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Reduced order, passive models for liquid transmission lines

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Abstract: - Models for the dynamic behaviour of transmission lines are essential for the understanding of wave propagation phenomena in hydraulic pipelines and hoses. If the Reynolds numbers are low enough to justify the assumption of laminar flow and if convective terms are negligible, the governing equations are linear and a very compact description of the input-output behaviour of a transmission line exists in the frequency domain. For a coupled simulation of networks of transmission lines interacting with other, possibly nonlinear components such as valves, there are numerous approaches for the approximation of the transcendental transfer functions arising from the transmission line modelling by finite dimensional models in the time domain. Discrete-time approaches such as the method of characteristics with a fixed grid are known to be very accurate but computationally ineffective due to the high number of state variables involved. This paper shows a method for the derivation of reduced order models with a trade-off between the degree of accuracy and the system order and with the additional feature that important properties like the passivity of the transmission line model is guaranteed.

Key-Words: - fluid transmission line, laminar pipe flow, passive model

1 Outline of the problem

This paper considers pipe flow in a long, straight transmission line with a constant and circular cross section filled with a weakly compressible Newtonian fluid. The pipe wall is assumed to be rigid and the flow is assumed to be laminar. The physical parameters describing the transmission line are the line length L, the internal tube radius R, the kinematic viscosity ν, mass density ρ and bulk modulus of compressibility E of the hydraulic fluid. The so called 'frequency-dependent friction' theory [2] of laminar pipe flow is used in the following. Two dimensions in space (axial coordinate x and radial coordinate r) are needed to describe laminar, axisymmetric pipe flow. However, the pressure is assumed to obey $\partial p/\partial r \ll \partial p/\partial x$ and a mean velocity or alternatively the volumetric flow rate $Q(x,t)$ is used instead of the velocity distribution. Together with a treatment of the viscous friction terms in the frequency domain this enables a formulation in one-dimensional space x and time t of the form

$$\frac{\partial q}{\partial \tau} + \varepsilon q \frac{\partial q}{\partial \xi} + \frac{\partial \psi}{\partial \xi} = f(q) \quad (1a)$$

$$\frac{\partial \psi}{\partial \tau} + \varepsilon q \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial \xi} = 0 \quad (1b)$$

where $q(\xi,\tau)$ is a scaled flow rate and $\psi(\xi,\tau)$ is a scaled pressure variable according to

$$\psi = \frac{p}{p_S}, \qquad q = \frac{Z_0}{p_S} Q$$

with the line impedance Z_0 and a characteristic pressure p_S which is chosen as either the maximum operating pressure of the transmission line or as a typical magnitude of transient pressure excitations acting at the boundary. The independent variables are scaled by

$$\tau = \frac{c_0}{L} t, \qquad \xi = \frac{x}{L}$$

with the pipe length L and the speed of wave propagation $c_0 = \sqrt{E/\rho}$. The system (1) contains a dimensionless parameter $\varepsilon = p_S/E$. For hydraulic systems, this ratio is a small number in the order of 10^{-2}. Therefore, the convective terms can be neglected and the system under consideration becomes

$$\frac{\partial q}{\partial \tau} + \frac{\partial \psi}{\partial \xi} = f(q), \quad (2a)$$

$$\frac{\partial \psi}{\partial \tau} + \frac{\partial q}{\partial \xi} = 0. \quad (2b)$$

The right hand side contains the friction term $f(q)$. Only for steady, laminar flow this term can be expressed as a simple linear function in q. For transient laminar flow, the term $f(q)$ represents a dynamical system mapping the flow rate q at a certain location along the pipeline to the frictional pressure loss per unit length at the same location. A precise model can be derived in the frequency domain in the form [5]

$$\hat{f} = s\left(\frac{J_0\left(\sqrt{-\frac{s}{D_n}}\right)}{J_2\sqrt{-\frac{s}{D_n}}} + 1\right)\hat{q} \qquad (3)$$

where s is the Laplace variable in scaled time. Boundary conditions for the equations system (2) are chosen as prescribed pressure at one end of the transmission line and prescribed flow rate at the other end, i. e.

$$\psi(\xi,\tau)|_{\xi=0} = \psi_0(\tau), \qquad q(\xi,\tau)|_{\xi=1} = -q_1(\tau).$$

