A Delay Independent Stability criterion for fpPID controller

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Abstract; The fpPID controller is a modified version of the well known PID controller, whose derivative component has been replaced by a fuzzy time series predictor. Such controller has shown good performance in simulation, but an important issue when modifying standard PID architecture is it's stability. In this paper we present delay independent conditions for fpPID controller stability[1]. Such conditions were obtained using the LMI approach and the Lyapunov Krasovskii Theorem.

keywords: Fuzzy logic; Lyapunov Krasovskii, stability; fpPID; Time series; LMI's.

1 Introduction

1.1 PID Controller (Proportional-Integral-Derivative)

The PID controller is by far the most common control algorithm. Most feedback loops are controlled by this algorithm or minor variations of it. It is implemented in many different forms, as a stand-alone regulator or as a part of a DDC package or hierarchical distributed process control system[2].

The equation for PID is given by:

$$u((t) = K_p[e(t) + \frac{1}{T_I}\int_0^t e(t)dt + T_D\frac{de(t)}{dt}] \quad (1)$$

Where:

$u(t)$	$\rightarrow$	Control signal
$e(t)$	$\rightarrow$	error signal, defined by: $e(t) = y(t) - r$
K_P	$\rightarrow$	Proportional constant
$\frac{K_P}{T_I}$	$\rightarrow$	Integral constant
$(\cancel{K}_P)(T_D)$	$\rightarrow$	Derivative constant

1.2 Fuzzy time series prediction

fpPID controller, uses a finite amount of past data, to predict $e(t+h)$. The prediction is given by:

$$\hat{e}_{t+h} = \frac{1}{k}\sum_{j=1}^{k} S_j e_{t_{j+h}}, \quad (2)$$

being h the ahead time step in the prediction; $e_{t_{j+h}}$ the control error measure; h the time length; t_j $j = 1, 2, \ldots, k$ are the neighbors used for prediction of $\hat{e}_{t+h}$ and S_j is the local similarity measure between each neighbor and the actual input. Local similarity is computed as follows: If $e_{t-m}\ldots e_t$ are the m time series points previous to actual time and $e_{t_{j-m}}$,...e_{t_j} any other sequence of points in the time series, then the local similarity of e_{t_j} to e_t is

$$S_j = \frac{1}{m+1}\sum_{j=0}^{m} \mu_{j,i}, \quad (3)$$

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where $\mu_{j,i}$ is the membership value of $e_{t_{j-i}}$ in a fuzzy set defined locally whose unique membership one point is located at e_{t-i}.

1.3 fpPID Controller

The fpPID controller, is obtained by replacing the derivative component in standard PID with a fuzzy *k*-nearest neighbors method for time series prediction[15], this architecture yields a better performance than PID because of using past dynamics of $e(t)$.

The equation for this controller is given by:

$$\begin{aligned} u(t) &= K_P(e(t)) + K_I \int_0^t e(t)dt \\ &\quad + \frac{K'_{fp}}{N} \sum_{j=1}^{N} \left\{ \frac{1}{m} \sum_{i=0}^{m-1} (\mu_{j,i}) e[t - h_j(t)] \right\} \end{aligned} \tag{4}$$

where:

K_P	$\rightarrow$	Proportional Constant.
K_I	$\rightarrow$	Integral Constant.
K'_{fp}	$\rightarrow$	Fuzzy Predictive Constant.
N	$\rightarrow$	Number of delays Constant.
m	$\rightarrow$	Number of points in mask. This mask defines the number of past points to considerate as reference for similarity function.
$\mu_{j,i}$	$\rightarrow$	membership value to the reference related.
$e(t)$	$\rightarrow$	Error signal at time t. $e(t) = r - x(t)$
$h_j(t)$	$\rightarrow$	Delay used to make the prediction

1.4 Stability analysis

While being of vital importance, closed loop stability analysis in Fuzzy Logic Controllers (FLC), it is not an easy task.

Since a system with a FLC is a nonlinear system, two different stability analysis should be undertaken: a local one around the operating point and a global one to check out if there are other equilibria or limit cycles.

