Modified Theories of Laminar and Turbulent Rotating Jets

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Abstract: Scale-invariant forms of mass, energy, and linear momentum conservation equations in chemically reactive fields are described. The modified equation of motion is then solved for the classical problem of axisymmetric laminar rotating free jet. The results are shown to be in agreement with the classical theories of *Loitsianskii* and *Görtler*. The problem of turbulent axisymmetric rotating jet is shown to have solutions identical to those of laminar jet. Temperature distributions of axisymmetric and two-dimensional turbulent jets are determined and the latter results are found to be in agreement with the observations of *Reichardt*.

Key-Words: Theory of laminar and turbulent cylindrically symmetric rotating jets. Decay of swirl.

1 Introduction

The universality of turbulent phenomena from stochastic quantum fields to classical hydrodynamic fields [1-26] resulted in recent introduction of a scale-invariant model of statistical mechanics and its application to the field of thermodynamics [27]. The implications of the model to the study of transport phenomena and invariant forms of conservation equations have also been addressed [28, 29]. In the present study, following the classical studies of *Loitsianskii* [39] and *Görtler* [40], the modified equation of motion is solved for the classical problem of laminar axisymmetric rotating jet. The predicted analytical solutions are found to reduce to the approximate solutions obtains from the classical form of the equation of motion. The solutions of the modified forms of the equation of motion and energy at the larger scale of laminar eddy-dynamics LED representing turbulent axi-symmetric rotating jet are also discussed.

2 Scale-Invariant Forms of the Conservation Equations for Reactive Fields

Following the classical methods [30-32], the invariant definitions of the density ρ_β, and the velocity of *atom* $\mathbf{u}_\beta$, *element* $\mathbf{v}_\beta$, and *system* $\mathbf{w}_\beta$ at the scale β are given as [29]

$$\rho_\beta = n_\beta m_\beta = m_\beta \int f_\beta du_\beta \quad , \quad \mathbf{u}_\beta = \mathbf{v}_{\beta-1} \tag{1}$$

$$\mathbf{v}_\beta = \rho_\beta^{-1} m_\beta \int \mathbf{u}_\beta f_\beta d\mathbf{u}_\beta \quad , \quad \mathbf{w}_\beta = \mathbf{v}_{\beta+1} \tag{2}$$

Also, the invariant definitions of the peculiar and the diffusion velocities are given as [29]

$$\mathbf{V}'_\beta = \mathbf{u}_\beta - \mathbf{v}_\beta \quad , \quad \mathbf{V}_\beta = \mathbf{v}_\beta - \mathbf{w}_\beta = \mathbf{V}'_{\beta+1} \tag{3}$$

Next, following the classical methods [30-32], the scale-invariant forms of mass, thermal energy, and linear momentum conservation equations at scale β are given as [29]

$$\frac{\partial \rho_\beta}{\partial t} + \nabla.(\rho_\beta \mathbf{v}_\beta) = \Omega_\beta \tag{4}$$

$$\frac{\partial \varepsilon_\beta}{\partial t} + \nabla.(\varepsilon_\beta \mathbf{v}_\beta) = 0 \tag{5}$$

$$\frac{\partial \mathbf{p}_\beta}{\partial t} + \nabla.(\mathbf{p}_\beta \mathbf{v}_\beta) = 0 \tag{6}$$

involving the *volumetric density* of thermal energy $\varepsilon_\beta = \rho_\beta h_\beta$ and linear momentum $\mathbf{p}_\beta = \rho_\beta \mathbf{v}_\beta$. Also, Ω_β is the chemical reaction rate and h_β is the absolute enthalpy [28].

