Modelling and Simulation of a Wheeled Mobile Robot in Configuration Classical Tricycle

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Abstract: - This paper addresses the problem of position control of a wheeled mobile robot. We present the kinematical and dynamical analysis for modelling a mobile robot in configuration classical tricycle. Simulation results are presented using a new position controller and it is supported by a rigorous analysis of asymptotic stability in agreement with the Lyapunov's direct method and the LaSalle's invariance principle.

Key-Words: - Wheeled mobile robot, Kinematics, Dynamics, Position Controller, Lyapunov function.

1 Introduction

Interest in mobile robots is growing rapidly because of the very broad range of their potential applications [1]. Wheeled Mobile Robots (WMR) constitute a class of mechanical systems characterized by kinematic constraints that are not integrable and cannot therefore be eliminated from model equations [2]. In recent years, the mobile robots have accomplished complex tasks, because they have the capability to adapt their environment by using an adequate system of sensors such as video camera, sonar, laser or infrared range-finders [3]-[8].

It is well known that feedback linearization through regular controllers has serious limitations for control of wheeled mobile robots. In particular, it does not allow a robot to be stabilized about a fixed point in the configuration space [2]-[3]. This paper presents the simulation results of a new position controller on a wheeled mobile robot using nonlinear dynamics. The objective of this paper is to propose a new control algorithm of position control with analysis of global asymptotic stability in Lyapunov sense of closed-loop system.

This paper is organized as follows: Section 2 and Section 3 recall the kinematic and dynamic models respectively, for a wheeled mobile robot in configuration classical tricycle. Section 4 presents the position control problem for mobile robots. In the Section 5, the new controller and its stability is presented. Section 6 contains the simulation results. Finally, some conclusions are offered in Section 7.

2 Robot kinematics

The kinematics studies the robot motion in function of its geometry. The kinematic model allows the behavior of wheeled mobile robot to be analysed within the framework of the theory of nonholonomic systems.

For the kinematical analysis we consider a robot capable of locomotion on a surface by the action of wheels mounted on the robot, therefore consider the next hypothesis:

- The wheeled mobile robot is moving on an horizontal flat surface, thus the potential energy $U(\xi)$ is constant.

- The guide axes are orthogonal with respect the surface.
- No exists flexible elements on the robot structure.
- The contact between the wheels and the surface is reduced to a single point (non-slip condition).

To determine the robot position is necesary to obtain the components of traslation and rotation of the attached coordinated system of the robot, this frame is mobile with respect to the inertial frame [9].

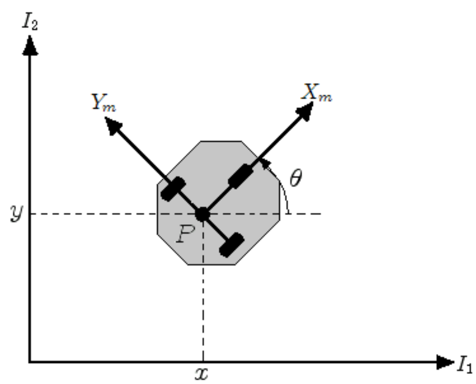

Figure 1: Robot plane.

The position of the mobile robot in the plane is described in Figure 1. All reference frames are Cartesian right-hand frames. Let Σ_I be an arbitrary inertial base frame and $[x,y]$ represents the coordinates of a point P with reference to Σ_I. P is the origin of the frame Σ_m which is attached to the wheeled mobile robot. The angle θ represents the rotation of frame Σ_m with respect to Σ_I. Hence, the robot posture is given by:

$$\xi = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \tag{1}$$

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

where $R(\theta) \in SO(3)$ is the rotation matrix [10].

An essential difference between wheeled mobile robots and other robotic systems are the kinematic restrictions. These restrictions are nonholonomics, thus the kinematical and dynamical analysis are more complicated than the holonomic systems [10]-[11].

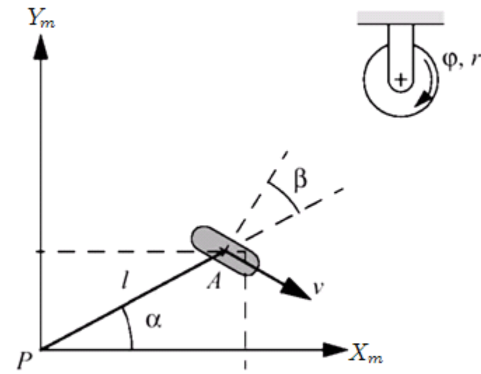

Figure 2: Conventional wheel.

