UNCERTAINTY ANALYSIS OF INSTRUMENTATION SYSTEMS: CLASSICAL & INTERVAL METHODS

B.VASUKI[1] AND M.SHANMUGAVALLI[2]

[1]Lecturer, [2] Research Scholars, Department of Instrumentation and Control
National Institute of Technology, Trichy-620 015
INDIA
http:// www.nitt.edu

Abstract: This paper presents uncertainty analysis of an instrumentation system. This has been carried out by classical and interval method. The Resistance Temperature Detector (RTD) based temperature measurement system is considered to illustrate the analysis.

Keywords: Uncertainty analysis, Worst case Analysis, Sensitivity Analysis, Monte-Carlo analysis, Interval method, Instrumentation system.

1 Introduction

Uncertainty analysis is a technique by which one can determine, with good approximation, whether a system will function within specification limits when the system parameters vary between their limits. The study of systems with parameters that lie within the prescribed limits can be approached by classical methods like: Worst-case, Probabilistic Transformation of Variables, Method of Moments and Monte Carlo. Worst-case analysis is the technique attempts to determine how much change there will be in a performance function if all the parameter variables are at their extremes and are combined in the worst possible manner.

The technique, Method of Moment is an approximate approach that allows generating the moments of reliability from the moments of parameter reliability. This technique is consistent but not generally an efficient estimator. Monte-Carlo method is a sampling based method which is widely used for uncertainty analysis. It is an algorithm for solving various kinds of computational problems by using pseudo random numbers as opposed to deterministic algorithm [1].

Interval Method is successfully used in many scientific and engineering applications like in chemical engineering, computer graphics and computer-aided design, electrical engineering, dynamical systems and chaos, control theory, remote sensing and geographic information systems and experts systems[3][4]. It is an alternative and valid technique to calculate how the system accuracy varies as parameters vary. Interval method applied to instrumentation system is able

(i) To consider simultaneously the effects of uncertainty of all the parameters on a system accuracy.

(ii) To provide strict bounds with only one parameter.

(iii) To perform uncertainty or sensitivity analysis.

Here the temperature measuring instrumentation system is used with RTD as a sensor. RTD is used in many industries including electronic, medical, aerospace and chemical. Pt-1000 type of RTD is a most popular one which is nearly linear over a wide range of temperature and has response times of fraction second. The advantages of using RTD are stable output for long period of time, ease of recalibration, accurate readings over relatively narrow temperature spans.

2. Analysis by classical and interval method

2.1 Worst-case analysis

If a system passes the worst-case analysis it will never fail as long as the inputs parameters are maintained within the tolerance limits established by the analysis [1][7].

To illustrate the worst-case method of analysis, let us consider temperature measurement system based on RTD bridge circuit, as shown in Fig 1. The output voltage (performance function) is:

$$E = Vs\left(\left[\frac{R1}{(R1+R4)}\right]-\left[\frac{R2}{(R2+R3)}\right]\right) \quad (1)$$

The tolerances associated with each component are:

Vs = (5 ± 0.0025) V = [4.9975, 5.0025] V

R3 = Pt1000 RTD

R1 = R2 = R4 = 1000 ± 10% = [900, 1100] Ω,

T = T ± 1.5 °C

The nominal output voltage for 50°C is 0.21167V. The worst case idea is to relate the minimum and maximum values of R1, R2, R3, R4 and Vs.