One of the first proposals for stability analysis of FLC was the use of the describing function method [1, 3, 4], the circle criterion [5, 6] and the related Popov criterion for analyze the stability of Mamdani-type fuzzy control systems [7, 8], for linear plant. In [9] the local stability of a direct neuro-fuzzy controller is analyzed in an input/output setting. Multivariable circle criterion has also been used to analyze the robust stability of a fuzzy feedback linearization regulator [10]. More recently, slide modes have gained some spread for analysis of FLC [11, 12]. The stability analysis techniques based on Lyapunov direct method have gained popularity in the last years.
The methods based on linear matrix inequalities (LMI) [13] approach have been popularized in recent years. From this approach delay independent and delay dependent stability conditions have been obtained. Delay independent stability conditions have been more conservatives than Delay dependent stability conditions.

Theories of robust control have been introduced for systems with unknown delays. Systems with constant unknown delay term but unlimited, i.e., $h \in [0, \infty)$, have been analyzed [14, 16, 17] providing delay-independent stability criteria. If an unknown delay term is constant and bounded, delay-dependent stability criteria [18, 19, 20, 21, 22] improve stability margins compared to delay-independent criteria. [23]

The stability analysis for a FLC which depends of past dynamics of error,modelled as bounded variant delays in time is presented in this paper. The LMI approach and the Lyapunov-Krasovskii functionals are used in order to derive delay independent conditions.

In this paper, the stability of such controller is studied.
The paper is distributed as it follows: Section 2 presents the general description of the problem; Section 3 the stability analysis; finally, Section 4 shows the illustrative examples

2 Problem Statement

Consider the following SISO system:

$$\dot{x}(t) = -ax(t) + bu(t), \tag{5}$$

where $x(t)$ is the state variable and $u(t)$ the control signal dependent on $e(t)$, a,$b > 0$ are numerical constants. $u(t)$ provided by a fpPID controller, and it is:
The fuzzy predictor in the fpPID controller, uses a finite number N of time variant delays $h_j(t)$. Each neighbor of the predictor algorithm vote with each $h_j(t)$, the

change reason $\Delta h_j(t)$ can not be longer than the time series step and $\Delta t > 0$.

Considering a constant reference r, the dynamics $x(t)$ is related to $e(t)$ as $x(t) = r - e(t)$.Replacing $x(t)$ and $u(t)$ in(5) we obtain:

$$\begin{aligned}\dot{e}(t) &= a(r - e(t)) + b\Big\{K_P(e(t)) + K_I \int_0^t e(t)dt \\ &+ \frac{K'_{fp}}{N}\sum_{j=1}^{N}\Big\{\frac{1}{m}\sum_{i=0}^{m-1}(\mu_{j,i})e[t-h_j(t)]\Big\}\Big\} \quad (6)\end{aligned}$$

In this controller we have two cases to describe stability criterium:

1. Prediction = 0. in this case PI controller acts on the plant and the stability can be demonstrated through classic techniques.

2. Prediction $\neq 0$. In this case fpPID controller acts on the plant and the stability can be demonstrate by means of LMI Approach.

We can see that equation (6) is of slowed down type, and assumed that $h_j(t) \leq D_j$ for $j = 1, \ldots, N$, where D_j are constants, and represents the amount of past time to considerate for prediction algorithm. As the equation (6) is retarded type since $h(t)$ does not grow faster than t, we have that $\dot{h}_j(t) \leq H_j < 1$,$H_j$ represents the superior top, for $j = 1, \ldots, N$.
For case (2), we have that initial conditions $\phi(t)$ for (6) are the data previously stored.

3 Stability analysis

We based our result in the next Theorem:

Theorem 3.1 (Lyapunov-Krasovskii) *The trivial solution of the system (6) is asymptotic stable [24] if and only if exists a functional $v(t, x_t)$ positive defined, and a function $V_1(x(t))$ positive defined such that*

1. $V_1(x(t)) \leq V(t, x_t) \forall t \geq 0$ *and* $||x_t||_h \leq H$,

2. $V(x(t))$ *decreases monotonically up to zero throughout the trajectories of the system(6) when* $t \to \infty$.