We use the conventional wheel as is depicted in Figure 2. A denotes the center of the wheel which is a fixed point of the frame Σ_m. The position of A is characterized by polar coordinates, i.e., the distance l from P to A and angle α. The orientation of the plane of the wheel with respect to l is represented by the angle β. The rotation angle of the wheel about its horizontal axle is denoted by $\varphi(t)$ and the radius of the wheel by r. With this description, the components of the velocity of the contact point on the wheel plane and orthogonal to the wheel plane respectively, are easily computed by:

$$[-\sin(\alpha+\beta) \quad \cos(\alpha+\beta) \quad l\cos\beta]\, R(\theta)\dot{\xi} + r\dot{\varphi} = 0 \tag{3}$$

$$[\cos(\alpha+\beta) \quad \sin(\alpha+\beta) \quad l\sin\beta]\, R(\theta)\dot{\xi} = 0, \tag{4}$$

satisfy both conditions of pure rolling and non-slipping along the motion. The fixed wheels and centered orientable wheels have the same constraints, except that in the centered orientable wheels the angle $\beta(t)$ is time varying.

The wheeled mobile robot in configuration classical tricycle (Type (1,1) [3], [2]) is showed in the Figure 3. This mobile robot possesses two conventional fixed wheels on the same axle and one conventional centered orientable wheel.

The constraints of any wheeled mobile robot can be written under the general matrix form [2]:

$$J_1(\beta_c, \beta_{oc})R(\theta)\dot{\xi} + J_2\dot{\varphi} = 0 \tag{5}$$

$$C_1(\beta_c, \beta_{oc})R(\theta)\dot{\xi} + C_2\dot{\beta}_{oc} = 0, \tag{6}$$

where β_{oc} is an angle for a conventional off-centered orientable wheel [12]. In the case of a tricycle and considering the parameters presented in the Table 1, we have that:

$$\begin{aligned}
J_1 &= \begin{pmatrix} J_{10} \\ J_{11} \\ J_{12} \end{pmatrix} \\
&= \begin{pmatrix} \cos\beta & \sin\beta & L\cos\beta \\ 0 & 1 & \frac{L}{2} \\ 0 & -1 & \frac{L}{2} \end{pmatrix} \\
J_2 &= \operatorname{diag}(r) \\
C_1 &= \begin{pmatrix} C_{10} \\ C_{11} \\ C_{12} \end{pmatrix} \\
&= \begin{pmatrix} \sin\beta & -\cos\beta & L\sin\beta \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \\
C_2 &= 0.
\end{aligned}$$

Table 1: Parameters of the wheels.

Wheel	α	β	l
$1c$	$\frac{3\pi}{2}$	-	L
$2f$	0	0	$\frac{L}{2}$
$3f$	π	0	$\frac{L}{2}$

3 Robot dynamics

Derivation of the dynamic model of a robot plays an important role for simulation of motion, analysis of robot structures, and design of control algorithms. The dynamics of a nonholonomic mobile robot in agreement with the Euler-Lagrange methodology [9], [13] is given for

$$\frac{d}{dt}\left(\frac{\partial\mathcal{K}}{\partial\dot{\boldsymbol{q}}}\right) - \frac{\partial\mathcal{K}}{\partial\boldsymbol{q}} = A(\boldsymbol{q})\lambda + B(\boldsymbol{q})\boldsymbol{\tau} \tag{7}$$

where:

a) $\mathcal{K}$ is the kinetic energy.

b) $B(\boldsymbol{q})\boldsymbol{\tau}$ is the set of generalized forces with $B(\boldsymbol{q})$ a $n \times p$ kinematic matrix and $\boldsymbol{\tau}$ the p-vector of applied torques.

c) $A(\boldsymbol{q})$ is the matrix associated with the nonholonomic constraints; λ is the m-vector of Lagrange multipliers.

The kinetic energy $\mathcal{K}$ [14] is defined as

$$\mathcal{K} = \frac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} \tag{8}$$

where $M(\boldsymbol{q})$ is the $n\times n$ definite positive symmetric inertia matrix.