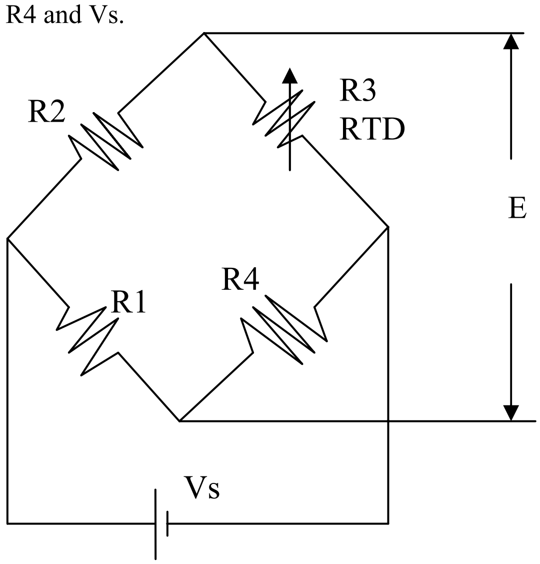

Fig.1 RTD Bridge Circuit

A simple analysis shows that minimum worst-case value is obtained when R1 minimum, R2 maximum, R3 minimum, R4 maximum and Vs minimum:

E_{min} = -0.16278 V

The maximum worst case is obtained when R1 maximum, R2 minimum, R3 maximum, R4 minimum and Vs maximum:

E_{max} = 0.59775 V

If the performance function $u = g(x_1, ..., x_n)$ is rather complicated involving many parameters, It may be hard to decide by inspection what combination of values gives the maximum and minimum value of the function. In such cases, approximating the performance function by the first few terms of a Taylor series expansion will simplify the problem. The general expression for the absolute variation of u is given by:

$$|\Delta u| = \sum_{i=1}^{N}\left|\frac{\partial u}{\partial x_i}\right|\left|\Delta x_i\right| \quad (2)$$

Then minimum and maximum values of the function are obtained by:

$u \pm |\Delta u|$ (3)

Returning to the RTD bridge circuit example:

$$|\Delta E| = \left|\frac{\partial E}{\partial Vs}\right||\Delta Vs| + \left|\frac{\partial E}{\partial R1}\right||\Delta R1| + \left|\frac{\partial E}{\partial R2}\right||\Delta R2| + \left|\frac{\partial E}{\partial R3}\right||\Delta R3| + \left|\frac{\partial E}{\partial R4}\right||\Delta R4| \quad (4)$$

We obtain ΔE = 0.380016 V. Then E is between 0.21167 ± 0.380016 or 0.16835 ≤ E ≤ 0.591686V.

2.2 Method of Moment

Exact solution of the probability transformation problem can become very difficult. The transformed density function tells the complete information about the problem. In many situations, information about the moments of the distribution is enough.

If one assumes independence and if u = g(x) is the performance function, then the expected value and the variance of u are given by [1]

$$E(u) \approx a \quad (5)$$

$$\text{var}(u) \approx \sum_{i=1}^{n} b_i^2\sigma_i^2$$

With:

a = g(x), x = nominal values

$$b_i = \frac{\partial u}{\partial x_i}$$

$$\sigma_i^2 = \text{variance of } x_i$$

It is reasonable to assume that 'a' is equal to the nominal value and that the tolerance is a certain number of σ units. The exact choice of σ in terms of the tolerance is a matter of judgment when data are not present. Shooman [5] recommends a value of 2 or 3.

Ranges for u can be calculated using:

$u = a \pm k\,\sigma_u$, k = 2 or 3 (6)

The variance of components is estimated from extreme values using:

$$\text{var}(x) = \left(\frac{X_{max} - X_{min}}{6}\right)^2 \quad (7)$$

Using (7) and the derivatives calculated previously, we obtain:

$$E(E) = 0.21167 \text{ V}$$

$$\text{var}(E) = 0.00518$$

Using (8) with k = 3, the range for E is

$$0.21167 \pm \left(3*\sqrt{5.184*10^{-3}}\right) \text{ or}$$

$$-0.00433 \leq E \leq 0.42767 \text{ V}$$

2.3 Monte-Carlo method

The Monte Carlo method provides complete and accurate frequency distribution of all output parameters provided that adequate statistical data for all input parameters are given [6].

An important question in uncertainty analysis is how to choose a particular uncertainty distribution from the incomplete knowledge that one has about the uncertain variable. Using the maximum entropy approach, if one only knows that the uncertain parameter takes values in (a, b), then the maximum entropy distribution is uniform on (a, b).

