An improved CA model with anticipation for one-lane traffic flow

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Abstract

A recent Cellular Automaton model (CA) with variable anticipation to simulate microscopic traffic flow for one-lane is modified. In this type of models, based on methods from statistical physics, vehicles follow a reduced set of rules. This allows simulation of large traffic networks with a reasonable computational effort.The goal is to obtain a quantitative agreement of the flux versus density relation in comparison with other existing CA models. For this purpose a very simple modification to the deceleration procedure is proposed. Simulation results show both a qualitative and quantitative coincidence of the relation derived from the density/flow curve with values taken from real traffic measurements for all density regimes. This modified model preserves the computational simplicity of CA models and the different flow phases observed in the CA model with anticipation of the velocity.

Key-Words : Traffic flow simulation, Cellular automata, Complex Systems, Traffic modeling

1 Introduction

In recent years, micro-simulation of traffic has become more popular as it can reproduce a large variety of phenomena observed in traffic. There is a recent trend on taking advantage of models developed originally in other research areas such as physics, mathematics, and computer science and applying them to traffic flow micro-simulation. In this address, Cellular Automata (CA) models [1, 2], that originated in statistical physics in the study of particle behavior, have shown the ability to capture the basic phenomena in real traffic flow [3]. Cellular automata are dynamic models in which space, time and state variables are discrete. When applied to traffic research, CA use cellular states to describe the position and velocity of each car, update every cell state with rules deduced from practical traffic experiences, and get the whole system's dynamical evolvement and final steady result. Compared with continuum models, CA traffic models are much simpler and more convenient for computer simulation. The most important aspect is that CA models can model the complexities of non-linear characters in traffic problems, and offer more intuitive physical images. Now, almost eleven years after the introduction of the first CA models[1], they have proved to be a realistic description of vehicular traffic in dense networks [4, 5].

One of the main characteristics of the driving behavior is the fact that drivers anticipate the maneuvers of the predecessor driver, and this is the most important fact in automatized driving. This last anticipation feature in automatized traffic model was introduced in the past [6]. Recently, a new single-lane probabilistic model based on the first CA model of Nagel and Schreckenberg [1] to describe the effects of several anticipation schemes in traffic flow was introduced [7]. In this model (hereafter cited as the LRS model) a new parameter in the deceleration process, so called *anticipatory driving parameter*, to estimate the velocity of the precedent vehicle is introduced [7]. This estimation, plus the real spatial distance to the leading vehicle, establishes a safe distance among vehicles. The addition of this parameter has been proved to be useful to describe different traffic situations of non-automated, automated, and mixed traffic. According to simulation results from the LRS model [7, 8], the parameters can be adjusted to reproduce empirical fundamental diagrams (flow vs. density curves) of real non-automated traffic. However, this fundamental diagram does not reproduce quantitatively well the characteristic curve obtained from empirical data for all density regimens. This problem is shared with other CA models. Thus, as in many other branches of the physics where the precise description of details is appreciated, some efforts have been addressed to modify several rules in order to describe quantitatively the fundamental diagram [9] [10].

In this paper, we study a modified LRS model aiming at a quantitative improvement of the fundamental diagram obtained. For that purpose we have developed through a deep understanding of the driver behavior a simple modification in the deceleration process of the LRS model. Concerning our simulation results, we want to claim that the suggested model variant agree with real data not only in a qualitative, but also in a quantitative form for all density regimes. Furthermore, the modified model still obey the "simplicity law" of CA modeling.

In section 2 of this paper a short review of the basic model is given and some limitations are discussed. Section 3 introduces our modification to the LRS model and gives results from our investigations. We focus on the shape of the fundamental diagram for non-automated traffic as obtained by CA simulations. Our aim is to achieve quantitative coincidence with empirical data of non-automated traffic instead of qualitative only. Finally, in section 4 we present the conclusions of our investigations.

2 The basic LRS model and some results

In this section a short review of the LRS model is given and some simulation results are discussed focussing on the fact that we want to reproduce details of the measured fundamental diagram.

2.1 The LRS model

The LRS model is a modification of the Nagel-Schreckenberg model [1] to better capture reactions of the drivers intended to keep safety on the highway. The model is defined on an one-dimensional lattice of L cells with periodic boundary conditions, which corresponds to a ring topology with the number of vehicles preserved. Each cell is either empty, or it is occupied by just one vehicle traveling with a discrete velocity v at a given instant of time. All vehicles have a velocity that ranges from $0, \ldots, v_{max}$. In addition, and for simplicity, only one type of vehicle is considered, this means that all vehicles are treated in the same manner. The time-step (Δt) is taken to be one second, therefore transitions are from $t \to t+1$. These aspects can also be easily modified. For convenience we use dimensionless (integer) variables, the real units being specified when needed.