The robot motion is completely described by the following vector de 7 generalized coordinates:

$$\boldsymbol{q} = [x \quad y \quad \theta \quad \beta \quad \phi_1 \quad \phi_2 \quad \phi_3]^T. \tag{9}$$

The dynamical equations of the tricycle have the following form:

$$\begin{aligned}
\frac{d}{dt}\left(\frac{\partial\mathcal{K}}{\partial\dot{\xi}}\right) - \frac{\partial\mathcal{K}}{\partial\xi} &= R^T(\theta)\left[J_1^T(\beta)\lambda + \right. \\
&\quad \left. C_{10}^T(\beta)\mu + C_{11}^T\nu\right] \\
\frac{d}{dt}\left(\frac{\partial\mathcal{K}}{\partial\dot{\beta}}\right) - \frac{\partial\mathcal{K}}{\partial\beta} &= B_1\tau_1 \\
\frac{d}{dt}\left(\frac{\partial\mathcal{K}}{\partial\dot{\phi}}\right) - \frac{\partial\mathcal{K}}{\partial\phi} &= J_2^T\lambda + B_2\tau_2
\end{aligned} \tag{10}$$

where $\lambda = (\lambda_1, \lambda_2, \lambda_3)$, μ and ν are the 5 Lagrange multipliers associated with the 5 independent kinematical constraints, $B_1 = 1$ and $B_2 = [0\ 1\ 1]^T$.

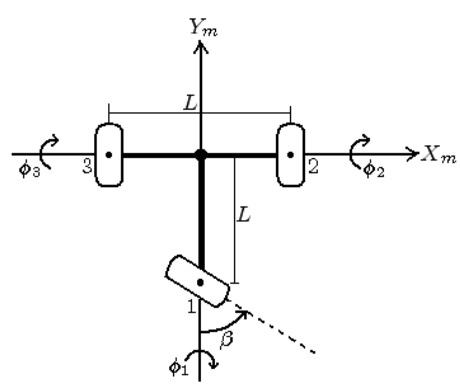

Figure 3: Mobile Robot.

The kinetic energy of the wheeled mobile robot (Figure 3) is expressed as the following symmetric quadratic form:

$$\mathcal{K} = \frac{1}{2}\dot{\boldsymbol{q}}^T \begin{bmatrix} R^T(\theta)M(\beta)R(\theta) & R^T(\theta)V(\beta) & 0 \\ V^T(\beta)R(\theta) & I_\beta & 0 \\ 0 & 0 & I_\phi \end{bmatrix} \dot{\boldsymbol{q}} \tag{11}$$

where $M(\beta)$ is a 3×3 symmetric matrix defined by:

$$\begin{aligned} M_{11}(\beta) &= M_{22}(\beta) = M + \sum_{i=1}^{3} m_i \\ M_{12}(\beta) &= M_{21}(\beta) = 0 \\ M_{13}(\beta) &= M_{31}(\beta) = -Ml_{c_2} - \sum_{i=1}^{3} m_i l_i \sin\alpha_i \\ M_{23}(\beta) &= M_{32}(\beta) = -Ml_{c_1} - \sum_{i=1}^{3} m_i l_i \cos\alpha_i \\ M_{33}(\beta) &= I_0 + M(l_{c_1}^2 + l_{c_2}^2) + \sum_{i=1}^{3} m_i l_i^2 \end{aligned} \tag{12}$$

$V(\beta)$ is a 3-vector defined as:

$$V(\beta) = \begin{bmatrix} 0 \\ 0 \\ I_\beta \end{bmatrix} \tag{13}$$

and I_ϕ have the following form:

$$I_\phi = \begin{bmatrix} I_{\phi_1} & 0 & 0 \\ 0 & I_{\phi_2} & 0 \\ 0 & 0 & I_{\phi_3} \end{bmatrix}. \tag{14}$$

In definitions (12)-(14), we present the next terms:

- M : mass of the trolley.
- m_i : mass of wheel i ; i=1,...,3.
- l_{c_1}, l_{c_2} : coordinates of the center of mass of the trolley in the frame attached to the trolley Σ_m.
- I_0 : inertia moment of the trolley around the vertical axes passing through its center of mass.
- I_β : inertia moment of the centered orientable wheel.
- I_{ϕ_i} : inertia moment of the wheel i, around its axis of rotation.

Using this kinetic energy, the dynamic model (10) is rewritten as:

$$\begin{aligned} R^T(\theta)M(\beta)R(\theta)\ddot{\xi} + f_1 &= R^T(\theta)\left[J_1^T(\beta)\lambda + C_{10}^T(\beta)\mu + \right. \\ &\quad \left. C_{11}^T\nu\right] \\ V^T(\beta)R(\theta)\ddot{\xi} + f_2 &= \tau_1 \\ I_\phi\ddot{\phi} &= J_2^T\lambda + B_2\tau_2 \end{aligned} \tag{15}$$

where f_1 and f_2 are respectively a 3-vector and 1-vector of functions of $\theta, \dot{\theta}, \beta, \dot{\beta}, \ddot{\beta}$. These functions are given by

$$\begin{aligned} f_1 &= R^T(\theta)\Big[\Big(2M(\beta)S_t - R(\theta)D(\dot{\xi}, \xi, \dot{\beta})\Big) \\ &\quad R(\theta)\dot{\xi} + V(\beta)\ddot{\beta} + S_t^T V(\beta)\dot{\beta}\Big] \end{aligned} \tag{16}$$

$$f_2 = V(\beta)^T S_t R(\theta)\dot{\xi} + I_\beta \tag{17}$$

where the time derivate $\dot{R}(\theta) = S_t R(\theta)$, the skew-symmetric matrix S_t [15] is

$$S_t = \begin{bmatrix} 0 & \dot{\theta} & 0 \\ -\dot{\theta} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{18}$$

