

RAMP: Fundamentals, Applications and New Developments

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Abstract: - Relaxation Adaptive Memory Programming (RAMP) is a new metaheuristic methodology recently introduced in [4]. The method integrates the concept of Adaptive Memory Programming (AMP), originated in Tabu Search, with mathematical relaxation procedures to produce a unified framework for the design of dual and primal-dual metaheuristics. This paper reviews the fundamentals of the method and discusses major advances that have yielding exceedingly promising results in a variety of applications.

Key-Words: - RAMP, Scatter Search, Surrogate Constraints, Lagrangean Relaxation, Cross-Parametric Relaxation, Subgradient Optimization, Adaptive Memory, Metaheuristics, Combinatorial Optimization.

1 Introduction

Adaptive memory programming (AMP) coupled with advances in neighborhood structures derived from dynamic and adaptive search constructions have been the source of numerous important developments in metaheuristic optimization throughout the last decade.

Because AMP originated in tabu search (TS), the terms TS and AMP have often been used interchangeably. However, more recently the principles of AMP have likewise been used to enhance other approaches such as genetic algorithms and evolutionary computation methods, notably including scatter search and its generalization, the path-relinking approach, whose origins share overlaps with tabu search.

On the other hand, relaxation techniques have been widely used in combinatorial optimization to provide bounds for tree search procedures (such as branch and bound) as well as to produce heuristic algorithms. These techniques are based on solving an auxiliary (or relaxed) problem derived from the original by dropping or diminishing the restrictiveness of some constraints.

Quite recently a new advance has occurred with the emergence of Relaxation Adaptive Memory Programming (RAMP), a method that integrates AMP with mathematical relaxation procedures to produce a unified framework for the design of dual and primal-dual metaheuristics that take full advantage of adaptive memory programming [4].

The purpose of this paper is to introduce fundamental principles of mathematical relaxation and adaptive memory programming, as embedded in the RAMP approach that is yielding new advances in the field of metaheuristics.

2 Cross-Parametric Relaxation

RAMP is designed to exploit primal-dual relationships by means of different types of relaxation techniques and advanced metaheuristic strategies. As far as the dual approach is concerned, the method also takes advantage of a special relaxation technique giving rise to a cross-parametric relaxation method (CPRM), introduced in [4]. CPRM combines Lagrangean and surrogate relaxations by using a Lagrangean based subgradient search within a surrogate constraint framework to generate good surrogate constraints. In terms of graph theory, the method can be defined as using a classical subgradient search with a *Lagrangean substitution* as a way to produce *parametric subgradients*, as defined in [2].

Throughout this manuscript we define specific problems by reference to their value functions. Following this convention, consider the general 0-1 integer linear programming problem defined by

$$v(P) = \text{Min} \{cx \mid Ax \leq b, Dx \leq e, x \in \{0,1\}\}$$

and assume that the constraints $Ax \leq b$ are the ones that make the problem difficult to solve.

The cross-parametric relaxation can be written as:

$$v(L_{\lambda}SP^w) = \text{Min} \{cx - \lambda w(Ax - b) \mid Dx \leq e, x \in \{0,1\}\},$$

where w is a vector of surrogate multipliers and λ is a scale factor representing the Lagrangean multiplier associated with the surrogate constraint.

The corresponding cross-parametric dual problem is the optimization problem in λ and w

$$v(D^{\lambda w}) = \text{Max} \{v(L_{\lambda}SP^w) \mid \lambda, w \geq 0\}.$$

It is immediate that by setting $\varphi = \lambda w$, problem $D^{\lambda w}$ is the Lagrangean dual D^{φ} , thus identifying the same optimal dual solution. The purpose of the cross-parametric relaxation is to approach a solution for D^{φ} through a *decomposition method* consisting of solving a sequence of locally optimal dual problems

$$v(D_{\lambda}^w) = \text{Max} \{v(L_{\lambda}SP^w) \mid \lambda \geq 0\},$$

in which w is a *parameter* associated with a vector of surrogate multipliers determined by the subgradient method (or some pre-defined vector used to initialize the method).

