Proceedings of the 5th WSEAS Int. Conf. on Power Systems and Electromagnetic Compatibility, Corfu, Greece, August 23-25, 2005 (pp39-43)

Mathematical Model of a Multi-phase Active Power Filter Based on Multi-phase Bridge Elements

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Abstract: - This paper presents theoretical development of a multi-phase bridge-element concept for application to the mathematical modelling of multi-phase active power filters with multi-stage inverter. For the given number of phases in an independent electric power supply system, and for the given number of phases for each B-element, equations for phase currents and phase voltages have been written in dq0 rotating coordinate system. The presented theoretical concept enables integrating of many elements with different number of phases into the multi-phase electric power supply system. The equations have been explained for the case of three-phase active power filter with two-stage inverter architecture, integrated in the three-phase electric power supply system.

Key-Words: - Electric power supply system, Active power filter, Mathematical modelling, Multi-stage inverter, Power quality, Electromagnetic compatibility.

1 Introduction

It is now recognized that advantages of multi-phase electric power supply systems at the number of phases $p > 3$ as compared to conventional three-phase systems are the following: lower installed power of AC machines at fixed dimensions, more compact power transmission line at equal carrying power, lower current loading per phase to result in lower-power semiconductor devices and more compact control equipment, wider range of speed control, and lower level of noise and vibration for electrical machines. Analysis and design methods for multi-phase electric power supply systems have been addressed by a number of authors, e.g. in [1, 2]. However, they are still not well developed for independent power supply systems where generated power and the consumed power are comparable. In such a system, electromagnetic compatibility (EMC) and power quality become urgent problems, which have to be solved in the early phase of electric power supply system design. The tendency to consider multi-phase systems in the variant design should be addressed in the modelling software enabling a common CAD environment for electric power supply systems consisting of elements with different number of phases in feeding lines.

This paper presents a so-called *m*-phase bridge-element concept (B-element concept) in the context of the mathematical modelling of multi-phase active power filters based on multi-stage inverter. An equivalent graph of B-element and a state scale for the graph edges are introduced. Theoretical fundamentals of this general concept are explained on the example of a three-phase active power filter with two-stage inverter architecture.

2 Multi-phase B-element Concept

It was shown in [3] that most of electric power conversion circuits can be represented by a combination of bridge elements or B-elements, independently of the number of electrical phases. Thus, an *m*-phase B-element can be defined both by an equivalent graph and by a state scale for the graph edges. The principle of how to construct the equivalent graph of a B-element is shown in Fig.1.

In general, the equivalent circuit of B-element contains $2m+2$ switching elements. Among them, $2m$ switching elements are the operating elements. Besides, there are a shunt switching element ($2m+1$) and a switching element referenced as $2m+2$, which is connected in series with the resistive-inductive load (R_{load}, L_{load}). Designations R^i and L^i stand for resistance and inductance of the feeding transformer winding, respectively; $R_{S_1}, R_{S_2}, \ldots, R_{S_{2m+2}}$ and $L_{S_1}, L_{S_2}, \ldots, L_{S_{2m+2}}$ are the resistances and inductances of typical branches of the B-element (see Fig.1). The bridge element is fed from the *p*-phase power distribution network or AC network (p is a prime integer). Here, $i = 1, 2, \ldots, m$ are the nodes where the bridge element is connected to the power distribution network. It is clear, that $m \le p$, and when a single-phase B-element is connected to the phase voltage, it is assumed that $m = 2$.

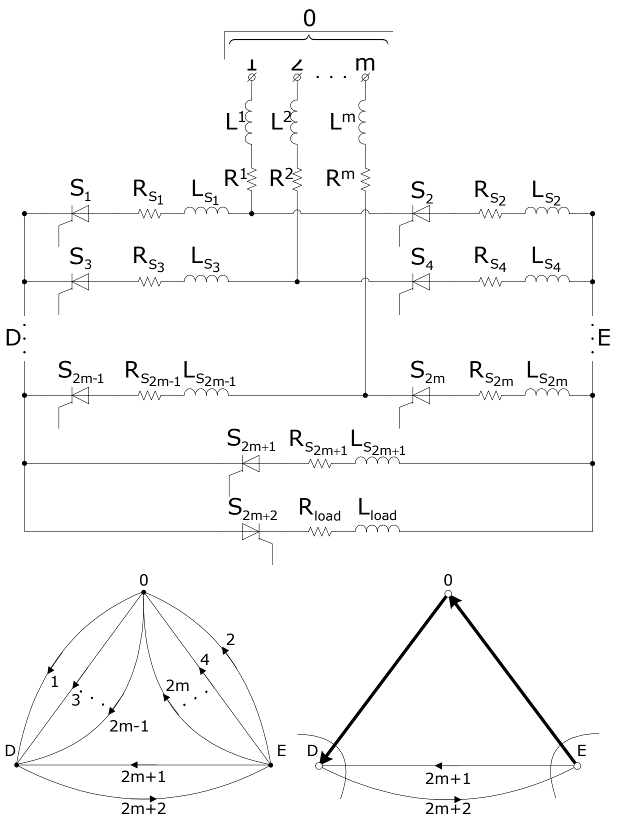

Fig.1. Equivalent circuit of m-phase B-element and its equivalent graph.