$D(\dot{\xi}, \xi, \dot{\beta})$ is a 3×3 matrix defined as

$$D(\dot{\xi}, \xi, \dot{\beta}) = \begin{bmatrix} [0 \quad 0 \quad 0] \\ [0 \quad 0 \quad 0] \\ \Big[M(\beta)S_p R(\theta)\dot{\xi} + S_p^T V(\beta)\dot{\beta}\Big]^T \end{bmatrix} \tag{19}$$

S_p have the following form

$$S_p = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{20}$$

and it is related to $\partial R(\theta)/\partial\theta = S_p R(\theta)$.

In compact form, the dynamic model (15) of the wheeled mobile robot is rewritten as:

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\dot{\boldsymbol{q}}, \boldsymbol{q})\dot{\boldsymbol{q}} = A^T(\boldsymbol{q})\lambda + B\boldsymbol{\tau} \tag{21}$$

where $M(\boldsymbol{q})$ is the $n \times n$ definite positive symmetric matrix, and $C(\dot{\boldsymbol{q}}, \boldsymbol{q})$ is the $n \times n$ matrix of centripetal and Coriolis torques.

4 Control problem formulation

The position control problem in wheeled mobile robots is to design a control law $\boldsymbol{\tau}$ in such a way that the robot posture $\xi = [x \ y \ \theta]^T$ reaches the desired robot posture $\xi_d = [x_d \ y_d \ \theta_d]^T$.

The error of position is defined as

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{\theta} \end{bmatrix} = \begin{bmatrix} x_d - x \\ y_d - y \\ \theta_d - \theta \end{bmatrix} \tag{22}$$

therefore, the control aim is to assure that $\lim_{t\to\infty}$ $[\tilde{x}(t) \ \tilde{y}(t) \ \tilde{\theta}(t)]^T = 0$, regardless the initial conditions $[\tilde{x}(0) \ \tilde{y}(0) \ \tilde{\theta}(0)]^T$.

5 Position controller

This section presents the new position controller and its stability analysis. We consider the following control algorithm:

$$\boldsymbol{\tau} = K_p \arctan \tilde{\boldsymbol{q}} - K_v \arctan \dot{\boldsymbol{q}} + g(\lambda, r) \quad (23)$$

where K_p represents the diagonal matrix of proportional gains, K_v represents the diagonal matrix of derivative gains, $\tilde{\boldsymbol{q}}$ is the position error, $\dot{\boldsymbol{q}}$ denotes the vector of velocities.

The closed-loop equation is given by the dynamic model of wheeled mobile robot (21) and the control law (23) as follows:

$$\frac{d}{dt}\begin{bmatrix} \tilde{\boldsymbol{q}} \\ \dot{\boldsymbol{q}} \end{bmatrix} = \begin{bmatrix} -\dot{\boldsymbol{q}} \\ M(\boldsymbol{q})^{-1}[B(K_p \arctan \tilde{\boldsymbol{q}} - K_v \arctan \dot{\boldsymbol{q}} \\ +g(\lambda, r)) + A^T(\boldsymbol{q})\lambda - C(\dot{\boldsymbol{q}}, \boldsymbol{q})\dot{\boldsymbol{q}}] \end{bmatrix} \quad (24)$$

then

$$\begin{aligned} g_1(\lambda, r) &= 0 \\ g_2(\lambda, r) &= -r \end{aligned}$$

the origin is the unique equilibrium point.

To prove the stability of equilibrium point, we propose the following Lyapunov function:

$$\begin{aligned} V(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}}) &= \frac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} + \sum_{i=1}^{n} k_{pi} \left[\tilde{\boldsymbol{q}} \arctan \boldsymbol{q} \right. \\ &- \left. \frac{1}{2}\ln\left(1 + \boldsymbol{q}^T\boldsymbol{q}\right)\right], \end{aligned} \quad (25)$$

The time derivate of (25) after to do some complex algebra is given by

$$\dot{V}(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}}) = -\dot{\boldsymbol{q}}^T K_v \arctan \dot{\boldsymbol{q}} \leq 0, \quad (26)$$

which is negative semidefinite function. Therefore, in agreement with the Lyapunov's direct method, the control law yields stable closed loop system.