The local velocity $\mathbf{v}_\beta$ in (4)-(6) is expressed in terms of the convective $\mathbf{w}_\beta = \langle \mathbf{v}_\beta \rangle$ and the diffusive $\mathbf{V}_\beta$ velocities [29]

$$\mathbf{v}_\beta = \mathbf{w}_\beta + \mathbf{V}_{\beta g} \quad , \quad \mathbf{V}_{\beta g} = -D_\beta \nabla \ln(\rho_\beta) \tag{7a}$$

$$\mathbf{v}_\beta = \mathbf{w}_\beta + \mathbf{V}_{\beta tg} \quad , \quad \mathbf{V}_{\beta tg} = -\alpha_\beta \nabla \ln(\varepsilon_\beta) \tag{7b}$$

$$\mathbf{v}_\beta = \mathbf{w}_\beta + \mathbf{V}_{\beta hg} \quad , \quad \mathbf{V}_{\beta hg} = -\nu_\beta \nabla \ln(\mathbf{p}_\beta) \tag{7c}$$

where ($\mathbf{V}_{\beta g}$, $\mathbf{V}_{\beta tg}$, $\mathbf{V}_{\beta hg}$) are respectively the diffusive, the thermo-diffusive, and the linear hydro-diffusive velocities.

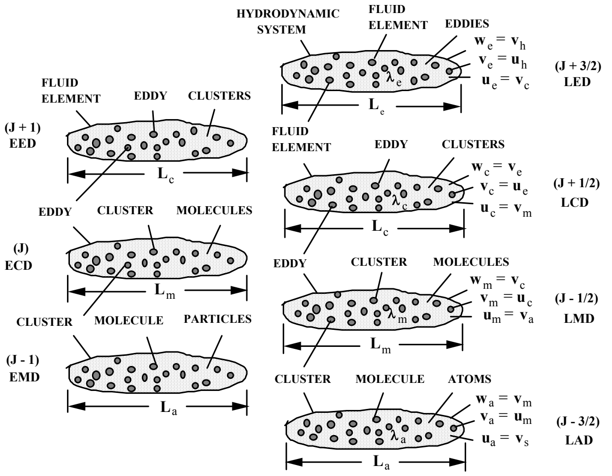

Fig.1 Hierarchy of statistical fields for equilibrium eddy-, cluster-, and molecular-dynamic scales and the associated laminar flow fields.

For unity *Schmidt* and *Prandtl* numbers, one may express

$$\mathbf{V}_{\beta tg} = \mathbf{V}_{\beta g} + \mathbf{V}_{\beta t} \quad , \quad \mathbf{V}_{\beta t} = -\alpha_\beta \nabla \ln(h_\beta) \tag{8a}$$

$$\mathbf{V}_{\beta hg} = \mathbf{V}_{\beta g} + \mathbf{V}_{\beta h} \quad , \quad \mathbf{V}_{\beta h} = -\nu_\beta \nabla \ln(\mathbf{v}_\beta) \tag{8b}$$

that involve the thermal $\mathbf{V}_{\beta t}$, and linear hydrodynamic $\mathbf{V}_{\beta h}$ diffusion velocities [29]. Since for an ideal gas $h_\beta = c_{p\beta} T_\beta$, when $c_{p\beta}$ is constant and $T = T_\beta$, Eq.(8a) reduces to the *Fourier* law of heat conduction

$$\mathbf{q}_\beta = \rho_\beta h_\beta \mathbf{V}_{\beta t} = -\kappa_\beta \nabla T \tag{9}$$

where κ_β and $\alpha_\beta = \kappa_\beta / (\rho_\beta c_{p\beta})$ are the thermal conductivity and diffusivity. Similarly, (8b) may be identified as the shear stress associated with diffusional flux of linear momentum and expressed by the generalized *Newton* law of viscosity [29]

$$\boldsymbol{\tau}_{ij\beta} = \rho_\beta \mathbf{v}_{j\beta} \mathbf{V}_{ij\beta h} = -\mu_\beta \partial \mathbf{v}_{j\beta} / \partial \mathbf{x}_i \tag{10}$$