The input-output behaviour can be written as

$$\begin{bmatrix} \hat{q}_0 \\ \hat{\psi}_1 \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\tanh(s\tilde{Z})}{\tilde{Z}} & -\frac{1}{\cosh(s\tilde{Z})} \\ \frac{1}{\cosh(s\tilde{Z})} & \tilde{Z}\tanh\left(s\tilde{Z}\right) \end{bmatrix}}_{\mathbf{G}(s)} \begin{bmatrix} \hat{\psi}_0 \\ \hat{q}_1 \end{bmatrix} \qquad (4)$$

with the scaled hydraulic impedance

$$\tilde{Z}(s) = s\sqrt{\frac{J_0\left(\sqrt{-\frac{s}{D_n}}\right)}{J_2\left(\sqrt{-\frac{s}{D_n}}\right)}}$$

and the dimensionless dissipation number [3] $D_n = \frac{\nu L}{c_0 R^2}$. It is important to note that the static gain of the the transfer matrix in (4) represents the pressure drop due to stationary Hagen-Poisseuille flow

$$\lim_{s\to 0}\mathbf{G}(s) = \begin{bmatrix} 0 & -1 \\ 1 & 8D_n \end{bmatrix} \qquad (5)$$

and the direct feedthrough term

$$\lim_{s\to\infty}\mathbf{G}(s) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (6)$$

exactly fulfils the Joukowsky relation for the scaled model. The goal is now to find a discrete-time state-space approximation

$$\mathbf{x}_{k+1} = \mathbf{A}\,\mathbf{x}_k + \mathbf{B}\begin{bmatrix} \psi_L \\ q_R \end{bmatrix}_k \qquad (7a)$$

$$\begin{bmatrix} q_L \\ \psi_R \end{bmatrix}_k = \mathbf{C}\,\mathbf{x}_k + \mathbf{D}\begin{bmatrix} \psi_L \\ q_R \end{bmatrix}_k \qquad (7b)$$

for the input-output behaviour described by eq. (4) assuming zero order hold on the inputs ψ_0 and q_1. The system (7) should exactly fulfill the limit properties (5) and (6), i.e.

$$\mathbf{C}(\mathbf{I}-\mathbf{A})^{-1}\mathbf{B} = \begin{bmatrix} 0 & -1 \\ 1 & 8D_n \end{bmatrix} \qquad (8)$$

$$\mathbf{D} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (9)$$

Furthermore, it has to reflect an important property (passivity) of the real system by meeting the criteria for discrete-time positive realness [11] and the approximation error - measured in an appropriate norm - between (7) and (4) should be as low as possible for a given system order.

2 Nonlinear programming problem

The problem laid out in the last section can be treated by a number of different approaches. Either a very accurate and passive model is found and reduced by a passivity preserving order reduction method, or passivity is enforced as a constraint in the reduction method. As the Zielke-Suzuki method used in this paper turned out not to guarantee model passivity, the second approach is tried by solving the following nonlinear programming problem.

The matrices $\mathbf{A} \in \mathbb{R}_n^n$, $\mathbf{B} \in \mathbb{R}_n^2$, $\mathbf{C} \in \mathbb{R}_2^n$, and $\mathbf{D} \in \mathbb{R}_2^2$ have to be found for a given system order n. The frequency response function for the discrete-time system (7) is defined by

$$\mathbf{G}_d(\theta) = \mathbf{C}\left(e^{j\theta}\mathbf{I}-\mathbf{A}\right)^{-1}\mathbf{B} + \mathbf{D}, \quad 0 \le \theta \le \pi. \qquad (10)$$

The transfer function matrix $\mathbf{G}(s)$ of eq. (4) is to be approximated by the discrete-time model (7). According to Parseval's theorem, a minimisation of the sum of squared errors between the frequency response function $\mathbf{G}_d$ and a reference function representing the real system behaviour will result in a minimisation of the error energy for an impulse input signal. Clearly, $\mathbf{G}(s)$ cannot serve as such a reference function because it is defined in continuous time. In this paper, the method of Zielke and Suzuki is used for the generation of a discrete-time reference function $\mathbf{G}_{ref}$ to be used in the cost function

$$\int_0^\pi tr\left(\mathbf{G}_d(\theta) - \mathbf{G}_{ref}(\theta)\right)\left(\mathbf{G}_d(\theta) - \mathbf{G}_{ref}(\theta)\right)^T d\theta. \qquad (11)$$

