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The Onset of Thermal Convection in Horizontal Fluid Layers Subjected to the Constant Heat Flux from Below

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Abstract: - The buoyancy-driven convection in an initially quiescent, horizontal fluid and also a fluid-saturated porous layer heated from below with a constant heat flux is analyzed in this study. When its bottom boundary is heated suddenly, the Boussinesq equation is solved numerically by using the finite volume method and the temporal growth rates of the mean temperature and its fluctuations are examined with time. Based on a new stability criterion, the critical time to mark the onset of intrinsic instability is found and its subsequent growth behavior is traced with time. In comparison with available experimental data the detection time of convective motion, that of manifest convection, and the undershoot time in the plot of the Nusselt number versus time are discussed. It is very interesting that under the Darcy flow the numerical results of the critical time and wavenumber are almost the same as those obtained from the propagation theory we developed.

Key-Words: - Buoyancy-Driven Convection, Temporal Growth Rate, Intrinsic Instability, Undershoot Time, Constant-Flux Heating, Propagation Theory, Rayleigh Number, Darcy-Rayleigh Number

1 Introduction

Natural convection is encountered in a number of industrial systems involving heat and mass transfer and also in nature. The related convective instabilities have been investigated extensively since 1900 [1, 2]. Consider an initially quiescent fluid layer. Starting from time t=0, the layer is heated rapidly from below. Its basic temperature profile of heat conduction develops with time. In this rapidly developing temperature field it is important to predict the critical time t_c to mark the onset of convective instability. Most of practical processes involve nonlinear, time-dependent temperature profiles. This transient problem may be called an extension of classical Rayleigh-Bénard problems. This instability problem still remains unresolved because of its inherent complexity.

For the case of nonlinear, developing temperature fields in horizontal fluid layers, Morton [3] first attempted to analyze the onset of convective instability by using the frozen-time model, and some modified models were proposed by Lick [4] and Currie [5]. Joseph [6], Homsy [7] and Wankat and Homsy [8] introduced the energy method, which suggests a lower bound of convective instability. Foster [9] determined the onset time of convection as that time when the amplification factor based on some initial velocity disturbances reaches a predetermined value. This amplification theory has been quite popular. Jhaveri and Homsy [10] proposed the stochastic model by introducing random force functions and solved the subsequent nonlinear equations. The above two models require the initial conditions at time t=0 and deal with the manifest convection. These analyses were followed by Gresho and Sani [11] and Kim and Kim [12], respectively. Tan and Thorpe [13] predicted the onset time of convection by using the maximum-Rayleigh-number criterion, which is the simplest model based on the conduction temperature. Choi et al. [14] developed a rather simple model called the propagation theory, which is based on linear theory and yields the critical time to mark the onset of a fastest growing instability.

The buoyancy-driven convection also sets in in porous media. When an initially quiescent, fluid-saturated porous layer is heated from below, convective motion is observed at a certain time. For the conduction system of a linear temperature field Horton and Rogers [15] and Lapwood [16] investigated the critical condition to mark convective instabilities. For a thermally developing system Beck [17], Kaviany [18], Yoon and Choi [19], and Tan et al.

[20] also analyzed the onset of motion by using the energy method, the amplification theory, the propagation theory, and the maximum-Rayleigh-number criterion, respectively.

The above models are not definitive and there is still a confusion among the characteristic times t_c, t_D, and t_u, which are, respectively, the onset time of intrinsic instability, the detection time of first visible motion, and the undershoot time in the plot of the heating rate versus time. Accordingly, we analyze the convective instability numerically by employing the finite volume method (FVM) and the above characteristic times are discussed in comparison with available experimental data. In the present study the onset of convective instability in both a horizontal fluid and a horizontal fluid-saturated porous layer subjected to a constant heat flux from below is analyzed. For this purpose a new stability criterion based on the growth rates of initiated disturbances is tested.