Substitutions from (7a)-(7c) into (4)-(6), neglecting cross-diffusion terms and assuming constant transport coefficients with $Sc_\beta = Pr_\beta = 1$, result in [29]

$$\frac{\partial \rho_\beta}{\partial t} + \mathbf{w}_\beta . \nabla \rho_\beta - D_\beta \nabla^2 \rho_\beta = \Omega_\beta \tag{11}$$

$$h_\beta \left[\frac{\partial \rho_\beta}{\partial t} + \mathbf{w}_\beta . \nabla \rho_\beta - D_\beta \nabla^2 \rho_\beta \right] + \rho_\beta \left[\frac{\partial h_\beta}{\partial t} + \mathbf{w}_\beta . \nabla h_\beta - \alpha_\beta \nabla^2 h_\beta \right] = 0 \tag{12}$$

$$\mathbf{v}_\beta \left[\frac{\partial \rho_\beta}{\partial t} + \mathbf{w}_\beta . \nabla \rho_\beta - D_\beta \nabla^2 \rho_\beta \right] + \rho_\beta \left[\frac{\partial \mathbf{v}_\beta}{\partial t} + \mathbf{w}_\beta . \nabla \mathbf{v}_\beta - \nu_\beta \nabla^2 \mathbf{v}_\beta \right] = 0 \tag{13}$$

In the first and second parts of (12) and (13), the *gravitational* versus the *inertial* contributions to the change in energy and momentum density are apparent. Substitutions from (11) into (12)-(13) result in the invariant forms of conservation equations in chemically reactive fields [29]

$$\frac{\partial \rho_\beta}{\partial t} + \mathbf{w}_\beta . \nabla \rho_\beta - D_\beta \nabla^2 \rho_\beta = \Omega_\beta \tag{14}$$

$$\frac{\partial T_\beta}{\partial t} + \mathbf{w}_\beta . \nabla T_\beta - \alpha_\beta \nabla^2 T_\beta = -h_\beta \Omega_\beta / (\rho_\beta c_{p\beta}) \tag{15}$$

$$\frac{\partial \mathbf{v}_\beta}{\partial t} + \mathbf{w}_\beta . \nabla \mathbf{v}_\beta - \nu_\beta \nabla^2 \mathbf{v}_\beta = -\mathbf{v}_\beta \Omega_\beta / \rho_\beta \tag{16}$$

An investigation of the system (14)-(16) in the presence of chemical reactions Ω resulted in a modified hydro-thermo-diffusive theory of laminar flames presented earlier [29]. It is emphasized here that in (16) the convective velocity $\mathbf{w}_\beta$ is different from the local fluid velocity $\mathbf{v}_\beta$.

3 Solution of the Modified Equation of Motion for Laminar Cylindrically Symmetric Rotating Jet

As examples of exact solutions of the modified equation of motion (16), the classical problems [32-41] of laminar [42] and turbulent [43] flow in cylindrically symmetric and two-dimensional jets were investigated. In the present study, the modified equation of motion is solved for the classical problem of axi-symmetric laminar rotating jet investigated by *Loitsianskii* [39] and *Görtler* [40]. Therefore, one looks for the local axial, radial and azimuthal velocities (v'_x, v'_r, v'_θ) along the corresponding coordinates (x', r', θ) and introduces the dimensionless quantities

$$(v_x,\ v_r, w_x,\ w_r) = (v'_x,\ v'_r, w'_x,\ w'_r)/w'_o$$

$$v_\theta = \frac{v'_\theta}{\sqrt{\nu\omega_o}} \quad , \quad w_\theta = \frac{w'_\theta}{\sqrt{\nu\omega_o}} \quad , \quad \theta = \frac{T - T_\infty}{T_o - T_\infty}$$

$$r = \frac{r'}{l_H} \quad , \quad x = \frac{x'}{l_H} \quad , \quad l_H = l_T = \frac{\nu}{w'_o} = \frac{\alpha}{w'_o} \tag{21}$$