While eq. (9) simply gives the solution for the direct feedthrough matrix $\mathbf{D}$, the gain condition (8) has to be built into the nonlinear programming problem. Due to the nature of the pipeline model with mixed boundary conditions, all poles of the transfer functions in $\mathbf{G}(s)$ have strictly negative real parts. Therefore, the poles of the discrete-time counterpart are now restricted to lie in the inner of the unit circle. With this assumption, the model is passive if and only [11] if

$$\mathbf{G}_d(\theta) + \mathbf{G}_d^*(\theta) \geq \mathbf{0} \tag{12}$$

for all real θ according to eq. (10).

3 Characteristics method as a starting point

The quality of the solution returned by the minimisation of the cost function (11) heavily depends on the initial values provided for the matrices $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$. The method of characteristics in the version due to Zielke and Suzuki together with an order reduction method is used for the generation of initial values. A standard method of characteristics with an equidistant grid according to Fig. 1(a) is used for the discretization of the equation system (2). The equations for the two boundary nodes "L" and "R" are

$$\psi_{0,k} = \psi_{L,k} \tag{13a}$$

$$\psi_{0,k+1} - \psi_{0,k} - q_{0,k+1} + q_{1,k} = 0 \tag{13b}$$

$$q_{N,k} = q_{R,k} \tag{13c}$$

$$\psi_{N,k+1} - \psi_{N-1,k} + q_{N,k+1} - q_{N-1,k} = 0 \tag{13d}$$

and the pressure and flow rate values at the inner nodes "P" are governed by

$$\psi_{j,k+1} - \psi_{j-1,k} + q_{j,k+1} - q_{j-1,k} = -f_{j-1,k} \tag{13e}$$

$$\psi_{j,k+1} - \psi_{j-1,k} = f_{j-1,k} \tag{13f}$$

The system of difference equations (13) gives a standard method of characteristics with the fluid friction represented by nodal friction models relating the flow rate q_j to the friction loss f_j at each node $j = 0, 1, \ldots, N$. A number of friction models approximating the transfer function given in eq. (3) are available in the literature [6]. A very accurate representation is the model due to Zielke and Suzuki et. al. [12, 10] which can be written as a discrete time state space system in the form

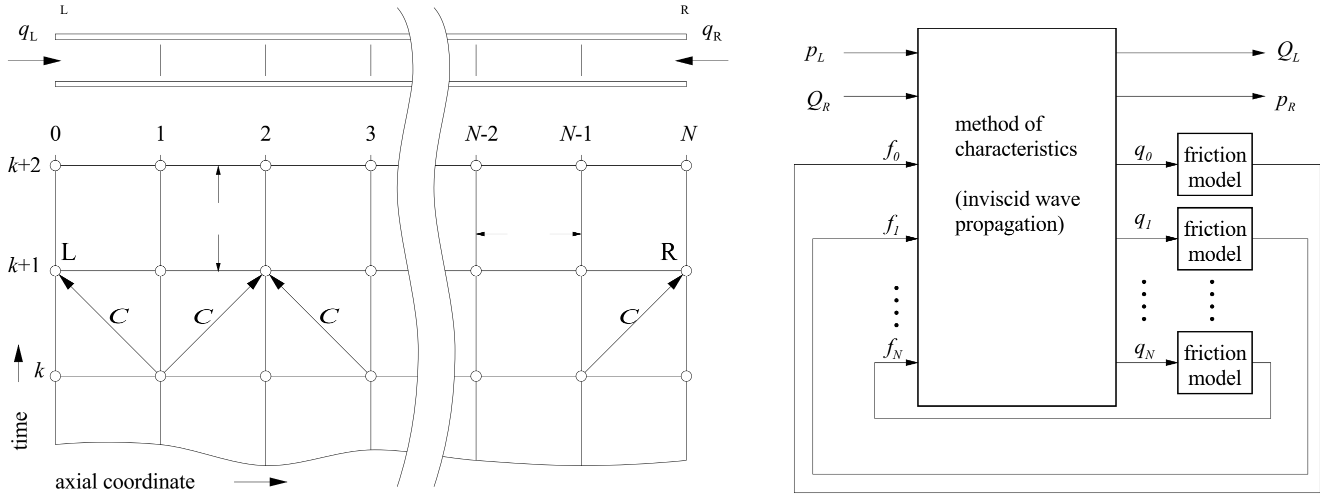
$$\mathbf{z}_{k+1} = \begin{bmatrix} 1 & 1 & 0 & \cdots & 0 \end{bmatrix}^T q_k + \begin{bmatrix} 0 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ -1 & \ddots & \ddots & & & & & & & \vdots \\ 0 & 1 & \ddots & \ddots & & & & & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & & & & & \vdots \\ \vdots & & \ddots & 1 & \ddots & \ddots & & & & \vdots \\ \vdots & & & 0 & b_1 & A_1 & \ddots & & & \vdots \\ \vdots & & & & b_2 & 0 & A_2 & \ddots & & \vdots \\ \vdots & & & & b_3 & \vdots & \ddots & A_3 & \ddots & \vdots \\ \vdots & & & & b_4 & \vdots & & \ddots & A_4 & 0 \\ 0 & \cdots & \cdots & 0 & b_5 & 0 & \cdots & \cdots & 0 & A_5 \end{bmatrix} \mathbf{z}_k \tag{14a}$$