In order to study asymptotic stability we can apply the LaSalle's theorem [16], in the region

$$\begin{aligned} \Omega &= \left\{ \begin{bmatrix} \tilde{\boldsymbol{q}} \\ \dot{\boldsymbol{q}} \end{bmatrix} \in \mathbb{R}^{14} : \dot{V}(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}}) = 0 \Longleftrightarrow \begin{matrix} \dot{\boldsymbol{q}} = 0 \\ \tilde{\boldsymbol{q}} \in \mathbb{R}^7 \end{matrix} \right\} \\ &= \left\{ \dot{\boldsymbol{q}} = 0 \wedge \tilde{\boldsymbol{q}} = 0 : \dot{V}(\dot{\boldsymbol{q}}, \tilde{\boldsymbol{q}}) = 0 \right\} \end{aligned}$$

the maximum invariant set is the state origin. Therefore we conclude that the equilibrium point is asymptotically stable.

6 Simulation results

We select the desired position in x-y plane as $[x_d\ y_d\ \theta_d]^T = [1.0[\text{m}]\ 0.0[\text{m}]\ 0[\text{degrees}]]^T$ and the following initial position $[x(0)\ y(0)\ \theta(0)]^T = [0.0[\text{m}]\ 0.0[\text{m}]\ 0[\text{degrees}]]^T$.

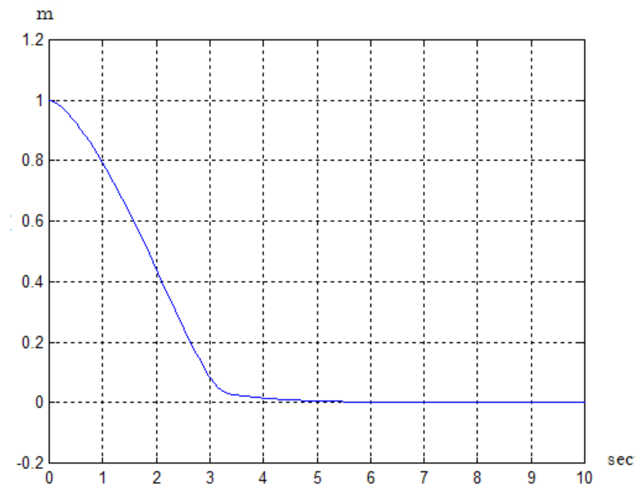

Figure 4: Position error $\tilde{x}$.

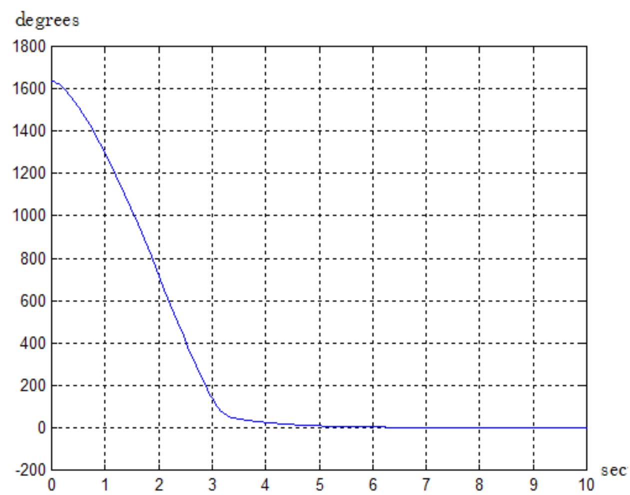

Figure 5: Position error $\tilde{\phi}$.

Figures 4-5 show the simulation results of the controller (23). The parameters of this controller were selected as $K_p = 1.0$ [Nm/degrees^2], $K_v = 1.0$ [Nm-sec/degrees]. Figure 4 depicts the time evolution of position error $\tilde{x}$. The transient response was around 5.5 sec. After transient, the position error tend asymptotically to a small neighborhood of zero.

The time evolution of angular position error $\tilde{\phi}$ is shown in figure 5. It can be observed that, in agreement with the tuning of the gains, the error tend asymptotically to a small neighborhood of zero.

7 Conclusions

In this paper we have presented the analysis of global asymptotic stability and simulation results of a new position controller for a wheeled mobile robot. The wheeled mobile robot belonging to the configuration classical tricycle. The position controller yield asymptotically stable closed–loop system. Also we present some simulation results.

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