The initial axial convective velocity w'_o at the origin of the jet is assumed to be known and signifies the strength of the jet. Also, the initial angular frequency ω_o at the origin of the jet is known and signifies the strength of the jet angular momentum. Hence, in (21) both the local and the convective dimensionless azimuthal velocities (v_θ, w_θ) involve this initial jet angular frequency. The parameters ν and α are the kinematic viscosity and thermal diffusivity and l_H and l_T are the characteristic hydrodynamic and thermal lengths. The steady dimensionless forms of the modified equation of motion (16), energy (15), and the continuity equation (4) for incompressible fluid with the usual boundary layer assumptions and in the absence of chemical reactions Ω = 0 reduce to

$$w_x \frac{\partial \mathbf{v_x}}{\partial x} + w_r \frac{\partial \mathbf{v_x}}{\partial r} = \frac{\partial^2 \mathbf{v_x}}{\partial r^2} + \frac{1}{r}\frac{\partial \mathbf{v_x}}{\partial r} \tag{22}$$

$$w_x \frac{\partial \mathbf{v}_\theta}{\partial x} + w_r \frac{\partial \mathbf{v}_\theta}{\partial r} + \frac{w_\theta \mathbf{v}_r}{r} = \frac{\partial^2 \mathbf{v}_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial \mathbf{v}_\theta}{\partial r} - \frac{\mathbf{v}_\theta}{r^2} \tag{23}$$

$$w_x \frac{\partial \theta}{\partial x} + w_r \frac{\partial \theta}{\partial r} = \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} \tag{24}$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_r}{\partial r} + \frac{v_r}{r} = 0 \tag{25}$$

subject to the boundary conditions

$$r = 0 \qquad \frac{\partial v_x}{\partial r} = \frac{\partial \theta}{\partial r} = v_r = 0 \tag{26a}$$

$$r \to \infty \qquad v_r = v_\theta = v_x = \theta = 0 \tag{26b}$$

To solve equation (22), the convective velocities (w_x, w_r) that are the average of the local axial and radial velocities (v_x, v_r) are needed. Because the jet momentum is initially only in the axial direction, the radial dispersion of the jet is entirely caused by diffusion of axial momentum in the radial direction. Therefore, the jet diameter d' at any axial position will be given by the radial diffusion length

$$d'^2 = 2\nu t' \tag{27}$$

where the local diffusion time t' is related to the axial position x' and the local convective velocity w'_x by

$$t' = x'/w'_x \tag{28}$$

The dimensionless jet cross sectional area A' is given by

$$A = A'/(\nu/\omega_o) = \pi d^2/4 = \\ = (\pi/2)\, t = (\pi/2)\, x/w_x \tag{29}$$

Hence, the mass flow rate at any axial position will vary as

$$Q = \rho A' w'_x \propto \rho A w_x \propto x \tag{30}$$

where the symbol (∝) in (30) denotes proportionality. On the other hand, the total axial momentum along the jet must remain constant [32] and hence

$$J = Q w'_x \propto x\, w_x = \text{constant} \tag{31}$$

leading to the dimensionless convective velocity [35]

$$w_x = \frac{1}{2x} \tag{32}$$

Solving the global continuity equation

$$\frac{\partial w_x}{\partial x} + \frac{\partial w_r}{\partial r} + \frac{w_r}{r} = 0 \tag{33}$$

after substitution from (32) results in

$$w_r = \frac{r}{4x^2} \tag{34}$$

Following *Schlichting* [32], one introduces the stream function Ψ and the similarity variable η

$$\Psi = x\, F(\eta) \quad , \quad \eta = \frac{r}{x} \tag{35}$$

leading to the axial velocity

$$v_x = \frac{F'(\eta)}{x} \tag{36}$$

Substitutions from (32), (34), (35), and (36) into (22) results in [42]

$$zF''' + (z^2+1)F'' + 2zF' = 0 \tag{37}$$

$$z = 0 \qquad F'' = 0 \tag{38}$$

$$z \to \infty \qquad F' = 0 \tag{39}$$

where $z = \eta/2$ and primes denote differentiation with respect to z. The bounded solution of (37)-(39) is

