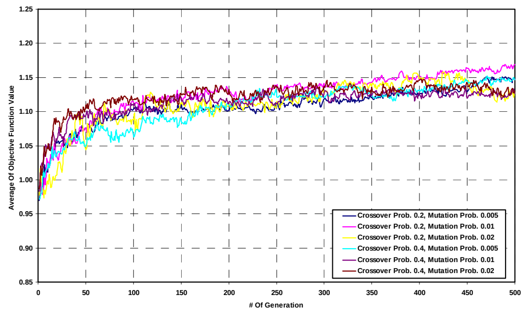


Fig. 8: sensitivity to crossover and mutation probability, average of objective function values (Standard GA)

Table 1 include the information such as minimum, average and maximum of obtained results through different operators of crossover and mutation.

Table 1: sensitivity analysis information in standard GA

Crossover Prob.	Mutation Prob.	Minimum	Average	Maximum	Standard Deviation	Coefficient Of Variation
0.2	0.005	1.077	1.148	1.188	0.035	0.030
0.2	0.01	1.146	1.169	1.192	0.015	0.013
0.2	0.02	1.109	1.127	1.153	0.019	0.017
0.4	0.005	1.103	1.146	1.188	0.032	0.028
0.4	0.01	1.057	1.126	1.163	0.032	0.028
0.4	0.02	1.097	1.130	1.158	0.019	0.017
0.6	0.005	1.081	1.136	1.185	0.032	0.028
0.6	0.01	1.125	1.154	1.175	0.015	0.013
0.6	0.02	1.072	1.125	1.173	0.025	0.022

The results demonstrate clearly that GAs are robust, with reasonable results being obtained by 1-point cut crossover with probability of 0.2 and mutation probability of 0.01. As it is seen, by this structure, it would be possible to obtain better results than previous works (Shafiei et al., 2005).

7 Results

As discussed above, consideration of GA formulation has been given to real-value coding with a range of probability between 0.2 and 0.6 for crossover and 0.0005 to 0.02 for mutation and the probability of 0.2 for crossover and 0.01 for mutation were selected. Figures 9 to 11 show obtained results for GA formulation.

The objective function values of accomplished runs is shown in figure 9. Figure 10 shows minimum, average and maximum of objective function values through the generations and finally the standard deviation and coefficient of variation's fluctuation along the generations is illustrated in figure 11.

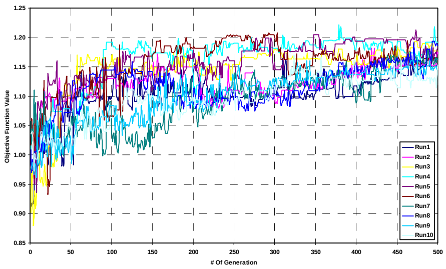


Fig. 9: Objective Function Value (Crossover Prob. 0.6 and Mutation Prob. 0.01)

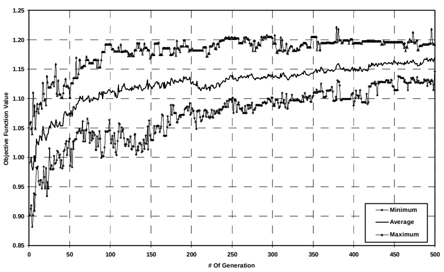


Fig. 10: Minimum,Average and Maximum of Objective Function Value (Crossover Prob. 0.6 and Mutation Prob. 0.01)

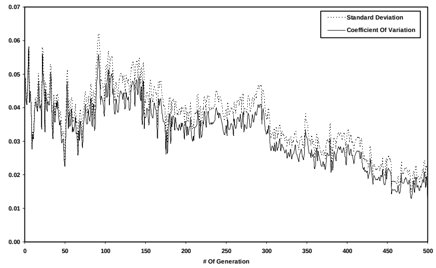
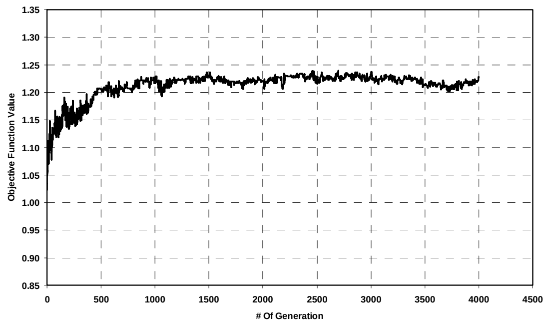


Fig. 11: Std. Deviation and Coeff. of Variation of Objective Function Value (Crossover Prob. 0.6 and Mutation Prob. 0.01)

Figure 12 illustrates objective function values through generations in final run. The population size of 400 through 4000 generations have been used in this step.



The longitudinal profile of river is also schematically shown in figure 12 including ground, levees base and top elevations and water level.

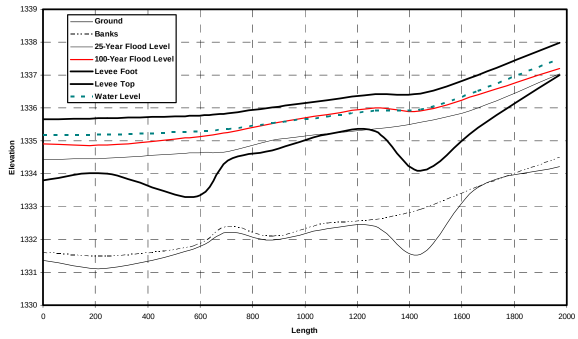


Fig. 12: Longitudinal Profile of Ajichai Study Reach

8 Conclusion

It has been demonstrated that GAs provide robust and acceptable solutions to the levees encroachment optimization problem. Several possible formulations have been considered, along with their sensitivity to various parameters. It is included that in real-value representation scheme, incorporating roulette wheel selection, elitism, 1-point cut crossover and uniform mutation with low probability will operate most efficiently and produce the best results.

Developing formulation of GA having different probabilities of operators, crossover probability of 0.2 and mutation probability of 0.01 are appropriate

for the problem presented here. For the levee problem, a more precise solution can be achieved within 4000 generations with a population of 400.
The results achieved indicate that there is potential for the application of GAs to large rivers levees optimization problems, where the objective function is complex and other techniques are difficult to apply.
The approach is easily applied to complex systems. A GA will generate several solutions that are very close to the optimum, and this gives added flexibility to an operator of a complex flood defense system.

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