Here, we employed the norm *h* is defined by

$$\|x_t\|_h = \sup_{t\in[t-h,t]} \|x(t)\| \quad (7)$$

The candidate equation to demonstrate the Stability under Lyapunov-Krasovskii criterium is as follow:

$$v(e_t) = P_0 e^2(t) + \sum_{i=1}^{N} P_i \int_{-h_{i(t)}}^{0} e^2(t+\theta)d\theta \quad (8)$$

where $P_0 > 0$, $P_i > 0$, $\quad \forall i, j = 1, \ldots, N$. We observe that N is defined positive and in addition satisfies:

$$V_1(e(t)) = \|e(t)\|, \qquad V_1(e(t) \leq V(e_t) \quad (9)$$

Differentiating (8) throughout the trajectories of (6) obtain

$$\begin{aligned}\dot{v}(e_t) &= 2P_0 e(t)\dot{e}(t) + \sum_{i=0}^{N} P_i\{e^2(t) - e^2[t-h_i(t)] \\ &[1 - \dot{h}_i(t)]\} \quad (10)\end{aligned}$$

Replacing (6) in (10), we obtain:

$$\begin{aligned}\dot{v}(e_t) &= 2P_0 e(t)\Big\{a(r-e(t)) - b\Big\{K_P(e(t)) + \\ &K_I\int_0^t e(t)dt \\ &+\frac{K'_{fp}}{N}\sum_{j=1}^{N}\{\frac{1}{m}\sum_{i=0}^{m-1}(\mu_{j,i})e[t-h_j(t)]\}\Big\}\Big\} \\ &+\sum_{i=1}^{N} P_i\Big\{e^2(t) - e^2[t-h_i(t)][1-\dot{h}_i(t)]\Big\}.\end{aligned}$$

Developing this equation we obtain:

$$\begin{aligned}\dot{v}(e_t) &= 2P_0 are(t) - (2P_0 a + 2P_0 bK_P)e^2(t) \\ &-2P_0 bK_I e(t)\int_0^t e(t)dt - \frac{2P_0 bK'_{fp}}{N}e(t) \\ &\sum_{j=1}^{N} P_j\Big\{\frac{1}{m}\sum_{i=0}^{m-1}(\mu_j, i)e[t-h_j(t)]\Big\} \\ &+\sum_{i=1}^{N} P_i\Big\{e^2(t) - e^2(t-h_j(t)\Big) \\ &\Big(1-\dot{h}_i(t)\Big)\Big\}. \quad (11)\end{aligned}$$

The term that describes delays into the system is:

$$\frac{2P_0bK'_{fp}}{N}e(t)\sum_{j=1}^{N}P_j\Big\{\frac{1}{m}\sum_{i=0}^{m-1}(\mu_j,i)e[t-h_j(t)]\Big\} \quad (12)$$

If we considered a single term for this sum, we obtain:

$$\frac{2P_0bK'_{fp}}{N}P_je(t)\Big\{\frac{1}{m}\sum_{i=0}^{m-1}(\mu_j,i)e[t-h_j(t)]\Big\} \quad (13)$$

To simplify notation, add two new term K_1 and $\theta(\mu_{j,i})$ as follow:

$$\begin{aligned} K_1 &= \frac{2P_0bK'_{fp}}{N} \\ \theta(\mu_{j,i}) &= \sum_{i=0}^{m-1}(\mu_j,i) \leq 1 \end{aligned} \quad (14)$$

Using this in (12), for N terms(delays) finally obtain:

$$\begin{aligned} & K_1e(t)\sum_{j=1}^{N}P_j\Big\{\frac{1}{m}\sum_{i=0}^{m-1}(\mu_j,i)e[t-h_j(t)]\Big\} \\ = \;& K_1e(t)\sum_{j=1}^{N}P_j\Big\{\frac{1}{m}\theta(\mu_j,i)e[t-h_j(t)]\Big\} \quad (15) \\ & \quad (16) \end{aligned}$$

replacing (15) in(11) obtain:

$$\begin{aligned} \dot{v}(e_t) &= 2P_0are(t) - \\ & (2P_0a + 2P_0bK_P + \sum_{i=1}^{N}P_i)e^2(t) \\ & -2P_0bK_Ie(t)\int_0^t e(\theta)d\theta \\ & +K_1e(t)\sum_{j=1}^{N}P_j\Big\{\frac{1}{m}\theta(\mu_j,i)e[t-h_j(t)]\Big\} \\ & -\sum_{j=1}^{N}P_ie^2(t-h_i(t))(1-\dot{h}_i(t)) \end{aligned} \quad (17)$$