2 Theoretical Analysis

2.1 Governing equations

The system considered here is a horizontal fluid layer or a fluid-saturated porous layer of thickness H, as shown in Fig. 1. For $t \geq 0$, the fluid layer is heated from below with a constant heat flux q_w and its upper boundary is kept at a constant initial temperature T_i. For a high q_w, the buoyancy-driven convection will set in at a certain time and the dimensionless governing equations of the flow and temperature fields can be expressed under the Boussinesq approximation by

$$\nabla \cdot \boldsymbol{u} = 0 \tag{1}$$

$$\{(1/\varepsilon)\partial/\partial\tau + (1/\varepsilon^2)\boldsymbol{u}\cdot\nabla\}\boldsymbol{u} = -\nabla p + (Pr/Da)\boldsymbol{u} + (Pr/\varepsilon)\nabla^2\boldsymbol{u} + Pr\,Ra\,\theta\boldsymbol{k} \tag{2}$$

$$\{(\partial/\partial\tau) + \boldsymbol{u}\cdot\nabla\}\theta = \nabla^2\theta \tag{3}$$

with the boundary conditions,

$$u = v = w = 0\,,\ \partial\theta/\partial z = -1 \ \text{ at } \ z = 0 \tag{4a, b}$$

$$u = v = w = 0\,,\ \theta = 0 \ \text{ at } \ z = 1 \tag{5a, b}$$

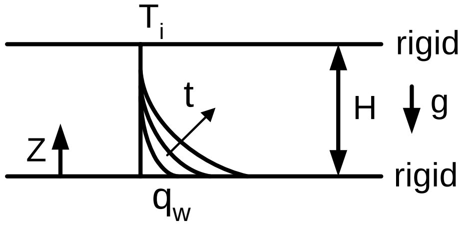

Fig. 1. Schematic diagram of the system considered

where $\boldsymbol{u}$ (=$\boldsymbol{i}u$+$\boldsymbol{j}v$+$\boldsymbol{k}w$), p, and τ repesent the the velocity vector, the pressure, and the time, respectively. The velocity vector has the scale of α/H, the presure that of $\rho\alpha^2/H^2$, and the time that of H^2/α, respectively. The dimensionless temperature is denoted by θ= $k(T-T_i)/q_wH$. Here α, ρ, ε, and k represent the thermal diffusivity, the fluid density, the porosity, and the thermal conductivity, respectively. The distances have the scale of H. Here z is the dimensionless vertical distance, $\boldsymbol{k}$ is the vertical unit vector, and ($\boldsymbol{i}$, $\boldsymbol{j}$) are the horizontal ones. The important parameters to describe the present system are the Prandtl number Pr, the Darcy number Da, and the modified Rayleigh number Ra based on q_w:

$$Pr = \frac{\nu}{\alpha}\,,\ Da = \frac{K}{H^2}\,,\ Ra = \frac{g\beta q_w H^4}{k\alpha\nu} \tag{6}$$

where ν, K, g and β denote the kinematic viscosity, the permeability, the gravitational acceleration constant and the thermal expansion coefficient, respectively.

In the present horizontal layer the simplest flow is a Darcy flow. With this flow Eq. (2) is simplified to

$$0 = -\nabla\bar{p} + R_D\theta\boldsymbol{k} \quad \text{with} \quad R_D = RaDa \tag{7}$$

where $\bar{p} = pDa/Pr$ and R_D is the Darcy-Rayleigh number.

With ε=1 Eq. (2) becomes the well-known Boussinesq equation, which corresponds to the case of $Da\to\infty$. For the basic conduction system heated with a constant heat flux Currie [5], Kim and Kim [12], Tan and Thorpe [21], and Choi et al. [22] conducted the instability analyses by using the afore-mentioned models. Nielsen and Sabersky [23], Chu [24], and Goldstein and Volino [25] observed experimentally cell-like patterns like Bénard cells over the heated bottom surface.

In the present study the problem is to find the critical time t_c to represent the onset time of a most energetic, fastest growing instability. For $t>t_c$ the incipient infinitesimal instabilities will grow faster than the conduction temperature, and the convective motion would be detected between t_c and t_u. Here both the Darcy flow in fluid-saturated porous media and the flow in non-porous layers will be considered for the theoretical analysis of convective instabilities.