Let v_i and x_i denote the current velocity and position of vehicle i, and v_p and x_p be the velocity and position of the vehicle ahead (preceding vehicle) at a fixed time; $d_i := x_p - x_i - 1$ denotes the distance (number of empty cells) in front of the vehicle in position x_i[a].

The dynamic of the system is defined with the following set of rules, which are applied to all N vehicles on the lattice each time-step:

R1: Acceleration

If $v_i < v_{max}$, the velocity of the car i is increased by one, i.e.,

$$v_i \to \min(v_i + 1, v_{max}).$$

R2: Randomization

If $v_i > 0$, the velocity of car i is decreased randomly by one unit with probability R, i.e.,

$$v_i \to \max(v_i - 1, 0) \quad \text{with probability } R.$$

[a] Bumper to bumper headway.

R3: Deceleration

If $d_i^s < v_i$, where

$$d_i^s = d_i + \left[(1-\alpha)\cdot v_p + \frac{1}{2}\right],$$

with a parameter $0 \leq \alpha \leq 1$, the velocity of car i is reduced to d_i^s. $[x]$ denotes the integer part of x, i.e. $[x+\frac{1}{2}]$ corresponds to rounding x to the next integer value. The new velocity of the vehicle i is therefore

$$v_i \rightarrow \min(v_i, d_i^s).$$

R4: Vehicle movement

Each car is moving forward according to its new velocity determined in steps 1-3, i.e.,

$$x_i \rightarrow x_i + v_i.$$

Rules $R1$, $R2$ and $R3$ are designed to update velocity of vehicles; rule $R4$ updates position. According to this, state updating is divided into two stages, first velocity (rules $R1, R2$ y $R3$), second position. In order to determine the velocity (v_i) consistently for all vehicles, rule $R3$ must be iterated at most (v_{max}) times.

Note that in rule $R3$ the distances between the ith and $(i+1)th$ vehicles, and their corresponding velocities are considered. Knowledge of the preceding vehicle's velocity is incorporated through the *anticipatory driving parameter* α with range $0 \leq \alpha \leq 1$. By only varying the parameter α in the term $d_i^s = d_i + [(1-\alpha)v_p + 1/2]$, different anticipatory driving schemes that require different safe braking distance with respect to the preceding vehicle can be modeled.

The LRS model is a minimal model in the sense that all four steps $R1$-$R4$ are necessary to reproduce the basic features of real traffic, however, additional rules may be needed to capture more complex situations [11] or as in our case to reproduce with more detail a real situation.

2.2 Some simulations results

The fundamental diagram characterizes the dependence of the vehicles flow on density and is one of the most important criteria to show that the model

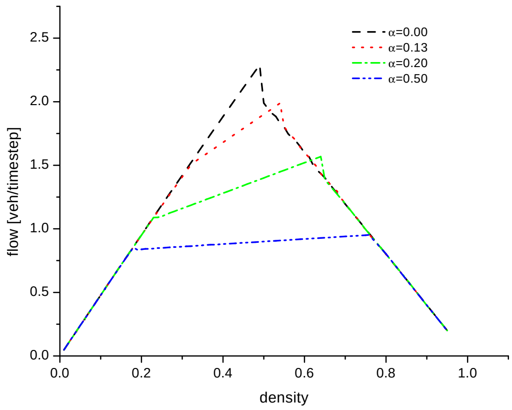

Figure 1: Fundamental diagram for different values of the anticipation parameter α. The legend indicates the effect of rounding in the estimation of the velocity of preceding car, defined in rule $R3$ (as presented in ref. [7]).

reproduces traffic flow behavior. Variation of the parameter α makes it possible to consider several anticipation strategies, e.g. non-automated, mixed and automated traffic flow, and so go beyond previous analysis.

To simulate the LRS model, the typical length of a cell is around 7.5 m [b]. With this value of the cell-size and a time-step of 1 s, the velocity of a vehicle $v_i = 1$ corresponds to moving from one cell to the downstream neighbor cell in one time-step, and that translates to 27 km/h in real units. The maximum velocity is set to $v_{max} = 5$, equivalent to 135 km/h. The density ρ is defined as $\rho = N/L$, where N is the number of cars on the highway. Initially, N vehicles are distributed randomly on the lane around the loop with an initial speed taking a discrete random value between 0 and v_{max}. Since the system is closed, the density remains constant with time.

In Fig. 1 the fundamental diagram resulting from the LRS model with a fixed value of the probability $R = 0.2$ and different values of α is shown. Simulations are carried out for $L = 10^4$ cells and $T = 15 * L$ time-steps. In order to analyze results, the first $10 * L$ time-steps of the simulation were discarded to let transients die out and the system reach its

[b] It is interpreted as the length of a vehicle plus the distance between cars in a dense jam, but it can be suitably adjusted according to the problem under consideration

steady state. Then the simulation data are averaged over the final $5 * L$ time-steps. The curve shows 98 points for ρ varying from 0.01 to 0.99 in steps of 0.01. Simulation results for driving schemes associated to intermediate-levels of anticipation with α from 0.13 to 0.5, exhibit phase separation in a certain density regime into a free-flow region and so-called v-platoons [7]. In these dense platoons vehicles move with the same velocity v and have vanishing headway. These states have been shown to be in agreement with different empirical observations in real traffic flow [7].