$$F' = \exp(-z^2/2)] = \exp(-\eta^2/8)] \tag{40}$$

that by (36) gives

$$v_x = \frac{\exp(-\eta^2/8)}{x} \tag{41}$$

From (35) and (41), the stream function that satisfies $\Psi = 0$ at $\eta = 0$ is obtained as

$$\Psi = 4x[1-\exp(-\eta^2/8)] \tag{42}$$

that in turn gives the radial velocity

$$v_r = \frac{1}{x}\left[\eta\exp(-\eta^2/8) - \frac{4}{\eta}\left(1-\exp(-\eta^2/8)\right)\right] \tag{43}$$

Since (24, 26) are similar to (22, 26), the distribution of temperature θ is similar to that of the axial velocity (41) and given by

$$\theta = \frac{\exp(-\eta^2/8)}{x} \tag{44}$$

Therefore, the ratio of the axial velocity v_x (temperature θ) to the centerline velocity v_{xc} (temperature θ_c) becomes

$$\frac{v_x}{v_{xc}} = \exp(-\eta^2/8) \tag{45}$$

$$\frac{\theta}{\theta_c} = \exp(-\eta^2/8) \tag{46}$$

The mass flow rate Q is given by [42]

$$Q = 2\pi\rho\int_0^\infty v'_x r' dr' = 8\pi\rho\nu x' \tag{47}$$

in exact agreement with the classical result of *Schlichting* [32]. Also, the axial momentum J

$$J = 2\pi\rho\int_0^\infty v'^2_x r' dr' = 4\pi\rho\nu^2 \tag{48}$$

is a constant independent of the jet strength w'_o in accordance with the classical results [32].

At large values of x for which the present similarity solution is valid, i.e. for small values of $z = \eta/2$, (41) can be expressed as

$$v_x = \frac{\exp(-z^2/2)}{x} = \frac{1}{x}\frac{1}{\exp(-z^2/2)}$$

$$= \frac{1}{x}\frac{1}{\exp(-z^2/2)} \approx \frac{1}{x}\frac{1}{\left(1+z^2/4\right)^2} \tag{49}$$

that is in exact agreement with the classical solution [32] except for some multiplicative constants. Therefore, it is expected that the excellent agreement of the classical theory of *Schlichting* [32] with the experimental observations also extend to the modified theory [42].

The above results are now used in the azimuthal momentum conservation equation (23) to determine the angular velocity. The radial velocity can be approximated as

$$v_r = \frac{1}{x}\left[\eta\exp(-\eta^2/8) - \frac{4}{\eta}\left(1-\exp(-\eta^2/8)\right)\right]$$

$$\approx \frac{1}{x}\eta\exp(-\eta^2/8) \tag{50}$$

since under the assumption x >>1 ($\eta \ll 1$) valid for the similarity solution being considered, the second term can be neglected. Substituting (50) along with the similarity form

$$v_\theta = \frac{H(\eta)}{x^2} \tag{51}$$

and the convective velocities (32), (34), and

$$w_\theta = \frac{\eta}{4x^2} \tag{52}$$

into (23) results in

$$\eta^2 H'' + \eta(1+\eta^2/4)H' + (\eta^2-1)H$$
$$-(\eta^3/4)\exp(-\eta^2/8) = 0 \tag{53}$$

$$\eta = 0 \qquad H = 0 \tag{54a}$$

$$\eta \to \infty \qquad H = 0 \tag{54b}$$

The solution of (53)-(54) is

$$H = \eta\exp(-\eta^2/8) \tag{55}$$

that by (51) leads to the angular velocity

$$v_\theta = \frac{\eta\exp(-\eta^2/8)}{x^2} \tag{56}$$

The rapid decay of the angular velocity with axial position in (56) is in accordance with the classical results [39-41].