$$f_k = \begin{bmatrix} -c_1 & c_2 & \cdots & c_{JT} & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \mathbf{z}_k + q_k \tag{14b}$$

While the friction model (14) is already in the standard form for a discrete-time LTI system, the method of charatcteristics model (13) needs to be restructured before fitting into the standard form. The set of all nodal pressures and flow rates with the exception of the values ψ_0 and q_N which are prescribed by boundary conditions seems to be a natural choice for the system state variables. However such a choice would not fit into a standard form like eqs. (7) because of the direct feedthrough from the inputs ψ_L and q_R to the flow rate q_0 and the pressure ψ_N according to the Joukowsky relation. In order to cast the system into standard form, a state vector is defined as

$$\mathbf{x} = \begin{bmatrix} q_0 - \psi_0 \\ \psi_1 \\ q_1 \\ \psi_2 \\ q_2 \\ \vdots \\ \psi_{N-1} \\ q_{N-1} \\ \psi_N + q_N \end{bmatrix}. \tag{15}$$

The method of characteristics model in standard form is given in eqs. (16) and (17). The input and output vectors have been augmented by friction terms f_j and flow rates q_j for all nodes. With these additional inputs and outputs, the model describing inviscid wave propagation is coupled with $N+1$ nodal friction models according to Fig. 1(b)



(a) Grid in the characteristic plane.

(b) Coupled model.

Figure 1: Method of characteristics.

$$
x_{k+1} = \begin{bmatrix}
0 & -1 & 1 & & & & & & & & & & & \\
\frac{1}{2} & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & & & & & & & & & \\
\frac{1}{2} & 0 & 0 & -\frac{1}{2} & \frac{1}{2} & & & & & & & & & \\
 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & & & & & & & \\
 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & -\frac{1}{2} & \frac{1}{2} & & & & & & & \\
 & & & \ddots & & & & \ddots & & & & & & \\
 & & & & \ddots & & & & \ddots & & & & & \\
 & & & & & & & \frac{1}{2} & \frac{1}{2} & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \\
 & & & & & & & \frac{1}{2} & \frac{1}{2} & 0 & 0 & -\frac{1}{2} & \frac{1}{2} & \\
 & & & & & & & & & \frac{1}{2} & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\
 & & & & & & & & & \frac{1}{2} & \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\
 & & & & & & & & & & & 1 & 1 & 0
\end{bmatrix} \mathbf{x}_k
$$