We have to remember that we are analyzing stability for the trivial solution of the system, therefore $r = 0$. Considering this for (17) finally we obtain:

$$\begin{aligned} \dot{v}(e_t) &= -2P_0a + 2P_0bK_P + \sum_{i=1}^{N}P_i)e^2(t) \\ & -2P_0bK_Ie(t)\int_0^t e(\theta)d\theta \\ & +K_1e(t)\sum_{j=1}^{N}P_j\Big\{\frac{1}{m}\theta(\mu_j,i)e[t-h_j(t)]\Big\} \\ & -\sum_{j=1}^{N}P_ie^2(t-h_i(t))(1-\dot{h}_i(t)) \end{aligned} \quad (18)$$

Now, we consider this:

$$P_i > 0\forall i, e^2(t+h_i(t)) \geq 0$$

Considering this, obtain:

$$\begin{aligned} \dot{v}(e_t) &= -(2P_0a + 2P_0bK_P + \sum_{i=1}^{N}P_i)e^2(t) \\ & -2P_0bK_Ie(t)\int_0^t e(\theta)d\theta \\ & +K_1e(t)\sum_{j=1}^{N}P_j\Big\{\frac{1}{m}\theta(\mu_j,i)e[t-h_j(t)]\Big\} \\ & -\sum_{j=1}^{N}P_je^2(t-h_j(t))(1-\dot{h}(t)) \end{aligned} \quad (19)$$

Considering the last term, and that we have a slowed down equation, we obtain:

$$(1-H_j) > 0 \quad (20)$$

One more time, applying sum quad in (19), for the second term, we obtain:

$$\begin{aligned} & -2P_0K_Ibe(t)\int_0^t e(\theta)d\theta \\ \leq \;& 4P_0^2b^2e^2(t) + K_I^2[\int_0^t e(\theta)d\theta]^2 \end{aligned} \quad (21)$$

And for the last term we obtain:

$$\begin{aligned} & -\{\sum_{j=1}^{N}P_je^2(\,t-h_i(t))\}(1-\dot{h}_j(t)) \\ \leq \;& \sum_{j=1}^{N}P_je^2(\,t-h_j(t))(1+H), \end{aligned} \quad (22)$$

Considering this, replacing (21) and (22) in (19), obtain:

$$\begin{aligned} \dot{v}(e_t) &\leq -(2P_0a + 2P_0bK_P + \sum_{j=1}^{N}P_j)e^2(t) \\ & +4P_0^2b^2e^2(t) \\ & +K_I^2[\int_0^t e(\theta)d\theta]^2 \\ & +K_1e(t)\sum_{j=1}^{N}P_j\Big\{\frac{1}{m}\theta(\mu_j,i)e[t-h_j(t)]\Big\} \\ & +\sum_{j=1}^{N}P_je^2(t-h_j(t))(1+H) \end{aligned} \quad (23)$$

simplifying this equation, finally obtain:

$$\begin{aligned}\dot{v}(e_t) &\leq -\Big\{2P_0[a+b(K_P+2b)]+\sum_{j=1}^{N}P_j\Big\}e^2\\ &+K_I^2\Big[\int_0^t e(\theta)d\theta\Big]^2\\ &+K_1\sum_{j=1}^{N}P_j\Big\{\frac{1}{m}\theta(\mu_j,i)e[t-h_j(t)]\Big\}\\ &+\sum_{j=1}^{N}P_je^2(t-h_j(t))(1+H) \qquad (24)\end{aligned}$$

In an space-state representation, obtain:

$$\dot{v}(e_t) \quad \leq \quad E^TME$$

where:

$$E \quad \leq \quad \begin{bmatrix} e(t)\\ e(t-h_1)\\ e(t-h_2)\\ \vdots\\ e(t-h_N)\\ \int_0^t e(\theta)d\theta \end{bmatrix}$$