The predictions of the modified theory are next compared with the classical results. As shown in (49), for z<<1 the axial velocity can be approximated as

$$v_x \approx \frac{1}{x\left(1+z^2/4\right)^2} \tag{57}$$

By substitution of (57) in the continuity equation (25) one obtains the approximate radial velocity

$$v_r \approx \frac{z(1-z^2/4)}{x(1+z^2/4)^2} \tag{58}$$

Finally, for z<<1 the angular velocity in (56) may be approximated by

$$v_\theta \approx \frac{2z}{x^2(1+z^2/4)^2} \tag{59}$$

Except for some multiplicative constants, the results (57)-(59) are in exact agreement with the classical solutions of *Loitsianskii* [39] and *Görtler* [40] discussed by *Crabtree* , *Kuchemann*, and *Sowerby* [41].

According to the classical results [39-41], the jet rotation has negligible influence on the axial and radial velocities. The weak effects occur indirectly through the pressure term that is in the parameter M signifying the total axial momentum. That is, the angular velocity v_θ in the angular momentum parameter L influences the pressure field p, that in turn influences linear momentum parameter M, thereby affecting (v_x, v_r) [41]. However, because of the small factor of 2^{-11} appearing in the expression for the dimensionless pressure in the classical theory [41], the effects of rotation on velocities (v_x, v_r) will be exceedingly small. According to the modified theory presented herein, the jet rotation does not influence either the axial or the radial velocity profiles.

4 Solution of the Modified Equation of Motion for Turbulent Cylindrically Symmetric Rotating Jet

The scale-invariant nature of the conservation equations (14)-(16) suggests that the problem of turbulent rotating jet will be governed by the same equations (14)-(16) except at the next larger scale of laminar-eddy-dynamics LED β = e. However, the transport properties such as the momentum diffusivity ν and the thermal diffusivity α will now correspond to eddy-diffusivity for momentum ν_e and heat α_e. Therefore, as discussed in a recent study [43], the solution to the problem of turbulent rotating jet will be identical to those already found for laminar jet (41)-(43). In particular, from (45), the ratio of the axial velocity to the centerline value v_{xc} for turbulent rotating jet will be [43]

$$\frac{v_x}{v_{cx}} = \exp(-\eta^2/8) \tag{60}$$

Therefore, the transverse position where v_x is half its maximum, i.e. centerline value v_{xc} becomes

$$\eta^* = \eta(v_{cx}/2) = [-8\ln(0.5)]^{1/2} \tag{61}$$

such that (60) may be expressed as

$$\frac{v_x}{v_{cx}} = \exp[\ln(0.5)\xi^2] \quad , \quad \xi = \frac{\eta}{\eta^*} \tag{62}$$

Similarly, from (46) one obtains

$$\frac{\theta}{\theta_c} = \exp[\ln(0.5)\xi^2] \tag{63}$$

where θ_c is the temperature at the jet centerline.

The predicted axial velocity profile calculated from (62) using *Mathematica* [44] is in excellent agreement with the experimental data of *Reichardt* [37-38] as shown in Fig.2.

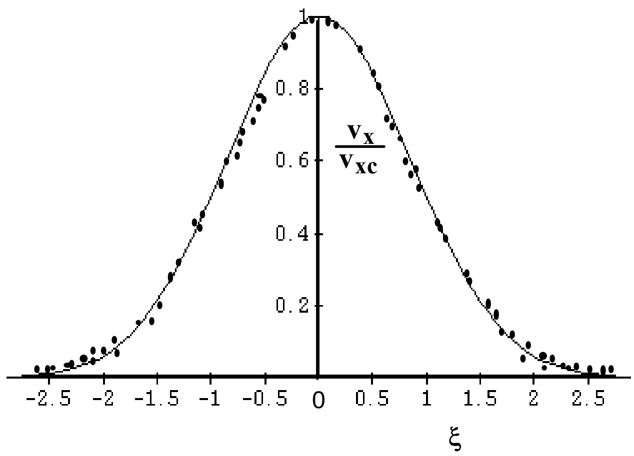

Fig.2 Comparisons of predicted velocity profile with the data of Reichardt [37] for turbulent axisymmetric jet.