$$
+ \begin{bmatrix}
0 & 0 & 0 & -1 & & & & & \\
1 & \vdots & -\frac{1}{2} & 0 & \frac{1}{2} & & & & \\
1 & \vdots & -\frac{1}{2} & 0 & -\frac{1}{2} & & & & \\
0 & \vdots & & -\frac{1}{2} & 0 & \frac{1}{2} & & & \\
\vdots & \vdots & & -\frac{1}{2} & 0 & -\frac{1}{2} & & & \\
\vdots & \vdots & & & \ddots & \ddots & \ddots & & \\
\vdots & \vdots & & & & -\frac{1}{2} & 0 & \frac{1}{2} & \\
\vdots & \vdots & & & & -\frac{1}{2} & 0 & -\frac{1}{2} & \\
\vdots & -1 & & & & & -\frac{1}{2} & 0 & \frac{1}{2} \\
\vdots & 1 & & & & & -\frac{1}{2} & 0 & -\frac{1}{2} \\
0 & 0 & & & & & & -1 & 0
\end{bmatrix}
\begin{bmatrix} \psi_L \\ q_R \\ f_0 \\ f_1 \\ f_2 \\ \vdots \\ f_{N-1} \\ f_N \end{bmatrix} \qquad (16)
$$

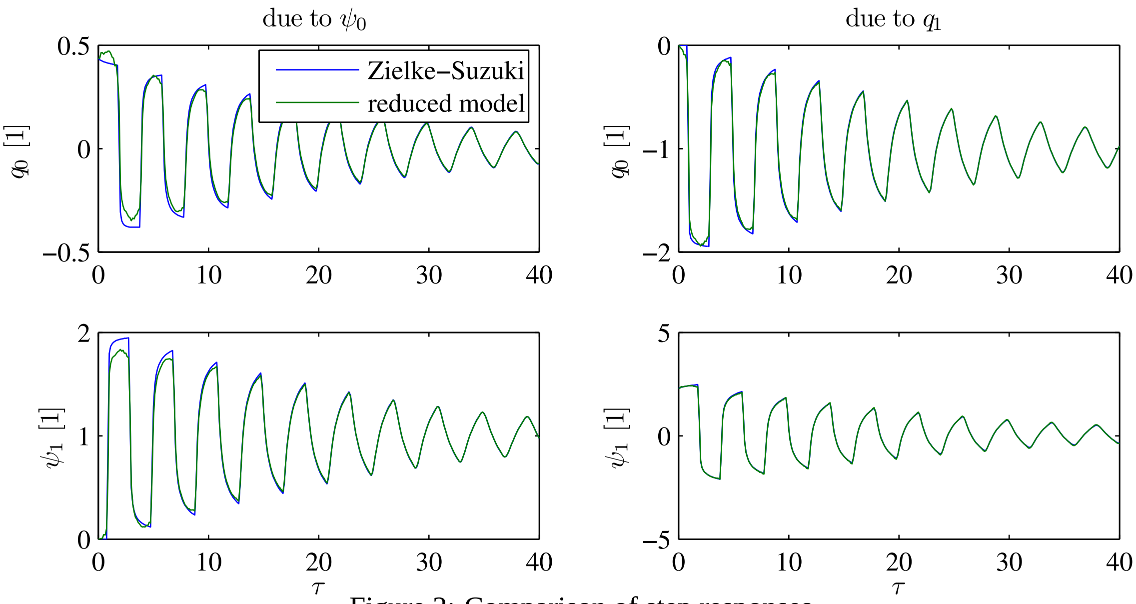


Figure 2: Comparison of step responses.

$$
\begin{bmatrix} q_L \\ \psi_R \\ q_0 \\ q_1 \\ \vdots \\ q_{N-1} \\ q_N \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & & 0 & 0 & 0 \\ 0 & 0 & 0 & & 0 & 0 & 1 \\ 1 & 0 & 0 & & 0 & 0 & 0 \\ 0 & 0 & 1 & & 0 & 0 & 0 \\ & & & \ddots & & & \\ 0 & 0 & 0 & & 0 & 1 & 0 \\ 0 & 0 & 0 & & 0 & 0 & 0 \end{bmatrix} x_k + \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 0 \\ 0 & \vdots \\ \vdots & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \psi_L \\ q_R \end{bmatrix} \qquad (17)
$$

4 Order reduction with passivity constraints

The model shown in Fig. 1(b) with the method of characteristics defined by eqs. (16, 17) and the nodal friction models according to eqs. (14a, 14b) is now used for computing both the inital values for the matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and the reference transfer function $\mathbf{G}_{ref}$ in the nonlinear programming problem of section 2.