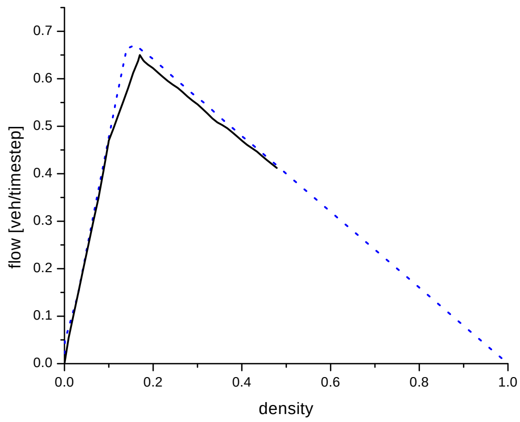

Figure 2: Typical form of an empirical fundamental diagram taken from [12] (solid line) in comparison with simulations of the proposed model for $R = 0.2$ and $\alpha = 0.75$ (dashed line). Each point corresponds to a fixed density.

2.3 Comparison with real non-automated traffic

When talking about fundamental diagrams from measured data one has to keep in mind that it is difficult to give the exact shape of such a curve. However a characteristic shape can be found [10]. For comparison with simulation data from the LRS model, the characteristic curve from measured data taken from [12], is plotted in Fig. 2 as a solid line. A maximum flow and critical density can be observed at $(\rho_c, q_{max}) = (0.17, 2340\ cars/h)$. Roughly speaking ρ_c separates the low density interval of free traffic flow (free-flow) from a jammed phase characterized by the persistence of jams (negative slope). Moreover, a change in the slope in the free flow region (positive slope) is identified at a density of 0.1. This fact corresponds to a reduction in the mean velocity of vehicles near the critical density. At this density interval the free flow velocity changes. Our interpretation of this fact is the following: Before traffic flow breaks down at the critical density of the fundamental diagram (where the maximum flow is reached) and therefore before first jams arise traffic flow organizes freely at a lower mean velocity. This means that drivers sacrifice speed in order to have less space to the preceding car without braking. As a consequence more cars fit on the road and the regime of free-flow can exist for high densities but with slower velocity.

For comparison with this characteristic curve, simulation data of the LRS model corresponding to an α value of 0.75 (cautious estimation of the preceding car's velocity, non-automated traffic) and $R = 0.2$ are also depicted in Fig. 2 (dash line).

As we can see from Fig. 2, the curve of the LRS model (dashed line) is consistent with the characteristic curve of the measured fundamental diagram (solid line). The values for the critical density and the maximum flux resulting from the LRS model are $(\rho_c, q_{max}) = (0.16,\ 2417\ cars/h)$. These values are closer to the corresponding ones of the empirical curve, in comparison with other existing models based on cellular automata, see [9]. However, the curve from simulations does not depict two slope in the free-flow region compared to real traffic measurements.

It would be even more satisfactory to reproduce the two slopes correctly, the improved LRS model to be introduced in the following tries to reproduce the quantitative characteristic of the fundamental diagram, i.e. the position of the maximum and the change in the slope before the maximum flow. We want to stress that recently other models have tried to reproduce the characteristic curve of the fundamental diagram presented here, in a quantitative way; however, the attempts complicate the models and do not completely succeed [9] [10].

3 Modification to the LRS model

In order to reproduce the quantitative characteristic curve of the fundamental diagram, we have understood how two different slopes arise in the real data based in the analysis performed in [10]. In the positive slope region of the fundamental diagram (free-flow regime) the value of the slope depends only on the free-flow velocity for the corresponding density. In the interval density from 0.1 to 0.165, there is a reduction in the mean velocity of vehicles near the critical density. In order to model this feature properly with the LRS model one would have to introduce a rule switching to a maximum velocity less than five for the density range from 0.1 to 0.165 but allowing to keep the positive slope. Such a switch would trigger a lower free flow velocity for that interval since the free flow velocity only depends on the maximum velocity and the value of the random braking parameter R with which speed is reduced by one in the randomization process. Therefore, in this paper we modify the deceleration process of the LRS model so that even for a gap of nine cells the maximum velocity is set to four. This means that very fast cars would only increase their velocity to maximum speed when there was a safety distance in average at least 67.5 meters in front of them. This physical restriction mimics the psychological drivers behavior of not reaching the maximum velocity with many cars around. This observation leads to a modification on the deceleration rule to LRS model as:

R3': If $v_i = 5$ (i.e. $v_i = v_{max}$) and $d_i^s \leq 9$ cells then

$$v_i \to \min(v_i - 1, d_i^s)$$

else (like rule *R3* of LRS model)

$$v_i \to \min(v_i, d_i^s).$$

In Fig. 3, the results obtained from the here proposed model are shown and compared with the experimental curve from real traffic data [12]. The curve resulting from the modified LRS model fits the real curve in both its increasing part and its decreasing part quite well. Furthermore, the results from the proposed model agree quantitatively with the experimental shape of the fundamental diagram in all of the density ranges. It is very important to claim that this agreement is only obtained by doing a very simple modification in the deceleration process of the LRS model. We need to stress one more time that attempts of other existing models in the literature to obtain this agreement did not succeed. Moreover, the modified LRS model preserves the simplicity of the models based in CA. Simulation results for different values of the anticipatory driving parameter

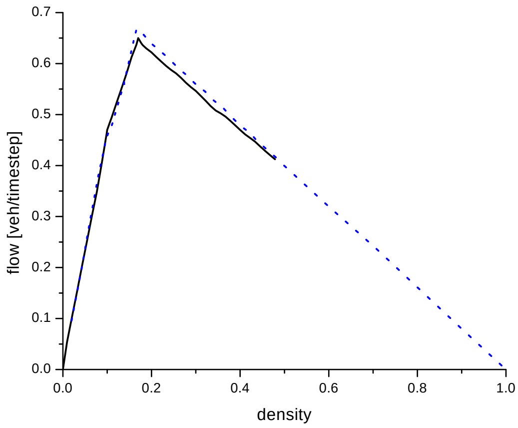

Figure 3: Fundamental diagram for the modified model. This diagram is in agreement with the experimental curve in a quantitative way.

(see Fig. 4) illustrate that this modified LRS model even captures the phases of traffic flow encoded in the fundamental diagram as it was observed in the LRS model, while preserving simplicity. This allows rapid simulation that can prove useful for application to large scale traffic networks. It is important to observe that a slight shift in the free-flow branch of the fundamental diagram appears. Such a shift is due to the modification on the deceleration rule, where a vehicle increases its velocity to v_{max} only when a safe distance is at least 9 cells, i.e., this change occurs beyond a density of 0.1. For this reason, around this density the straight line of free-flow regime is broken in two parts for all the α-values. For $\alpha \leq 0.5$, while the first branch has a slope of $v_{max} - R$ the second branch shows a slope of 4.0. This velocity is the "effective" maximum velocity according to the modification of $R3$.

4 Conclusions

In this paper we introduced simulation results which we obtained by modifying the LRS [7] model for traffic flow simulation. We suggest a modified deceleration process aiming at a quantitative improvement of the fundamental diagram obtained for all density regimes. The new deceleration process is still highly local and does not affect the run time of model computations. The modified model preserves the simplic-

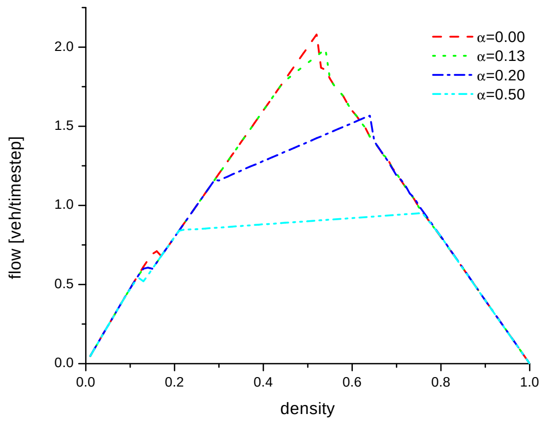

Figure 4: Fundamental diagram for different values of the anticipation parameter α resulting from the modified LRS model.

ity law of CA modeling. Concerning our simulation results corresponding to the cautious estimation of the preceding car's velocity ($\alpha = 0.75$) of the modified model, the relations obtained from the density vs. flow curve are in agreement with the characteristic curve of the measured fundamental diagram. Moreover, this agreement is not only qualitative but also quantitative for all density ranges, and it has not been attained with other CA models. The improved LRS model introduced in this paper reproduces the quantitative characteristic of the fundamental diagram, i.e. the position of the maximum and the change in the slope before the maximum. We want to stress that recently other models have tried to reproduce the characteristic curve of the fundamental diagram presented here, in a quantitative way. However, the attempts to improve the correlation of simulation and measurement did not succeed. Furthermore, we want to claim that within a small variant in the free-flow regimen, simulation results for different driving schemes associated to intermediate-levels of anticipation exhibit the same phase separations observed in the fundamental diagram for the original LRS model [7] opening new questions in phase separation in driven diffusive phenomena.

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