It is important to note that, as opposed to the classical theories discussed in [32], the agreement with experimental data shown in Fig.2 is achieved without any empirically adjustable constants.

Comparisons between the predicted temperature profile and experiments performed on turbulent jets will be discussed next. Because the experimental data on temperature distribution in turbulent jets performed by *Reichardt* [38] involve two-dimensional jets, the problem of turbulent two-dimensional jet [43] will also be considered. Following steps parallel to (21)-(25), one arrives at the conservation equations

$$w_x \frac{\partial v_x}{\partial x} + w_y \frac{\partial v_x}{\partial y} = \frac{\partial^2 v_x}{\partial y^2} \tag{64}$$

$$w_x \frac{\partial \theta}{\partial x} + w_y \frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} \tag{65}$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \tag{66}$$

that are subject to the boundary conditions

$$y = 0 \qquad \frac{\partial v_x}{\partial y} = \frac{\partial \theta}{\partial y} = v_y = 0 \tag{67a}$$

$$y \to \infty \quad v_x = v_y = \theta = 0 \tag{67b}$$

Similar to the treatment of axisymmetric jet in (27)-(34) one obtains the convective field

$$w_x = \frac{1}{6x^{1/3}} \quad , \quad w_y = \frac{y}{18x^{4/3}} \tag{68}$$

Following the classical studies [32, 36], one introduces for the two-dimensional laminar jet the stream function Ψ and a similarity variable ζ as

$$\Psi \equiv x^{1/3} G(\varsigma) \quad , \quad \varsigma \equiv \frac{y}{3x^{2/3}} \tag{69}$$

that lead to the stream function

$$\Psi = \sqrt{\pi} x^{1/3} \operatorname{erf}(\varsigma / 2) \tag{70}$$

and hence the velocity components [43]

$$v_x = \frac{\exp(-\varsigma^2 / 4)}{x} \tag{71}$$

$$v_y = \frac{\sqrt{\pi}}{3x^{2/3}} \left[(2\varsigma / \sqrt{\pi}) \exp(-\varsigma^2 / 4) - \operatorname{erf}(\varsigma / 2) \right] \tag{72}$$

Since under the assumption $\nu_e = \alpha_e$, the momentum and temperature equations (64) and (65) are identical and subject to identical boundary conditions (67), the temperature distribution is similar to the axial velocity (71) and given by

$$\theta = \frac{\exp(-\varsigma^2 / 4)}{x} \tag{73}$$

The results (71) and (73) lead to

$$\frac{v_x}{v_{xc}} = \exp(-\varsigma^2 / 4) \tag{74}$$

$$\frac{\theta}{\theta_c} = \exp(-\varsigma^2 / 4) \tag{75}$$

The predicted velocity profile (74) is in excellent agreement with the experimental observations of *Förthmann* [35, 43]. Also, the result (75) has exactly the same form as the classical solution presented by *Reichardt* [32, 38] on the basis of an entirely different theory. The classical theory is based on the assumption that the eddy diffusivity for heat is twice that for momentum

$$\alpha_e = 2\nu_e \tag{76}$$

This assumption results in the square root relation between the temperature and the axial velocity ratios [32, 38]

$$\frac{\theta}{\theta_c} = \left(\frac{v_x}{v_{xc}} \right)^{1/2} = \exp(-\eta^2 / 4) \tag{77}$$

However, according to the simplified kinetic theory of ideal gas [45], (α_e, ν_e) are expected to be identical [28]

$$\alpha_e = \nu_e = l_e u_e / 3 = \lambda_c v_c / 3 \tag{78}$$

Of course, (α_e, ν_e) are expected to be variables in the non-equilibrium region of transition from one scale to another (Fig.1).