The method progresses by maintaining an appropriate interaction between the multipliers λ and w in such a way that changes in λ leading to surrogate dual solutions implicitly induce changes in w . This interaction materializes by cross-linking the Lagrangean and surrogate approaches, using Lagrangean based subgradient search within a surrogate constraint relaxation and using subgradient directions (inferred by surrogate dual values and the corresponding projected primal solution values) to determine new surrogate multipliers.

3 Fundamentals of RAMP

The RAMP method [4] is presented in two stages, representing different levels of sophistication. The first level focuses on exploiting a dual framework, which is confined to creating an adaptive memory relaxation, making use of tabu search and path-relinking strategies. The second level of sophistication embodies the primal-dual framework, which establishes the relationship between the dual approach (defined in the first level of sophistication) and a primal approach represented by scatter search and path-relinking. The role of path-relinking at this second level is different from the one represented in the first level.

The *Simple Relaxation AMP* considers frequency-based tabu search memory to construct a primal solution from the dual solution and also to improve this solution further after reaching primal feasibility. Lower and upper bounds are then updated based on the new values of the dual and primal solutions respectively, and these values are used to compute a new subgradient direction from the dual, if it applies.

A more *Advanced Relaxation AMP* retains a collection of *elite* primal solutions selected among those projected from the dual to define a reference set for a *path-relinking projection method*, generating paths between dual solutions and their primal counterpart elite solutions.

The complete RAMP method (embodying its primal-dual form as well as its more basic form) is organized to take advantage of a number of existing methods, using them as components or building blocks to form more advanced search strategies. The resulting procedure affords advantages that cannot be obtained by more customary metaheuristic approaches alone, or even by current hybrid methods. The component methods within RAMP are strategically articulated in a unified design rather than used as independent add-on components to be called whenever other components are unable to make progress in finding improved solutions. The cross-parametric relaxation method, the path-relinking projection method (in connection with subgradient optimization) and the integration of surrogate constraints and scatter search constitute primary factors of such an articulated methodology. Such comprehensive form of the method uses cross-parametric relaxation in the primal side and scatter search in the dual side, though other variations of the method are possible. For example, scatter search can be naturally replaced by its path-relinking generalization. Ultimately a hybrid approach can be created by using other evolutionary approaches such as genetic algorithms (GAs) or evolution strategies (ESs), preferentially provided with adaptive memory components. Likewise, traditional surrogate constraint and Lagrangean relaxation can be used as stand alone dual procedures rather than combined in a cross-parametric approach. As far as the improvement method is concerned, there are advantages for using the adaptive memory orientation of tabu search. In addition, as described earlier path-relinking is also particularly relevant to create a projection method to bridge the gap between dual and primal solutions.

4 Assessment and Conclusion

A fundamental advantage of the RAMP method is its capability to exploit duality as well as primal-dual relationships that have been largely neglected in the field of metaheuristics. Moreover, the method includes a learning process that relies on adaptive memory rather than kicking-off or re-starting the search through randomized-based processes. Although memory is explicitly structured by reference to solution attributes, the RAMP method implicitly includes an automatic learning process embodied in an evolutionary framework that is enhanced by a cohesive integration of primal and dual approaches.

We identify specific combinations of strategies to operate at primal and dual levels, based on Lagrangean and surrogate constraint relaxation on the dual side and on scatter search and path relinking on the primal side, in each instance joined with appropriate guidance from adaptive memory processes. The framework invites the use of alternative procedures for both its primal and dual components, including other forms of relaxations and evolutionary approaches such as genetic algorithms and other procedures based on metaphors of nature.

Recent applications of relatively simple forms of RAMP have already proved highly effective in solving a number of recognizably difficult problems in Combinatorial Optimization such as the Generalized Assignment Problem (GAP) and its extension, the Multi-Resource GAP (MRGAP), the Set Covering Problems (SCP), and more recently, the Capacitated Minimum Spanning Tree problem (CMSTP). The results obtained for the GAP and MRGAP [5] clearly demonstrates the superiority of the RAMP method comparatively to the current state-of-the-art algorithms for the solution of these problems. Likewise, preliminary results in solving the SCP [1] and the CMST [3] indicate that similar performance should be expected.

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