First of all, the order n of the reduced system is chosen arbitrarily with a lower bound for n given by the order of the inviscid model, i.e. $n \geq 2N$. Then, the coupled model is reduced to order n. Kung's algorithm [7] as implemented in [9] is used for this purpose. This algorithm computes a model of arbitrary order via a singular value decomposition of a Hankel matrix generated from impulse response data. The impulse response could be generated without the reformulation of the Zielke-Suzuki method in the form of eqs. (16, 17, 14a, 14b), yet this approach enables the calculation of the passivity criterion for the coupled model.

The choice of a time domain identification algorithm is due to the fact that the Zielke-Suzuki method is already known to generate models reducable to a large extent [8] without a loss of accuracy. For a fine grid (large N) and for transmission lines with weak damping (small viscosity or large diameter) this results in a situation where a truncated impulse response sequence is much easier to handle than the huge sparse system given by the aforementioned reformulation of the Zielke-Suzuki method.

The reduced model of order n is now transformed into the modal form. In this canonical form, the matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ contain a number of $5n$ parameters to be optimized in the nonlinear programming problem. This problem is solved with the sequential quadratic programming code SNOPT [4]. Figure 2 shows a comparison of step responses of the original Zielke-Suzuki model and of a reduced model of order ρ for a dimensionless dissipation number of $D_n = 3.08 \cdot 10^{-3}$ and a grid size of $N = 8$. The minimal eigenvalue associated with the criterion (12) is shown over the frequency range $0 \leq \theta \leq \pi$ in Fig. 3. The reduced model derived with the proposed method is passive while an order reduction using Kung's algorithm results in a model that clearly fails to be passive.

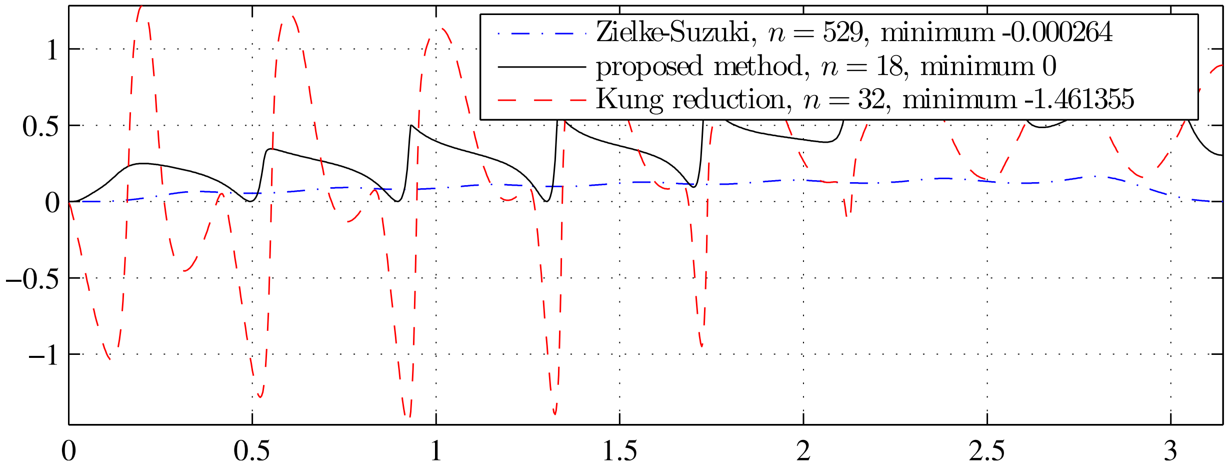


Figure 3: The minimum eigenvalue of $\mathbf{G}_d(\theta) + \mathbf{G}_d^*(\theta)$ over $0 \leq \theta \leq \pi$.

5 Conclusions

A method for the calculation of a discrete-time, reduced order, passive model for laminar, transient pipe flow has been proposed. While the presented example is a single line with one pressure and one flow-rate boundary condition, the method is also suitable for the reduced order modelling of compound fluid line systems as long as only linear elements are connected together.

The proposed method will be compared against established methods for passive constrained order reduction like Nevanlinna-Pick interpolation [1] in future work.

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