2.2 Onset of convective instability

The velocity and temperature fields are decomposed into the horizontal mean and its fluctuations:

$$\theta = \langle\theta\rangle + \theta' \tag{8}$$

$$\boldsymbol{u} = \langle\boldsymbol{u}\rangle + \boldsymbol{u}' \tag{9}$$

where $\langle\cdot\rangle$ and $'$ represent the horizontal mean and its fluctuations, respectively.

In the present system, thermal convection sets in due to the buoyancy force and its local magnitude F_B is represented by

$$F_B = \rho g \beta |T - T_i| \tag{10}$$

which is produced by temperature variations. The buoyancy forces based on the mean temperature and its fluctuations can be written as $(F_{B,0}, F_{B,1}) = (\langle\theta\rangle, \theta')\rho g \beta q_w H/k$, where $F_B = F_{B,0} + F_{B,1}$. In order to examine the temporal behavior of thermal instabilities, the following temporal growth rates are defined:

$$r_{0,\mathrm{T}} = \frac{1}{\langle\theta\rangle_{rms}} \frac{d\langle\theta\rangle_{rms}}{d\tau} \tag{11}$$

$$r_{1,\mathrm{T}} = \frac{1}{\theta'_{rms}} \frac{d\theta'_{rms}}{d\tau} \tag{12}$$

$$r_{1,\mathrm{V}} = \frac{1}{\boldsymbol{u}'_{rms}} \frac{d\boldsymbol{u}'_{rms}}{d\tau} \tag{13}$$

where $r_{0,\mathrm{T}}$, $r_{1,\mathrm{T}}$, and $r_{1,\mathrm{V}}$ are the temporal growth rates of the mean temperature, the temperature fluctuations, and the velocity fluctuations, respectively. Here the subscript *rms* refers to the root-mean-square quantity, *i.e.*, $(\cdot)_{rms} = [\int_V |(\cdot)|^2 dV/V]^{1/2}$, where V represents the volume of the system considered.

With $r_{1,\mathrm{T}} < r_{0,\mathrm{T}}$, temperature fluctuations are expected to be several orders of magnitude smaller than that of the mean temperature. Here it is assumed that the system is unstable only when the temperature fluctuations are growing faster than the mean temperature. With $r_{1,\mathrm{T}} > r_{0,\mathrm{T}}$, the fluctuations will grow to a measurable magnitude. Therefore, the marginal stability criteria are suggested as

$$r_{1,\mathrm{T}} = r_{0,\mathrm{T}} \quad \text{with } r_{1,\mathrm{V}} \ge 0 \quad \text{at } \tau = \tau_c \tag{14}$$

which represents the onset condition of intrinsic instability at the earliest time τ_c with the dimensionless critical wavenumber a_c. It is expected that fluctuations are first driven thermally.

A more convenient measure to exhibit the incipient nonlinear effects is the undershoot time τ_u in the plot of the Nusselt number versus time. In the present study the Nusselt number Nu with the characteristic length of H is defined as follows:

$$Nu = \int_S \left(1/\theta\right)_{z=0} dS \Big/ S \tag{15}$$

where S is the surface area of the bottom plate. With thermal convection, Nu deviates from its conduction solution and it has the minimum at $\tau = \tau_u$. The undershoot time τ_u is frequently used as the characteristic time to represent the manifestation of thermal convection.

2.3 Propagation theory

The convective instabilities at the marginal state are well illustrated by the propagation theory, which employs the normal mode analysis under linear theory. This model is based on the assumption that in deep-pool systems the incipient temperature disturbances are propagated mainly within the thermal penetration depth Δ_T near the onset time of thermal instability. Therefore, all the variables and parameters having the length scale are rescaled with Δ_T. The self-similar transformations are forced and the stability criteria obtained easily. This model satisfies the condition of $r_{1,\mathrm{T}} = r_{0,\mathrm{T}}$ in Eq. (14). For the fluid layer subjected to a constant heat flux the resulting τ_c-values were obtained by Choi et al. [22]. In the present study their τ_c-values are referred to τ_c^*. For the case of the porous layer the critical time τ_c^* was obtained by following their procedure. Its stability equations and resulting stability criteria are obtained easily by following Chung et al.'s [26] work..