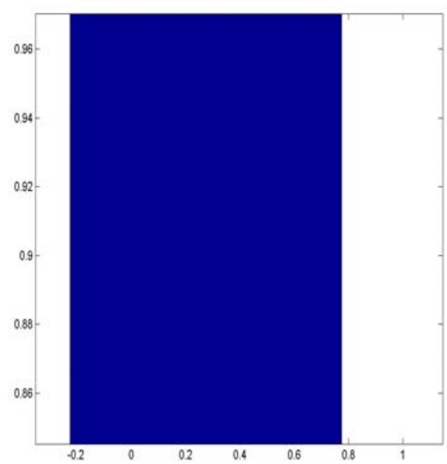

Fig.2 Histogram for 50°C

In our RTD bridge circuit example, if the parameters are uniformly distributed, we obtain, after 1000 trials, the V density distribution, with extreme values of [-0.2, 0.76] V from Fig.2.

2.4 Interval Method

The basic concept of interval analysis is that of an interval. An interval is a bounded segment of the real number. Since interval analysis deals with intervals rather than points, it is ideally suited for handling electric circuit problems whose initial data are allowed to take on values from some prescribed intervals [3]. Interval arithmetic originates from the recognition that there is frequently uncertainty associated with the parameters used in a computation [8]. This form of mathematics uses interval "numbers", which are actually an ordered pair of real numbers representing the lower and upper bound of the parameter range.

In our RTD bridge circuit, the range of E is obtained, evaluating (1) using interval arithmetic

To avoid overestimation of interval, (1) is evaluated as

$$E = Vs\left[\frac{1}{\left(1+\left(\frac{R4}{R1}\right)\right)} - \frac{1}{\left(1+\left(\frac{R3}{R2}\right)\right)}\right] \quad (8)$$

Using interval arithmetic, we obtain

$$E=[4.9975, 5.0025]*\left\{\left[\frac{1}{1+\frac{[900,1100]}{[900,1100]}}\right] - \left[\frac{1}{1+\frac{[1179.45,1190.55]}{[900,1100]}}\right]\right\}$$

E = [-0.16293, 0.59775]

In many practical tolerance problem is difficult to obtain an expression in which each variable occurs not more than once. In these cases a single computation of the interval extension of f(x) only yields an interval F(x) that is wider than the tolerance f(x) on the output variable. However, by the inclusion property, the interval F(x) is guaranteed to enclose f(x). Thus F(x) in some cases serves as an initial rough estimate of the output variable tolerance providing infallibly outward bounds on it [1].

Sensitivity analysis is performed using interval arithmetic by assigning bounds to some or all the input parameters and observing the effects on the final interval outcome, which contain all possible solutions due to the variations in input parameters.

Temperature	50°C	100°C
Nominal output	0.21167	0.390295
Practical output	0.22	0.41
Worst case	[-0.16835, 0.591686]	[0.0.321, 0.76738]
Method of moment	[-0.00433, 0.42767]	[0.175537, 0.606055]
Monte-Carlo	[-0.2, 0.76]	[-0.1, 0.85]
Interval	[-0.16293, 0.59775]	[0.01824, 0.77284]

Table3.Comparison of variability analysis technique

3 Conclusions

Although Monte-Carlo method gives the most realistic estimate of true worst case it requires the use of computer as the result is a histogram of circuit attributes probability distribution. Also computer must be capable of generating uniform random numbers. Monte-Carlo method assumes no restriction on the shape of the parameter distribution. The Monte Carlo method requires more computer time than the others techniques.

In the method of moments, it is assumed that the output parameters have s-normal (Gaussian) distribution.

Comparing the interval with the worst case method the interval results coincides with the worst case results. Therefore interval method is an alternative and valid technique to calculate how system accuracy varies as parameters varies.

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