The equivalent directed graph has three vertices O, D, and E connected with $2m+2$ edges. The graph is obtained by removing some of the nodes from the equivalent circuit in order to combine the B-element typical branches (including the switching elements $S_1, \ldots, S_{2m}$) with the branches representing the feeding transformer windings. The state scale is a row-matrix consisting of $2m+2$ elements. Each element characterizes the state of a single switching element, and equal to either 1 (closed) or 0 (open). The state scale should be related to equivalent graph's edges, with 1 indicating a conducting edge and 0 indicating a non-conducting edge.

Let us consider application of the B-element concept to modelling of a p-phase active power filter, which is based on the multi-stage inverter, Fig.2. The active power filter can be considered as a system of k interconnected and interacting subsystems or B-subsystems, with each subsystem comprising single-phase B-elements B_i^1, B_i^2, …, B_i^p, $i = 1, 2, 3, \ldots, k$.

3 Model of a 3-phase Power Filter

For simplicity, let us assume $p = 3$ and $k = 2$, i.e. consider the example of a three-phase filter with a two-stage inverter. The choice of example is dictated by wide practical use of three-phase electric power supply systems, with the possibility to extend the theoretical results to multi-phase active power filters, at any number of stages, k. In Fig.3, an equivalent circuit of the three-phase active power filter is provided, where the electrical phases are assigned commonly used symbols, a, b, and c. In the example, the subsystems B_1 and B_2 including three single-phase B-elements (B_1^a, B_1^b, B_1^c) and (B_2^a, B_2^b, B_2^c), respectively, are connected to the nodes of the three-phase power distribution network with phase voltages U_{a1}, U_{b1}, U_{c1} and U_{a2}, U_{b2}, U_{c2}, respectively. The subsystem B_3 includes a 3-phase B-element, B_3^{abc}, representing the thyristor controlled rectifier supplying the power for the inverters. A multi-winding transformer is considered as a part of the power distribution network, and the connection of its windings resembles "star connection".

In Fig.4, the equivalent graph of the equivalent filter circuit is presented where the equivalent graphs of B-elements are combined. Additional chords $6_{1,2}^a$, $6_{1,2}^b$, $6_{1,2}^c$, $8_{1,2}^a$, $8_{1,2}^b$, $8_{1,2}^c$, and edges 9 and 10 of the directed graph (chord is an edge of the graph which is not included in the spanning tree), were obtained by splitting of the equivalent circuit. Voltages related to the graph edges in Fig.4 can be described by Equation (1), as follows:

$$\begin{bmatrix} V \\ V_X \end{bmatrix} = \begin{bmatrix} U \\ 0 \end{bmatrix} + Z \cdot \begin{bmatrix} I \\ I_X \end{bmatrix} - N \cdot \begin{bmatrix} U_{abc} \\ 0 \end{bmatrix}, \qquad (1)$$

where $V = \left[V_1^T, V_2^T, V_3^T\right]^T$ is the vector of voltages related to the edges of the graph included in B-elements, V_1, V_2, V_3 are the vectors of voltages related to the edges corresponding to the first, second and third B-subsystems, respectively; V_X is the vector of voltages related to the edges of the graph not included in B-elements; $U = \left[U_1^T, U_2^T, U_3^T\right]^T$ is the vector of voltages on the switching elements, U_1, U_2, U_3 are the vectors of voltages on the switching elements of the first, second and third B-subsystems, respectively; $I = \left[I_1^T, I_2^T, I_3^T\right]^T$ is the vector of currents related to the edges of the graph included in B-elements.