Because of the close agreement between the theory and experimental data achieved by the classical theory [32, 38], it is interesting to examine other possible reasons for the occurrence of the factor 2 in the classical model. According to the classical theory of *Reichardt* [32, 38], rather than solving the equation of motion directly, the first integral of this equation is considered in the form

$$\frac{\partial}{\partial x}\left(\frac{\overline{p}}{\rho} + v_x^2 \right) + \frac{\partial}{\partial y}(\overline{u_x u_y}) = 0 \tag{79}$$

It is suggested here that rather than (79), the integral of the equation of motion should lead to the energy, i.e. *Bernoulli* equation in the presence of turbulent diffusion expressed as

$$\frac{\partial}{\partial x}\left(\frac{\overline{p}}{\rho} + \frac{1}{2} v_x^2 \right) + \frac{\partial}{\partial y}(\overline{u_x u_y}) = 0 \tag{80}$$

With the *Boussinesq-Reynolds* stress expressed as

$$\overline{u_x u_y} = -\nu_e \frac{\partial v_x^2}{\partial y} \tag{81}$$

and the assumption $\overline{p} = 0$ for the free turbulent jet one obtains from (80)

$$\frac{\partial v_x^2}{\partial x} = 2\nu_e \frac{\partial v_x^2}{\partial y^2} \tag{82}$$

It is therefore suggested that the factor of 2 leading to (77) in the classical theory [32, 38] should be attributed to difference of factor ½ between (79) and (80) rather than the difference between the eddy-diffusivity for heat versus momentum (76). Another words, the factor of 2 in (82) leads to the occurrence of $1/\sqrt{2}$ in the similarity variable $\eta = y/\sqrt{2x}$ used in the analysis of the momentum as compared to the temperature field thus leading to the square root relation in (77). According to the present theory, on the other hand, the velocity and temperature ratios for axisymmetric (45)-(46) and two-dimensional (74)-(75) turbulent jets are identical.

The characteristic lengths for "atom", element, and system (l_e, λ_e, L_e) for the scales of laminar molecular-, cluster-, and eddy-dynamics (Fig.1) will be about

$$\begin{aligned} &\text{LMD} \quad (l_m = 10^{-9},\ \lambda_m = 10^{-7},\ L_m = 10^{-5}\ \text{m}) \\ &\text{LCD} \quad (l_c = 10^{-7},\ \lambda_c = 10^{-5},\ L_c = 10^{-3}\ \text{m}) \qquad (83) \\ &\text{LED} \quad (l_e = 10^{-5},\ \lambda_e = 10^{-3},\ L_e = 10^{-1}\ \text{m}) \end{aligned}$$

The field of conventional fluid mechanics will be identified as ECD scale β = c, composed of a spectrum of cluster sizes moving with Brownian motion with velocities $\mathbf{u}_c$ [29]. In a stationary fluid, molecular-clusters will be in equilibrium with suspended particles within the fluid that also undergo *Brownian* motions [28].

For LED field (Fig.1), the *Kolmogoroff* length and velocity, and the dissipation rate will be associated with the "atomic" scale and hence identified as [21-23, 26, 46, 47]

$$\eta_k = l_e = \lambda_c \quad , \quad v_k = u_e = v_c \tag{85}$$

and

$$\varepsilon \propto \frac{u_e^3}{l_e} \tag{85}$$

The stationary field of isotropic turbulence is identified as equilibrium-eddy-dynamic EED field shown in Fig.1 and its temperature will be expressed as [27]

$$3kT_e = m_e < u_e^2 > \tag{86}$$

Therefore, in an isolated system, as the convective and the local velocities (w_e, v_e) vanish, their kinetic energy will be dissipated into the "atomic" scales and hence manifested as heat at the temperature defined in (86).

5 Concluding Remarks

The solution of the modified equation of motion for the classical problem of laminar axisymmetric rotating jet studied by *Loitsianskii* and *Görtler* was determined. The analysis was also extended to velocity and temperature distributions in turbulent rotating jets as well as two-dimensional turbulent jet. The predicted velocity profiles were found to be in close agreement with the experimental observations of *Reichardt*. Also, the predicted temperature profile of two-dimensional turbulent jet was found to be in agreement with the observations of *Reichardt.*

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