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} & \cdots & M_{1N} & 0\\ M_{21} & M_{22} & 0 & \cdots & 0 & 0\\ M_{31} & 0 & M_{33} & \cdots & 0 & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & 0\\ M_{N1} & 0 & 0 & \cdots & M_{NN} & 0\\ 0 & 0 & 0 & 0 & 0 & M_{N+1,N+1} \end{bmatrix}$$

The values for M are:

$$\begin{aligned} M_{11} &= -\big\{P_0\big(2bK_P-P_0(1-4b^2)\big)\\ &\quad +2P_0a+\textstyle\sum_{j=1}^{N}P_i\big\}\\ M_{21} &= -\tfrac{1}{2}K_1P_1\big(\tfrac{1}{m}\theta(\mu_j,i)\big)\\ M_{31} &= -\tfrac{1}{2}K_1P_2\big(\tfrac{1}{m}\theta(\mu_j,i)\big)\\ &\vdots\\ M_{N1} &= -\tfrac{1}{2}K_1P_N\big(\tfrac{1}{m}\theta(\mu_j,i)\big)\\ M_{12} &= -\tfrac{1}{2}K_1P_1\big(\tfrac{1}{m}\theta(\mu_j,i)\big)\\ M_{13} &= -\tfrac{1}{2}K_1P_2\big(\tfrac{1}{m}\theta(\mu_j,i)\big)\\ &\vdots\\ M_{1N} &= -\tfrac{1}{2}K_1P_N\big(\tfrac{1}{m}\theta(\mu_j,i)\big)\\ M_{22} &= K_1^2P_1(1-H_1)\\ M_{33} &= K_1^2P_2(1-H_2)\\ &\vdots\\ M_{NN} &= -K_1^2P_N(1-H_N)\\ M_{N+1,N+1} &= K_I^2 \end{aligned}$$

By the previous thing, we can to establish the following result:

Theorem 3.2 (fpPID Stability) *The trivial solution of the system (6) is locally asymptotic stable if and only if matrix M is defined negative.*

4 Answer of some plants simulated with the controller fpPID

In the following graphs, the plots drawn up with dotted line represent the fpPID controller output, and the plots drawn up with solid line represent a standard PID.

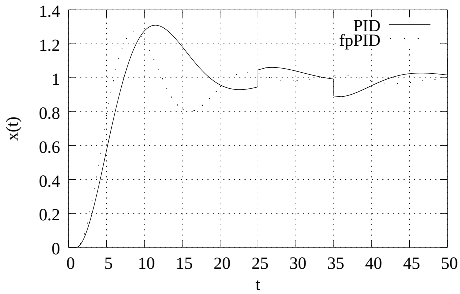

Fig. 1: Plant 1

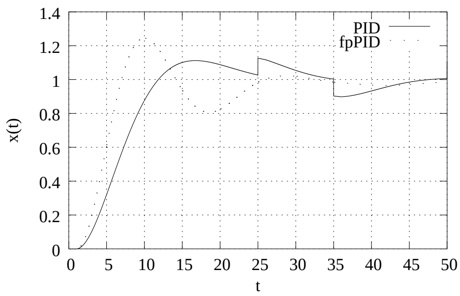

Fig. 2: Plant 2

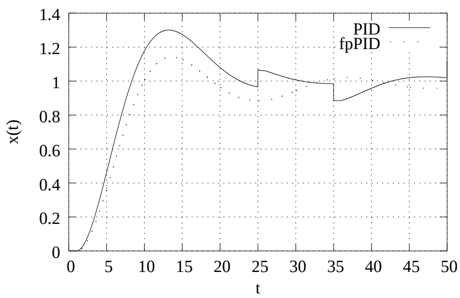

Fig. 3: Plant 3

5 Conclusions

In the Asymptotic Stability analysis for fpPID controller in closed loop, in spite of of being a system nonlinear (that in addition depends on last values), It was possible to find sufficient conditions for this stability system by means of LMI approach.

The Stability analysis was made with Lyapunov-Krasovskii's functionals.

with the use of Lyapunov-Krasovskii's functionals, the possibility fits of analyzing robustness schemes.

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