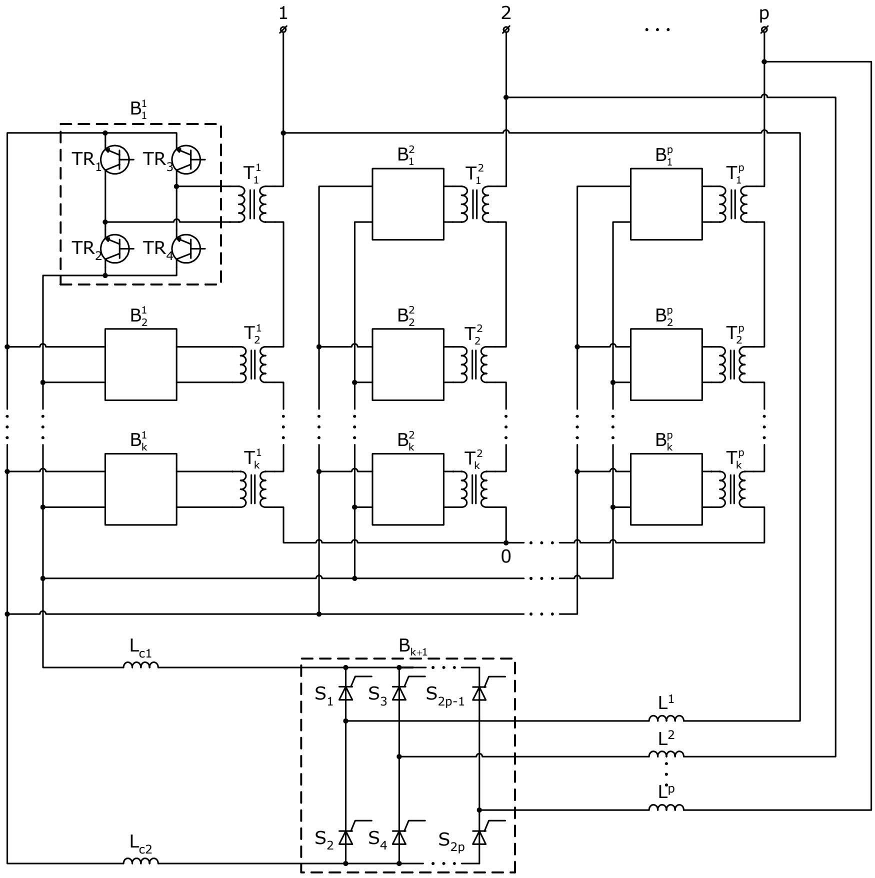

Fig.2. Circuit of a multi-phase active power filter based on a multi-stage inverter.

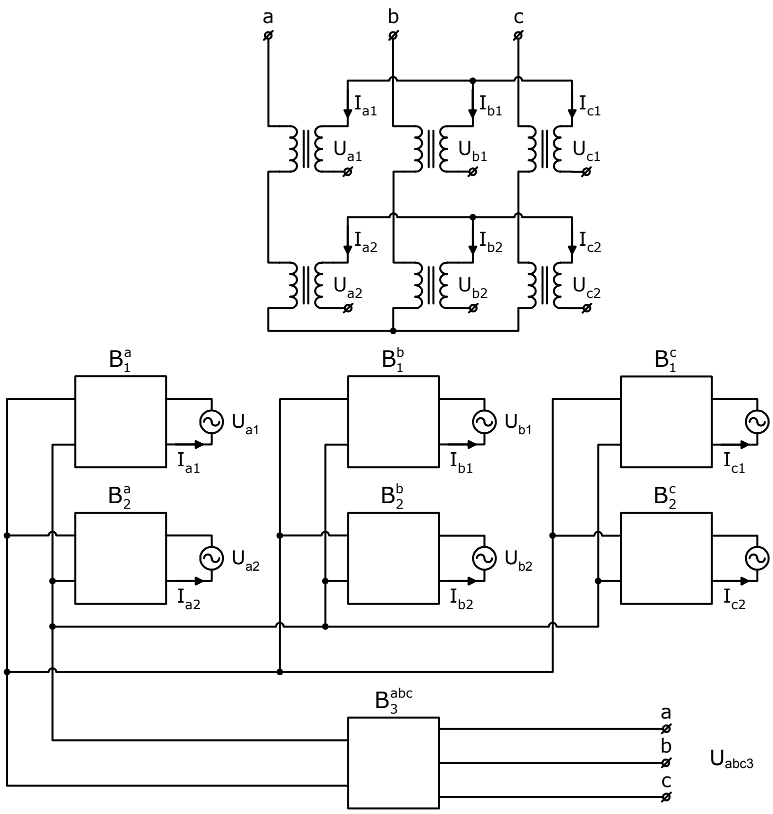

Fig.3. Equivalent circuit of the three-phase active power filter.