3 Numerical Method

3.1 Finite volume method

The governing equations (1)-(3) were discretized by using the FVM introduced by Patankar [27]. The non-uniform arrangement of meshes was forced along the z-direction and the nodes of velocities were staggered from those of scalar values, θ and p, along the y- and z-directions. The no-slip and slip conditions were imposed on the top and bottom boundaries of the fluid layer and the fluid-saturated porous layer, respectively. For the present, horizontally infinite layers, only one two-dimensional roll was considered and the stress-free and adiabatic conditions were introduced to its vertical free boundaries.

For the treatment of coupling between the pressure and velocity the SIMPLE algorithm was used. To solve the convection and diffusion terms the hybrid scheme was employed. In order to solve the present time-dependent problem, the implicit method was employed and the first-order time increment was used. The number of meshes was 42×60 and finer meshes were used near the top and bottom boundaries. To ensure the numerical stability the time step of $\Delta\tau = 10^{-7}$ was used. At each time step the iteration continued until the relative error between the present value and the previous one reached 10^{-6}.

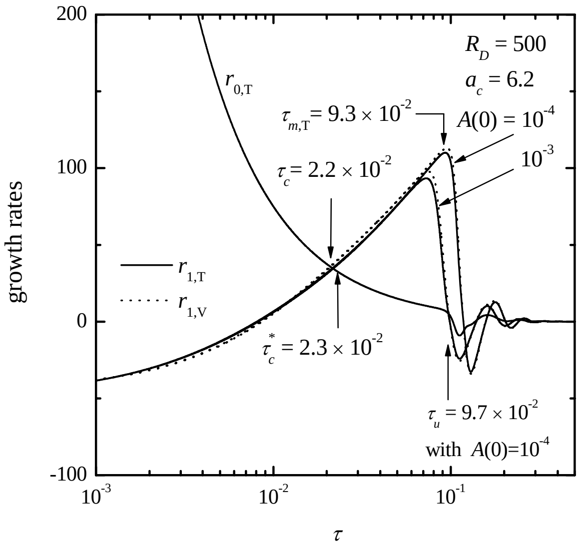

Fig. 2. Temporal growth rates for R_D=500

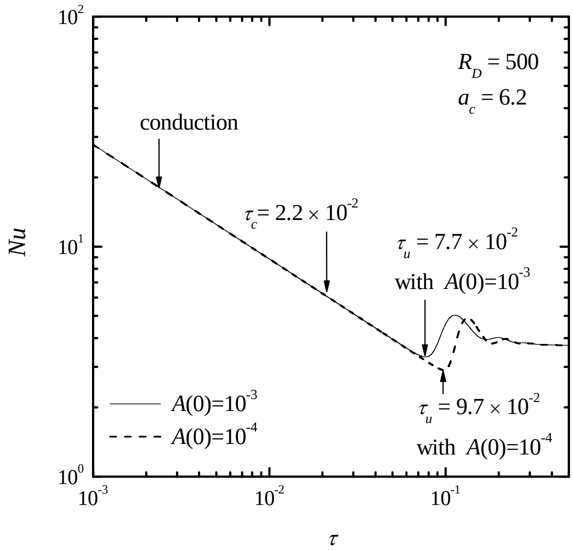

Fig. 3. Nusselt number versus time for the Darcy flow

3.2 Simulation

For two-dimensional rolls the periodic, initial fluctuations at τ=0 are constructed as $\theta'= A(0)\theta_*(z)\cos(ay)$, $v'= -B(0)((\partial w_*(z)/\partial z)/a)\sin(ay)$ and $w'=B(0)w_*(z)\cos(ay)$. Here $A(0)$ and $B(0)$ are the initial magnitudes. The profiles of θ_*(z) and w_*(z) were chosen to be unique such that the arbitrary initial ones forced were finally converged to the same patterns through iteration for $0\leq\tau\leq\tau_c$. The first, initial patterns forced were given as the two-dimensional roll patterns obtained from the propagation theory.

With the proper $A(0)$- and $B(0)$-values, the present system was simulated numerically for a given *Ra*, *Pr* and *Da*. The proper $A(0)$-value was chosen in comparsion with existing experimental data. In the present study, the numerical simulation was conducted for the Darcy flow using Eq. (7) and also for the fluid flow of ε=1, $Da\to\infty$ and $Pr\to\infty$ in Eq. (2). Then the τ_c- and τ_u-values to satisfy Eq. (14) were obtained.