Furthermore, I_1, I_2, I_3 are the vectors of currents related to the edges corresponding to the first, second and third B-subsystems, respectively; I_X is the vector of currents related to the edges of the graph not included in B-elements; $U_{abc} = \left[U_{abc1}^T, U_{abc2}^T, U_{abc3}^T\right]^T$ is the vector of phase voltages, $U_{abc1}, U_{abc2}, U_{abc3}$ are the vectors of phase voltages in the nodes of connection of the first, second and third B-subsystems, respectively; Z is the block diagonal impedance matrix related to the edges of the graph, $Z = \begin{bmatrix} diag(Z_1, Z_2, Z_3) & \\ & Z_X \end{bmatrix}$. Here, Z_1, Z_2, Z_3 are the impedance matrices related to the edges included in B-elements corresponding to the three B-subsystems; Z_X is the impedance matrix related to the edges of the graph not included in B-elements.

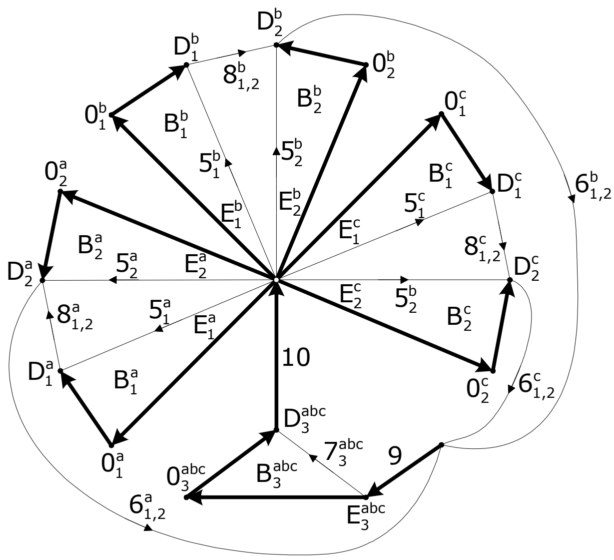

Fig.4. Equivalent graph of the filter equivalent circuit.

In Equation (1), $N = \begin{bmatrix} diag(N_1, N_2, N_3) & \\ & N_X \end{bmatrix}$ where matrix $N_1 = \left[\left[N_{B1}^a\right]^T, \left[N_{B1}^b\right]^T, \left[N_{B1}^c\right]^T\right]^T$ and matrix $N_2 = \left[\left[N_{B2}^a\right]^T, \left[N_{B2}^b\right]^T, \left[N_{B2}^c\right]^T\right]^T$. Matrix N_X is the 8×9 null matrix. Furthermore,

$$N_3 = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } N_{B1}^a = \begin{bmatrix} 1/2 & 0 & 0 \\ -1/2 & 0 & 0 \\ -1/2 & 0 & 0 \\ 1/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Matrices N_{B1}^b, N_{B1}^c, N_{B2}^a, N_{B2}^b, and N_{B2}^c, have a structure similar to N_{B1}^a [3].

In order to determine the currents related to the conducting edges of the graph, and the voltages related to the non-conducting edges, a matrix operator C is constructed, as follows:

$$C = \left[\begin{array}{ccccccc|c} \multicolumn{7}{c|}{C_M} & \mathbf{0} \\ \hline C_{X1}^a & C_{X1}^b & C_{X1}^c & C_{X2}^a & C_{X2}^b & C_{X2}^c & C_{X3} & C_X \end{array}\right]$$

where C_M is the block-diagonal matrix, comprising the circuit matrices for independent directed cycles corresponding to B-elements. These matrices are constructed based on state scales Λ_{t1}^a, Λ_{t1}^b, Λ_{t1}^c, Λ_{t2}^a, Λ_{t2}^b, Λ_{t2}^c, Λ_{t3} of the tree edges corresponding to the B-elements. Here, matrices

$$C_X = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix},$$

$$C_{X1}^a = \left[\left[\Lambda_{t1}^a\right]^T, \mathbf{0}, \mathbf{0}, \left[-\Lambda_{t1}^a\right]^T, \mathbf{0}, \mathbf{0}\right]^T,$$

$$C_{X1}^b = \left[\mathbf{0}, \left[\Lambda_{t1}^b\right]^T, \mathbf{0}, \mathbf{0}, \left[\Lambda_{t1}^b\right]^T, \mathbf{0}\right]^T,$$

$$C_{X1}^c = \left[\mathbf{0}, \mathbf{0}, \left[\Lambda_{t1}^c\right]^T, \mathbf{0}, \mathbf{0}, \left[-\Lambda_{t1}^c\right]^T\right]^T,$$

$$C_{X2}^a = \left[\left[-\Lambda_{t2}^a\right]^T, \mathbf{0}, \mathbf{0}, \left[\Lambda_{t2}^a\right]^T, \mathbf{0}, \mathbf{0}\right]^T,$$

$$C_{X2}^b = \left[\mathbf{0}, \left[-\Lambda_{t2}^b\right]^T, \mathbf{0}, \mathbf{0}, \left[\Lambda_{t2}^b\right]^T, \mathbf{0}\right]^T,$$