4 Results and Discussion

Based on the propagation theory, the present results on the onset condition of convective instability in a fluid-saturated porous layer is represented by

$$R_D\tau_c^* = 11.7 \quad \text{with} \quad a_c\tau_c^{*1/2} = 1.01 \quad \text{for } R_D>100 \qquad (16)$$

which corresponds to the case of deep-pool systems. According to this model the system would be unstable for $\tau > \tau_c^*$.

The results of the numerical simulation for the Darcy flow by the FVM are shown in Fig. 2. For R_D= 500 the numerically predicted $r_{1,\mathrm{T}}$- and $r_{1,\mathrm{V}}$-curves of two-dimensional rolls are seen with time. Under the Darcy model the relation of $r_{1,\mathrm{T}}\cong r_{1,\mathrm{V}}$ is obtained. With increasing $A(0)$, the $\tau_{m,\mathrm{T}}$-value representing the maximum $r_{1,\mathrm{T}}$-value becomes smaller but the resulting $r_{1,\mathrm{T}}$-curves are all the same for $0\leq\tau\leq\tau_c$. According to Eq. (14), it is stated that the intrinsic instability sets in at τ_c = 2.2 × 10^{-2} with a_c = 6.2 for R_D = 500. The undershoot time τ_u in the plot of *Nu* versus τ is shown in Fig. 3. With conduction only, it is well-known that $Nu = 1/\sqrt{\pi\tau}$ for small time. With convection, the *Nu*-value deviates from that of conduction and the minimum *Nu*-value appears at τ=τ_u. It is known that $\tau_c<\tau_{m,\mathrm{T}}\leq\tau_u$. The predicted $\tau_{m,\mathrm{T}}$- and τ_u- values depend on the $A(0)$-value. The effect of $B(0)$ is negligible.

The present numerical results τ_c, τ_c^* and τ_u with $A(0)$=10^{-3} and 10^{-4} are compared in Fig. 4. The τ_c- and τ_c^*-values are nearly the same and the former one represents the onset time of a fastest growing instability among the initiated two-dimensional rolls. The numerical results of Tan et al. [20] exist between the present τ_u-values with $A(0)$=10^{-3} and 10^{-4}. In the present porous layer heated with a constant heat flux thermal convection exists when R_D is larger than $R_{D,c}$, *i.e.* $R_{D,c}$=27.1 with a_c=2.33, which is well illustrated by Nield [28]. At this well-known critical condition of $\tau_c\to\infty$, the numerically predicted $r_{1,\mathrm{T}}$- and $r_{0,\mathrm{T}}$-values approach zero with time but for $R_D<R_{D,c}$ the system is absolutely stable with $r_{1,\mathrm{T}}$, $r_{1,\mathrm{V}}$<0. This verifies the stability condition (14) and the accuracy of the present FVM to a certain degree.

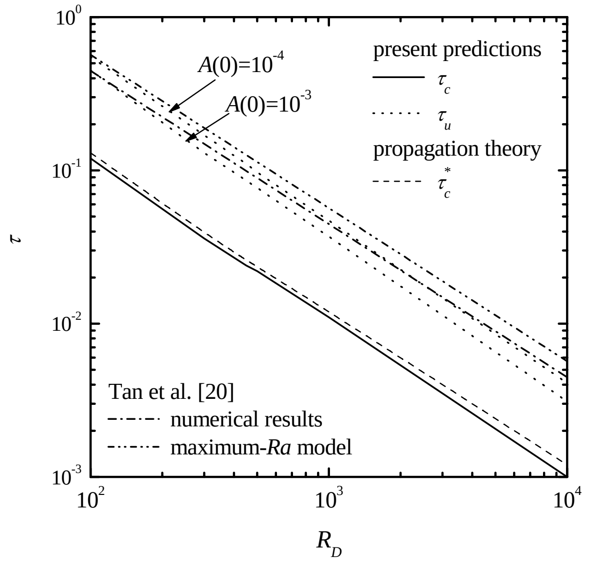

Fig. 4. Characteristic times for the Darcy flow

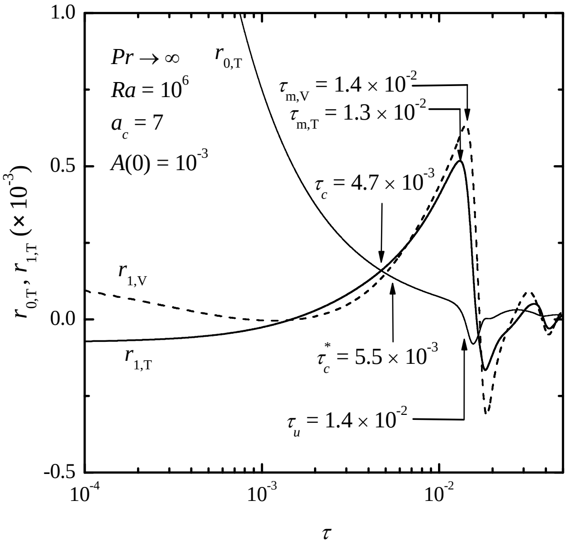

Fig. 5. Growth rates for the non-porous layer

For a non-porous fluid layer with Ra=10^6, $Pr\rightarrow\infty$ and A(0)=10^{-3} the numerically predicted, temporal growth rates are illustrated in Fig. 5. Here $r_{1,\mathrm{T}}$ and $r_{1,\mathrm{V}}$ are more distinguishable than those for a porous layer. Also, the difference between τ_c and τ_c^* is a little larger. It is known that $\tau_{m,\mathrm{T}}\cong\tau_u$. It is clear that manifest convection should exist at $\tau=\tau_u$. The overall trend of τ_c-values is illustrated as a function of Ra and Pr in Choi et al.'s [22] work based on the propagation theory.

The experimental data of Nielsen and Sabersky [23] and Chu [24] are compared with the present numerical τ_u-values in Fig. 6. Most of experimental τ_u-values are placed between the numerical results for A(0)=10^{-3} ~ 10^{-5}. Here the τ_c-value is also the invariant like that for the above porous layer but the $\tau_{m,\mathrm{T}}$- and τ_u-values are dependent upon the A(0)-value.

It is very difficult to define the detection time τ_D both experimentally and theoretically. Chu's [24] experimental results show that the first visible motion is detected at t=t_D earlier than t_u. Detection of motion depends on experimental apparatus to a certain degree [25]. According to experimental observations [24, 25], it is suggested that $\tau_c\leq\tau_D\leq\tau_u$.

5 Conclusion

The present numerical simulation by the FVM reveals that there exists the unique τ_c-value when the growth rates are considered. The intrinsic instability would set in under the condition of $r_{1,\mathrm{T}} = r_{0,\mathrm{T}}$ at the earliest time because instabilities would be first driven thermally. Their growth period is required until they are detected. It is suggested here that $\tau_c\leq\tau_D\leq\tau_u$. The present results complements Choi et al.'s [29] work.

Acknowledgements

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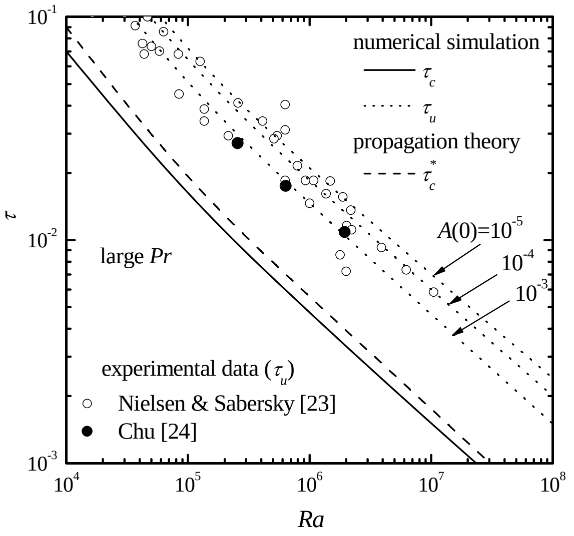

Fig. 6. Characteristic times for the non-porous layer

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