$C_{X2}^c = \left[\mathbf{0}, \mathbf{0}, \left[-\Lambda_{t2}^c\right]^T, \mathbf{0}, \mathbf{0}, \left[\Lambda_{t2}^c\right]^T\right]^T$, where bold zeros stand for 4×1 column vectors. Besides, $C_{X3} = \left[\mathbf{0}, \mathbf{0}, \mathbf{0}, \left[\Lambda_{t3}\right]^T, \left[\Lambda_{t3}\right]^T, \left[\Lambda_{t3}\right]^T\right]^T$, where bold zeros indicate 6×1 column vectors.

By rearranging the rows, matrix C can be presented by two blocks, i.e. by matrix C_L (for independent directed cycles, containing only conducting edges) and by matrix C_0 (for independent directed cycles, containing at least one non-conducting edge). Currents I_L through the conducting edges and voltages U_0 across the non-conducting edges are determined from the following systems of differential and algebraic equations:

$$\begin{cases} \frac{d}{dt} I_L = B_L \cdot I_L + D_L \cdot U_{abc}; \\ U_0 = F_0 \cdot I_L + G_0 \cdot U_{abc}, \end{cases} \quad (2)$$

where $B_L = -X_L^{-1} \cdot R_L$; $D_L = -X_L^{-1} \cdot N_L$;
$F_0 = X_0 \cdot X_L^{-1} \cdot R_L - R_0$;
$G_0 = X_0 \cdot X_L^{-1} \cdot N_L - N_0$; $X_L = C_L \cdot X \cdot C_L^T$;
$X_0 = C_0 \cdot X \cdot C_L^T$; $R_L = C_L \cdot R \cdot C_L^T$;
$R_0 = C_0 \cdot R \cdot C_L^T$; $N_L = C_L \cdot N$; $N_0 = C_0 \cdot N$.
Here, R is the matrix of resistances related to the graph edges; X is the matrix of reactances related to the graph edges. Elements of matrices R and X can be extracted from matrix Z. Phase currents of the i-th B-subsystem are expressed from Equations (3), as follows:

$$\frac{d}{dt} I_{abc,i} = B_{abc,i} \cdot I_L + D_{abc,i} \cdot U_{abc}, \; i = 1, 2, 3, \quad (3)$$

where $B_{abc,i}$ and $D_{abc,i}$ are constructed from the rows of matrices B_L and D_L, respectively.

Rewriting Equations (3) in the dq0 rotating coordinate system obtained by the Park's transformation, gives Equations (4).

$$\frac{d}{dt} I_{dq0,i} = B_{dq0,i} \cdot U_{dq0,i} + D_{dq0,i}, \; i = 1, 2, 3. \quad (4)$$

Equations (4) can be used for modelling of independent power supply systems containing an active power filter with two-stage inverter architecture.

In [4] a transformation matrix from the p-phase (p is a prime integer) stationary coordinate system was introduced, which is similar to the conventional Park's transformation matrix for a three-phase coordinate system. For the given number of phases in the electric power supply system, p, and for the given m for each B-element, Equations (3) for phase currents can be transformed to Equations (4), by the mentioned transformation matrix. Equations (4) provide the mathematical model of the multi-phase active power filter based on multi-stage inverter architecture.

4 Conclusion

The multi-phase bridge-element concept was presented and applied for mathematical modelling of a multi-phase active power filters based on multi-stage inverter. The proposed concept enables modelling of active power filters at any number of phases in the electric power supply system, at any number of inverters, and at the same time being independent of the number of phases of the B-elements. The mathematical model of the three-phase active power filter with two-stage inverter was derived using the B-element concept. This filter circuit has a wide practical application. The mathematical model of the active power filter circuit was obtained in dq0 coordinates. Since mathematical models of the other power supply system elements, such as generator, rectifier bridges and various loads, can be reduced to Equations (4), the presented approach enables an easy way for integrating many multi-phase elements into a multi-phase electric power supply system. In future, this would enable developing a general CAD environment for multi-phase power supply systems.

Acknowledgement
The authors would like to acknowledge financial support for this publication received from The Royal Swedish Academy of Sciences, within the project "Mathematical modelling of independent power supply systems for power quality and EMI robustness".

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